THE DESIGN OF ALTERNATING CURRENT MACHINERY

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PREFACE

THIS book was written by the late Mr. James R. Barr, and was intended to be a companion volume to his book on *Direct Current Electrical Engineering.*

Owing to the unfortunate death of the author, before the book was revised for publication, I undertook the task of revision and correction of the proofs. In doing so I have altered matter only where the author's meaning was not perfectly clear and where mistakes arose. References have been given where possible, and symbols arranged so that as far as possible each quantity shall have a separate one.

For the chapters on alternators and rotary converters Mr. Barr was much indebted to the excellent treatise *Die Wechselstromtechnik*, by E. Arnold and J. La Cour, on whose methods of design these chapters are largely founded.

Though clockwise and anti-clockwise rotation have been used indiscriminately in the vector diagrams, this has not been thought a disadvantage, and arrows have been added indicating the direction of rotation in all the figures in which there might be ambiguity.

Many thanks are due to the various firms, both here and abroad, which have given detailed drawings and designs, and other useful information. Personally, my thanks are due to Mr. E. O. Turner, of Heriot-Watt College, who very kindly looked over my corrections and made valuable suggestions.

ROBERT D. ARCHIBALD.

DUNDEE TECHNICAL COLLEGE,
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Design of Rotary Converters

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LIST OF SYMBOLS EMPLOYED IN THE BOOK

A_a	= Cooling surface of armature in sq. dcms. (p. 334).
A_m	= Cooling surface of field magnet coils in sq. dcms. (p. 333).
A	= Cross-section of field winding space in sq. dcms. (p. 391).
AC	= Ampère-conductors per cm. of armature periphery (p. 379).
AT . . .	= Ampère-turns (suffix *c* for core, *g* for gap, etc.).
AT_{CM} . . .	= Cross-magnetising ampère-turns per pole (p. 270).
AT_{DM} . . .	= Demagnetising ampère-turns per pole (p. 269).
AT_o . . .	= Field ampère-turns at no-load per pole.
at	= Ampère-turns per cm. (p. 228).
a_a	= Cross-section of armature winding (p. 319).
a_f	= Cross-section of field winding (p. 319).
a_p	= Cross-section of primary winding (p. 151).
a_s	= Cross-section of secondary winding (p. 151).
B	= Flux density in lines/sq. cm. (B_c=flux density in armature core, etc.).
b_i	= Ideal pole arc in cms.
b_s	= Pole arc in cms.
C	= Number of conductors in q slots (p. 253).
C_1	= Constant (p. 146).
C_2	= Constant (p. 146).
C_o	= Ratio of weight of iron / weight of copper (p. 146).
C_s	= Conductors per slot (p. 386).
D	= Diam. of armature in cms. (p. 379).
D_c	= Diam. of commutator in cms. (p. 477).
Δ	= Drop of volts due to brush contact resistance (p. 452).
δ	= Radial depth of air gap in cms.
E	= D.C. terminal voltage of a rotary converter (p. 420).
E_i	= A.C. induced voltage.
E'_i . . .	= D.C. induced voltage of a rotary converter (p. 476).
E_n	= Voltage between neutral and line.
E_o	= D.C. voltage at no-load of a rotary converter (p. 447).
E_p	= Induced voltage in each phase of the armature.
E_s	= Slip ring voltage of a rotary (p. 421).
E_{so}	= Voltage between slip rings of a rotary at no-load (p. 446).
E_t	= Voltage of transformer (p. 443).

e_r . . . . . = Voltage drop due to resistance $= I_a r_a$ (p. 394).
e_x . . . . . = Voltage drop due to reactance $= I_a r_x$ (p. 394).
$\mathcal{E}$ . . . . . = Stored energy of flywheel (p. 364).
ϵ . . . . . = Efficiency (p. 170).
η . . . . . = Steinmetz's hysteresis constant.
= Ratio of nett length to gross axial length of armature core (p. 234).

F_c . . . . . = Copper space-factor.
F_i . . . . . = Iron space-factor.

I . . . . . = Current (R.M.S. value).
I_a . . . . . = Armature R.M.S. current of an alternator (p. 277).
= Total R.M.S. current per armature phase of a rotary converter including (p. 429)—
hI_{ao} = Power and loss component of current.
kI_{ao} = Wattless component of current.
I_e . . . . . = Core loss current (p. 163).
I_m . . . . . = Magnetising current (p. 163).
I_o . . . . . = No-load current (p. 163).
= Normal short circuit current of an alternator (p. 302).
I_s . . . . . = R.M.S. current per slip ring of a rotary (p. 445).
= Synchronising current (p. 356).
I_{so} . . . . . = Outside or line current of a rotary (p. 423).
I_{sw} . . . . . = Watt component of slip ring current (p. 445).
I_{swl} . . . . = Wattless component of slip ring current (p. 445).
I_w . . . . . = Watt current per phase.
I_{wl} . . . . . = Wattless current in each armature circuit at normal-load.
I'_{wl} . . . . = Wattless current in each armature circuit at no-load.
i . . . . . = Instantaneous value of current.

K_3 . . . . . = Constant (p. 273).
k_0 . . . . . = Value given on (p. 273).
k_1 . . . . . = Form factor (p. 216).
k_2 . . . . . = Breadth factor (p. 218).
k_3 . . . . . = Gap factor (p. 237).
k_{1c} . . . . . = Form factor (p. 271).
k_{2c} . . . . . = Breadth factor (p. 271).

L_c . . . . . = Mean length of magnetic path of armature core (p. 242).
L_g . . . . . = Gross axial length of armature core (p. 230).
L_n . . . . . = Nett axial length of armature core (p. 230).
L_p . . . . . = Length of pole (radially) (p. 242).
L_t . . . . . = Length of teeth (radially) (p. 242).
l_a . . . . . = Length of armature coil (p. 319).
l_i . . . . . = Ideal axial length of core (p. 240).
l_{in} . . . . . = Length of armature conductor embedded in slot per turn (p. 253).
l_s . . . . . = Length of pole (axially) (p. 245).

M . . . . . = Moment or torque.
M_s . . . . . = Synchronising torque in kg. metres (p. 257).
m . . . . . = Number of phases of an alternator.
= Number of slip rings of rotary converter (p. 423).
μ . . . . . = Permeability.
= Coefficient of friction.

N_s	= Number of commutator segments (p. 475).
n	= Number of slots per pole pitch (p. 221).
n_c	= Number of cycles of mechanical oscillation of rotor per revolution (= number of engine cranks not in line) (p. 365).
	= Number of coils in series (p. 259).
n_d	= Number of air ducts in armature core.
n_s	= Number of commutator segments covered by brush (p. 471).
P_a	= Permeance of teeth and air-gap (p. 229).
P_c	= Price of copper (p. 146).
P_i	= Price of iron (p. 146).
P	= Permeance of leakage paths (p. 229).
P_m^s	= Permeance (p. 253).
p	= $2\pi\sim$.
	= Pairs of poles.
p_c	= Copper losses ÷ total losses of transformer (p. 146).
p'	= Price of copper ÷ price of (copper + iron) (p. 147)
p_i	= Iron losses ÷ total losses of transformer (p. 146).
p'_i	= Price of iron ÷ price of (copper + iron) (p. 147).
Q	= Current density at brush contact.
q	= Number of circuits in parallel in an armature (p. 476).
	= Reaction quotient (p. 366).
	= Slots per pole per phase (p. 386).
R	= Revolution per minute.
R_a	= Resistance of converter armature from D.C. side.
R_s	= Resistance of series winding.
r_a	= Resistance of armature winding (p. 319).
r_f	= Resistance of field winding (p. 319).
r_x	= Reactance.
r_{xc}	= Reactance of choking coil (p. 445).
s	= Width of slot.
s_o	= Width of opening of slot.
σ	= Ratio of pole arc to pole pitch $= b_s/\tau$ (p. 268).
	= Leakage coefficient (p. 244).
	= Coefficient of speed variation (p. 360).
T	= Time of one complete period.
T_a	= Armature turns per pole (p. 289).
T_f	= Field turns per pole (p. 289).
T_p	= Primary turns of a transformer (total).
T_s	= Secondary turns of a transformer (total).
T_1	= Armature turns per pair of poles per phase (p. 264).
T_2	= Time of a forced oscillation in seconds (p. 360).
t_p	= Tooth pitch in cms.
τ	= Pole pitch in cms. (p. 152).
τ_c	= Pole pitch in cms. at commutator (p. 470).
θ_e	= Displacement in electrical degrees (p. 364).
v	= Peripheral speed in metres per second (p. 384).
v_c	= Peripheral speed of commutator in metres per second (p. 470).

W_a . . . . = Armature I^2R loss in rotary (p. 452).
W_c . . . . = Copper loss in armature slots (p. 452).
W_{ca} . . . = Armature copper loss (p. 319).
W_{ce} . . . = Copper loss in exciting coils (p. 319).
W_{cf} . . . = Watts loss due to brush friction on commutator.
W_{ct} . . . = I^2R losses at contact of brushes and commutator.
W_e . . . . = Eddy-current losses.
W_h . . . . = Hysteresis losses (p. 320).
W_{hc} . . . = Hysteresis losses in core.
W_{ht} . . . = Hysteresis losses in teeth.
W_i . . . . = Total iron losses $= W_h + W_e$ (pp. 323 and 454)
W_m . . . = Mechanical losses (p. 321).
= Watts loss per coil of field magnet (p. 332).
W_o . . . . = Constant or no-load losses (p. 322).
W_{sf} . . . = Watts loss due to brush friction on slip rings.

DESIGN OF ALTERNATING CURRENT MACHINERY

CHAPTER I

COMPLEX WAVE FORMS AND HARMONIC ANALYSIS

THE elementary theory of alternating currents is usually based upon the assumption that the waves of electromotive force and current are simple sine curves. This assumption, although facilitating the analytical and graphic treatment of alternating current problems, is often not realised in practice. In some cases the curves may approximate very nearly to a sine wave, whereas in others the deviation may be considerable. When a periodic function does not follow a simple sine law it can, however, be resolved into a number of components, each of which is a simple harmonic, having a frequency which is a whole multiple of the frequency of the complex function. The components are known as the harmonics of the alternating quantity under consideration.

The harmonic which has the same frequency ($=\sim$) as the complex wave is the regular curve to which the former approximates, and is called the *fundamental*. The other components, having a frequency of $2\sim$, $3\sim$, $4\sim$, $5\sim$, etc., are termed the second, third, fourth, fifth, etc., harmonics respectively. The expression for the instantaneous value of an alternating E.M.F. which follows the sine law is

$$e_1 = \mathrm{E}_{max} \sin 2\pi\sim t = \mathrm{E}_{max} \sin \theta.$$

If the frequency be doubled, *i.e.* increased to $2\sim$, then

$$e_2 = \mathrm{E}_{max} \sin 2\pi \,.\, 2\sim \,.\, t = \mathrm{E}_{max} \sin 2\theta.$$

In general, when they all start from zero at the same instant, the

component terms of any complex periodic function may be expressed as follows:—

$y_1 = F_1 \sin \theta$	1st harmonic or fundamental
$y_2 = F_2 \sin 2\theta$	2nd harmonic
$y_3 = F_3 \sin 3\theta$	3rd ,,
$y_4 = F_4 \sin 4\theta$	4th ,,
. . . . .	. . . .
$y_n = F_n \sin n\theta$	nth harmonic,

where $F_1, F_2, F_3, F_4, \ldots F_n$ are the amplitudes of the various harmonics. The relations between the fundamental wave and the second, third, and

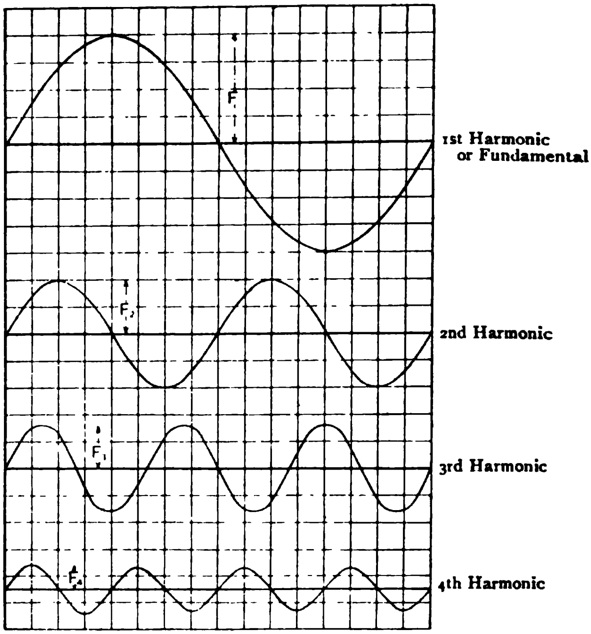


FIG. 1.

fourth harmonics of the above series are shown graphically in Figure 1 for a case in which $F_2 = 0.5\ F_1$, $F_3 = 0.4\ F_1$, and $F_4 = 0.2\ F_1$.

Even Harmonics.—In Figure 2 the fundamental and second harmonic of the previous figure have been plotted on a common reference axis as shown by the curves F and H_2 respectively. By adding the ordinates of the curve H_2 to those of the fundamental there is obtained the resultant curve R, the equation to which is

$$y = F_1 \sin \theta + F_2 \sin 2\theta.$$

Since the component curves do not attain their maximum values at

the same instant, the amplitude of the complex curve will be less than $F_1 + F_2$.

In some cases the harmonics may not start from zero at the same instant as the fundamental, but may either lag or lead with respect to the latter. If a second harmonic of amplitude F_2 and lagging ϕ degrees

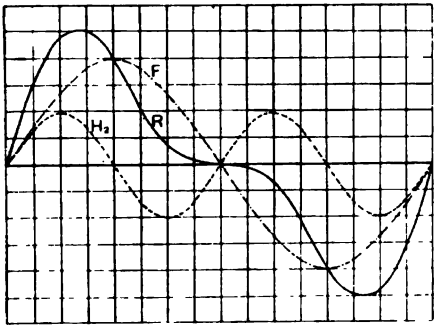


FIG. 2.—Fundamental and second harmonic.

behind the fundamental, be superimposed on a fundamental whose maximum value is F_1, the resultant curve R (Figure 3), having a shape considerably different from that of the previous figure, is represented by the equation

$$y = F_1 \sin \theta + F_2 \sin (2\theta - \phi).$$

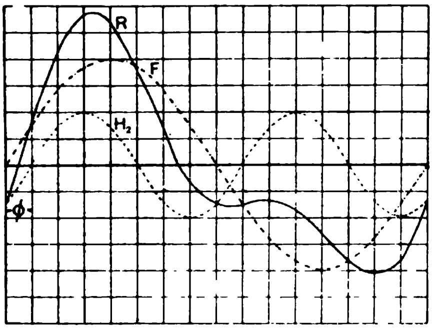


FIG. 3.—Second harmonic lagging behind fundamental.

Figure 4 shows the form of the complex curve obtained by superimposing a fourth harmonic on the fundamental, both curves starting from zero at the same instant. The equation for the complex curve is

$$y = F_1 \sin \theta + F_2 \sin 4\theta.$$

From these curves it will be seen that when a second or fourth harmonic is superimposed on the fundamental, successive loops of

the resultant curve are dissimilar, *i.e.* the shape of the curve when rising positively from a zero value is different from its shape when rising negatively from another zero value. Such will always be the case when a complex curve contains an even harmonic, no matter of what order. Now, the wave form of E.M.F. or current generated in an alternator must be a symmetrical curve with the positive half-waves exactly similar to the negative half-waves. This follows from

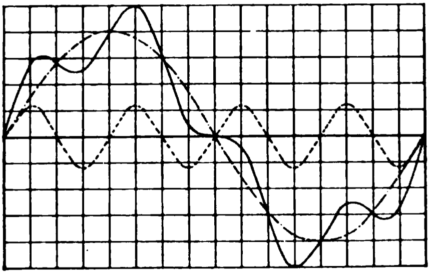


FIG. 4.—Fourth harmonic superposed on fundamental.

the fact that the poles of the alternator are all alike and therefore generate similar electromotive forces. A want of symmetry in the E.M.F. wave could only be produced by making alternate pairs of poles of different shapes. Hence the E.M.F. and current waves of alternators can contain no even harmonics, and the problem of complex wave forms is considerably simplified, as only odd harmonics need be considered.

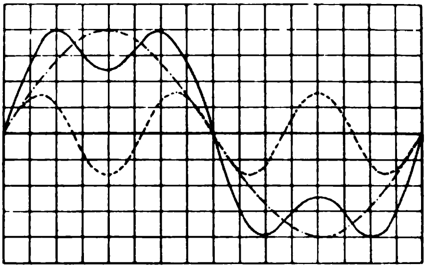


FIG. 5.—Third harmonic superposed on fundamental.

Odd Harmonics.—Next consider the form of wave produced by superimposing a third harmonic upon the fundamental. The shape of the complex curve will again depend upon the relative amplitudes of the fundamental and the harmonic, and also upon the phase relation between the two curves. If they are coincident in phase when the fundamental passes through its zero value the equation for the resultant wave will be

$$y = F_1 \sin \theta + F_3 \sin 3\theta$$

and the form of this curve when $F_3 = 0.4\ F_1$ will be as shown in Figure 5, the resultant curve having a flattened top. When the amplitude of the harmonic is negative the harmonic starts from zero

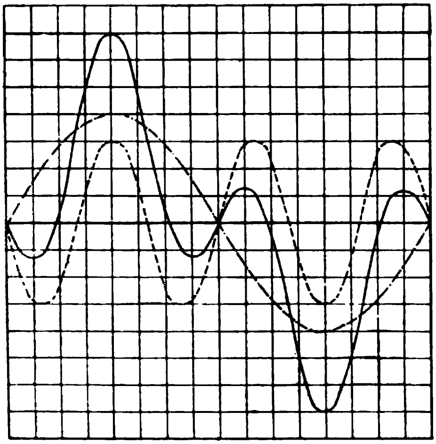


FIG. 6.—Third harmonic superposed on fundamental.

in the opposite direction to the fundamental, and the resultant curve, expressed by

$$y = F_1 \sin \theta - F_3 \sin 3\theta$$

will be peaked as shown in Figure 6, where $F_3 = 0.75\ F_1$. If the harmonic does not attain its zero value until the fundamental has

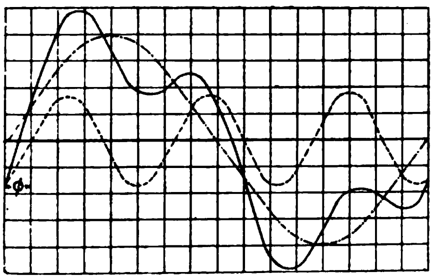


FIG. 7.—Lagging third harmonic superposed on fundamental.

advanced ϕ degrees ($\phi < 90°$) the resultant wave becomes peaked in one part of the half period and flattened in another, as shown in Figure 7. From a comparison of Figures 5 to 7 it will be seen that very different complex waves can be obtained with the same harmonics, depending upon the phase relation between the harmonic and the fundamental.

The effect of superimposing the fifth harmonic on the fundamental is shown in Figures 8 and 9, the harmonic being negative in the latter. Figure 10 indicates the shape of the curve obtained when a third and fifth harmonic are superimposed on the fundamental. Since the third harmonic lags ϕ degrees behind this, and the fifth harmonic

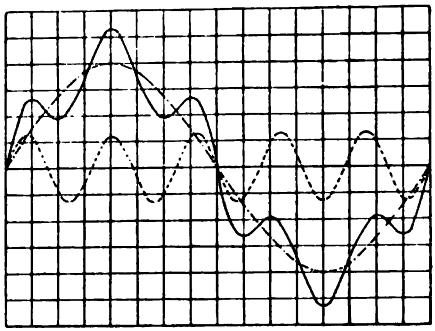


FIG. 8.—Fifth harmonic and fundamental.

starts from zero in the opposite direction to the fundamental, the equation to the complex curve is

$$y = A \sin \theta + A_3 \sin (3\theta - \phi) - A_5 \sin 5\theta.$$

An examination of the above wave forms, containing third and fifth harmonics, will show that (1) The number of ripples in a complex curve indicates the order of the harmonics; for example,

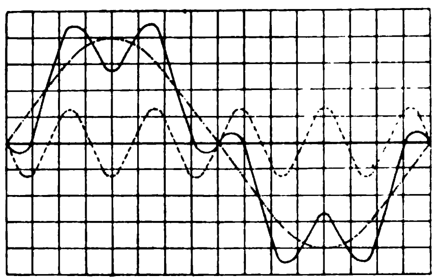


FIG. 9.—Fifth harmonic and fundamental.

when there is a third harmonic the resultant curve will have three ripples. (2) When the harmonics start from zero at the same instant as the fundamental each half wave is symmetrical about a vertical line bisecting it. (3) The result of superimposing a third harmonic on the fundamental is to alter the peak that occurs in the middle of a half period of the fundamental. A third harmonic lowers the peak (see Figure 5), whereas a fifth harmonic raises it (Figure 8). When the

amplitudes of the harmonics are negative, then the third harmonic raises the peak (Figure 6), while the fifth harmonic lowers it (Figure 9). Similarly, a seventh harmonic with a positive amplitude lowers the peak, whereas the peak is raised when the amplitude is negative. The ninth harmonic distorts the fundamental in the same way as the fifth.

In practice seventh harmonics do not occur very frequently, but higher ones (*e.g.* eleventh, thirteenth, and seventeenth) are often met with, and are caused by the pulsation of magnetic flux as the poles of the generator move past the armature teeth.

Harmonic Analysis.—In the preceding it has been shown how complex wave forms are obtained by adding together any number of simple harmonic curves. By a method due to Fourier it is also

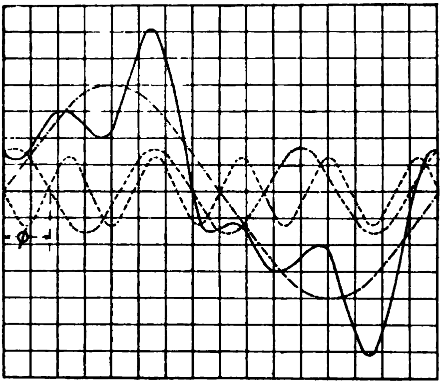


FIG. 10.—Third and fifth harmonic superposed on fundamental.

possible to resolve any complex periodic curve into its fundamental and various harmonics.

Fourier's theorem states that any periodic curve, however complex—provided it represents a single-valued function—can be resolved into a series of simple harmonic curves, the first harmonic or fundamental having the same periodic time as the given function. In mathematical language this means that any single-valued periodic function can be expressed analytically as the sum of a series of terms, each of which is a sine or cosine of an angle multiplied by a constant. If y denote the magnitude of the ordinate of the complex function, then

$$y = F_o + F_1 \sin(\theta + \phi_1) + F_2 \sin(2\theta + \phi_2) + F_3 \sin(3\theta + \phi_3) + \ldots \ldots + F_n \sin(n\theta + \phi_n)$$

where ϕ is the phase of the harmonic at the instant of time zero.

The constant F_o is introduced when the abscissa reference axis has not been drawn so that the mean ordinate is zero, but in current and

E.M.F. waves the ordinates are measured from the mean axis, and F_o is therefore zero.

Since $\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$, each of the sine functions in the equation for y is thus the sum of a sine and a cosine term, and the series may be written in the form

$$
\begin{aligned}
y &= A_1 \sin\theta + B_1 \cos\theta + A_2 \sin 2\theta + B_2 \cos 2\theta \\
&\quad + \ldots \quad . \quad . \quad . + A_n \sin n\theta + B_n \cos n\theta \\
&= A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + \ldots . \\
&\quad + B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + \ldots .
\end{aligned}
$$

where $A_1 = F_1 \cos\phi_1$, $A_2 = F_2 \cos\phi_2$, etc.
and $B_1 = F_1 \sin\phi_1$, $B_2 = F_2 \sin\phi_2$, etc.

The process of determining the values of the coefficients A_1, A_3 . . . . A_n and B_1, B_3 . . . . B_n is called harmonic analysis. The number of odd terms in the expansion will depend on the extent to which the function deviates from a simple sine curve. In certain cases an infinite number of simple harmonic terms may have to be taken to express the function with absolute exactness. The number of terms into which the function is analysed depends upon the degree of accuracy it is desired to obtain. In most cases analysis as far as the fifth harmonic is quite sufficient, but under certain circumstances it may be expedient to determine the coefficients up to the eleventh and seventeenth order respectively. It is very seldom that higher harmonics come into consideration.

Several methods have been proposed for determining the value of the constants in Fourier's series, but only the two more important ones will here be discussed. They are both based upon three theorems of the integral calculus

$$(1) \int_0^\pi \sin\theta \, d\theta = 2$$

$$(2) \int_0^\pi \sin^2\theta \, d\theta = \frac{\pi}{2}.$$

$$(3) \begin{cases} \int_0^\pi \sin a\theta \,.\, \sin .\, b\theta \,.\, d\theta = 0 \\ \int_0^\pi \cos a\theta \,.\, \cos b\theta \,.\, d\theta = 0 \\ \int_0^\pi \sin a\theta \,.\, \cos b\theta \,.\, d\theta = 0 \end{cases}$$

where a and b are positive integers and a is not equal to b.

Perry's Method.*—The value of the constants in Fourier's series are derived one at a time, by multiplying the equation throughout by

* *Electrician*, 1898, vol. xxviii. p. 362.

the sine which occurs in the particular term of which it is desired to find the coefficient. Consider the equation

$$y = A_1 \sin \theta + A_3 \sin 3\theta + A_5 \sin 5\theta + \dots\dots$$
$$+ B_1 \cos \theta + B_3 \cos 3\theta + B_5 \cos 5\theta + \dots\dots$$

To determine A_1, multiply both sides of the equation by $\sin \theta$ and integrate between the limits of o and π, then

$$\int_o^\pi y \sin \theta \, d\theta = \int_o^\pi A_1 \sin^2 \theta \, . \, d\theta + \int^\pi A_3 \sin 3\theta \, . \, \sin \theta \, . \, d\theta + \dots\dots$$
$$+ \int_o^\pi B_1 \cos \theta \, . \sin \theta \, . \, d\theta + \int_o^\pi B_3 \cos 3\theta \, . \sin \theta \, . \, d\theta + \dots$$

All the integrals of the right-hand side of the equation vanish except the first, which $= \frac{\pi}{2} A_1$.

$$\text{Hence } A_1 = \frac{2}{\pi} \int_o^\pi y \sin \theta \, . \, d\theta,$$

that is, the coefficient A_1 is equal to twice the mean value of the product y and $\sin \theta$ throughout the half-period. Similarly, by multiplying the equation by $\sin 3\theta$, $\sin 5\theta$, etc., and by $\cos \theta$, $\cos 3\theta$, etc., the other coefficients are as follows:—

$$A_3 = \frac{2}{\pi} \int_o^\pi y \, . \sin 3\theta \, . \, d\theta = 2 \times \text{mean value of } y \sin 3\theta$$

$$A_5 = \frac{2}{\pi} \int_o^\pi y \, . \sin 5\theta \, . \, d\theta = 2 \times \text{mean value of } y \sin 5\theta$$

. . . .

$$B_1 = \frac{2}{\pi} \int_o^\pi y \, . \cos \theta \, . \, d\theta = 2 \times \text{mean value of } y \cos \theta$$

$$B_3 = \frac{2}{\pi} \int_o^\pi y \, . \cos 3\theta \, . \, d\theta = 2 \times \text{mean value of } y \cos 3\theta$$

$$B_5 = \frac{2}{\pi} \int^\pi y \, . \cos 5\theta \, . \, d\theta = 2 \times \text{mean value of } y \cos 5\theta$$

etc. etc.

The value of the constants in the equation

$$y = A_1 \sin \theta + A_3 \sin 3\theta + A_5 \sin 5\theta + \dots\dots$$
$$+ B_1 \cos \theta + B_3 \cos 3\theta + B_5 \cos 5\theta + \dots\dots$$

having been determined by the above process the equation may be put into the form

$$y = F_1 \sin (\theta + \phi_1) + F_3 \sin (3\theta + \phi_3) + F_5 \sin (5\theta + \phi_5) + \dots\dots$$

where $F_1 = \sqrt{A_1^2 + B_1^2}$, $F_3 = \sqrt{A_3^2 + B_3^2}$, $F_5 = \sqrt{A_5^2 + B_5^2}$, etc.

and $\tan \phi_1 = \frac{B_1}{A_1}$, $\tan \phi_3 = \frac{B_3}{A_3}$, $\tan \phi_5 = \frac{B_5}{A_5}$, etc.

TABLE I.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.
Angle θ.	e.	Sin θ.	e Sin θ.	Cos θ.	e Cos θ.	Sin 3θ.	e Sin 3θ.	Cos 3θ.	e Cos 3θ.	Sin 5θ.	e Sin 5θ.	Cos 5θ.	e Cos 5θ.
10	100	0.174	17	0.985	98	0.500	50	0.866	87	0.766	77	0.643	64
20	154	0.342	53	0.939	145	0.866	134	0.500	77	0.985	152	-0.174	-27
30	215	0.500	107	0.866	184	1.000	215	0.000	000	0.500	107	-0.866	-186
40	270	0.643	174	0.766	205	0.866	235	-0.500	-135	-0.342	-92	-0.939	-253
50	310	0.766	240	0.643	200	0.500	155	-0.866	-270	-0.939	-290	-0.342	-106
60	340	0.866	295	0.500	170	0.000	0	-1.000	-340	-0.866	-290	0.500	170
70	340	0.939	320	0.342	120	-0.500	-170	-0.866	-295	-0.174	-59	0.985	335
80	310	0.985	305	0.174	53	-0.866	-270	-0.500	-155	0.643	200	0.760	237
90	260	1.00	260	0.000	0	-1.000	-260	-0.000	-000	1.000	260	0.000	000
100	230	0.985	225	-0.174	-40	-0.866	-200	0.500	115	0.643	149	-0.766	-175
110	220	0.939	205	-0.342	-75	-0.500	-110	0.866	190	-0.174	-38	-0.985	-215
120	230	0.866	200	-0.500	-115	-0.000	0	1.000	230	-0.866	-200	-0.500	-115
130	250	0.766	190	-0.643	-160	0.500	125	0.866	215	-0.939	-235	0.342	85
140	270	0.643	174	-0.766	-205	0.866	235	0.500	135	-0.342	-92	0.939	255
150	260	0.500	130	-0.866	-225	1.000	260	0.000	000	0.500	130	0.866	225
160	210	0.342	72	-0.939	-195	0.866	180	-0.500	-105	0.985	207	0.174	36
170	125	0.174	20	-0.985	-125	0.500	62	-0.866	-110	0.766	96	-0.643	-80
180	0	0.000	0	-1.000	0	0.000	0	-1.000	0	0.000	000	-1.000	0
			Sum of 18 terms = 2992 $A_1 = 2 \times$ mean $= 2 \times \frac{2992}{18} = 332$		Sum of 18 terms = 50 $B_1 = 2 \times$ mean $= 2 \times \frac{50}{18} = 6$		Sum of 18 terms = 655 $A_3 = 2 \times$ mean $= 2 \times \frac{655}{18} = 73$		Sum of 18 terms = -303 $B_3 = 2 \times$ mean $= 2 \times \frac{-303}{18} = -34$		Sum of 18 terms = 88 $A_5 = 2 \times$ mean $= 2 \times \frac{88}{18} = 9.5$		Sum of 18 terms = 249 $B_5 = 2 \times$ mean $= 2 \times \frac{249}{18} = 28$

Example.—As an illustration of this method, the complex periodic curve of Figure 11, showing the variation in voltage during one-half of a period, will be resolved into its harmonics.

Divide the abscissa reference axis into eighteen equal parts each of which will correspond to an angular interval of 10 degrees. Hence, starting from 0, θ will be successively 10, 20, 30, 40, etc. Measure from the curve the value of each ordinate for successive values of θ, and tabulate as shown in Columns I. and II. of Table I.

In Column III. are the values of $\sin\theta$ as obtained from a book of mathematical tables. The data in Column IV. is then obtained by multiplying together the values of e and $\sin\theta$ as given in Columns II.

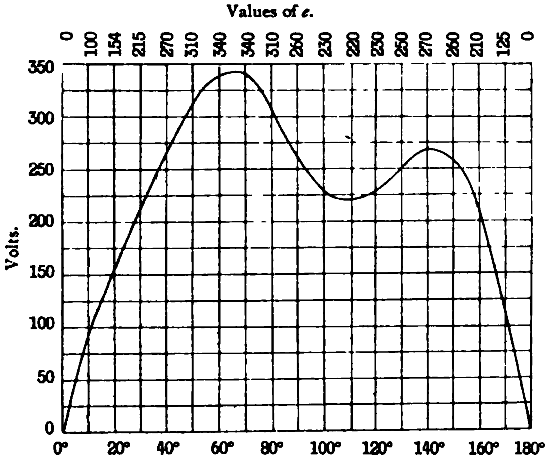


FIG. 11.—$e=332 \,.\, \sin\theta+80\sin(3\theta-25)+30\sin(50+72)$.

and III. respectively. Columns V. to XIV. are filled up in a similar manner, and the constants A_1, A_3, etc. derived as follows:—

$A_1 = 2 \times$ mean value of Column IV. $= 332$.
$B_1 = 2 \times$ mean value of Column VI. $= 6$.
$A_3 = 2 \times$ mean value of Column VIII. $= 73$.
$B_3 = 2 \times$ mean value of Column X. $= -34$.
$A_5 = 2 \times$ mean value of Column XII. $= 9.5$.
$B_5 = 2 \times$ mean value of Column XIV. $= 28$.

The equation to the E.M.F. wave in Figure 11 is therefore

$$\begin{aligned} e = {} & 332\sin\theta + 73\sin 3\theta + 9.5\sin 5\theta + \dots \\ & + 6\cos\theta - 34\cos 3\theta + 28\cos 5\theta \quad + \dots \end{aligned}$$

When the coefficients have been obtained the accuracy of the

results can be tested as follows: If the origin be taken where the curve cuts the abscissa reference axis, the sum

$B_1+B_3+B_5+B_7+$ . . . . should equal zero; and
$A_1-A_3+A_5-A_7+$ . . . should equal the ordinate at 90°.

$$\text{Now } F_1=\sqrt{A_1^2+B_1^2}=\sqrt{332^2+6^2}=332$$
$$F_3=\sqrt{A_3^2+B_3^2}=\sqrt{73^2+(-34)^2}=80$$
$$F_5=\sqrt{A_5^2+B_5^2}=\sqrt{9.5^2+28^2}=30$$
$$\phi_1=\tan^{-1}\frac{B_1}{A_1}=\tan^{-1}\frac{6}{332}\cong 0$$
$$\phi_3=\tan^{-1}\frac{B_3}{A_3}=\tan^{-1}\frac{-34}{73}=-25^\circ$$
$$\phi_5=\tan^{-1}\frac{B_5}{A_5}=\tan^{-1}\frac{28}{9.5}=72^\circ.$$

The equation for e can now be written in the simpler form

$$e=332\sin\theta+80\sin(3\theta-25^\circ)+30\sin(5\theta+72^\circ).$$

Thompson's Method.—The analysis of complex wave forms by the previous method is rather laborious, especially when harmonics of the 9th or higher order have to be determined. Several other processes have been suggested for a more rapid determination of the coefficients A_1, A_3, A_5 . . . . and B_1, B_3, B_5, ... The method now to be described is due to Professor S. P. Thompson,* and of the several methods so far evolved is certainly the simplest. This method also requires the multiplication by sines of angles and the subsequent integration by simple addition, but the labour involved in the calculations is shortened by a process of grouping. The ordinates are grouped by sums and differences successively taken, in such a way that the work of multiplying through by the sines of angles is not on individual ordinates, but on assemblages of them.

A half-period of the wave form to be analysed is divided into $2m$ equal parts so as to give $(2m-1)$ equidistant ordinates, $(2m-1)$ being the order of the highest harmonic that has to be determined. These ordinates are then grouped together in pairs so that there may be found the sum and difference of the ordinate of any angle and the ordinate of its supplement. The sum will contain sine components only, and the difference cosine components only. For, since $\cos n\theta = -\cos(180^\circ-n\theta)$ and $\sin n\theta=\sin(180^\circ-n\theta)$, it follows that by adding the pair of supplemental ordinates all cosines are eliminated, whereas subtracting them from one another eliminates all the sines. The values of the ordinates are therefore written down in two rows under one another, the first row of m members from right to left, and the second

* *Electrician*, 1905, vol. lv. p. 79.

row of $(m-1)$ members from left to right, and the sums and differences taken. All the cosine terms are thus separated from the sine terms. The sums are used for determining the coefficients A_1, A_3, A_5, etc., of the sine terms; and the differences for determining those of the cosine terms B_1, B_3, B_5, etc.

Analysis of a Periodic Curve up to the Eleventh Harmonic. —Suppose it is desired to analyse a periodic curve up to the eleventh harmonic, then the half-period is divided into twelve equal parts and the eleven ordinates $y_1, y_2 \ldots . y_{11}$, arranged thus (the ordinates y_0 and y_{12} being zero):

	y_1	y_2	y_3	y_4	y_5	y_6
	y_{11}	y_{10}	v_9	y_8	y_7	
adding	a_1	a_2	a_3	a_4	a_5	a_6
subtracting	b_1	b_2	b_3	b_4	b_5	

where a_1 stands for the sum of y_1 and y_{11}, and b_1 for the difference between y_1 and y_{11}.

A further grouping is possible for the higher harmonics of those orders for which the ordinal number is a factor of the number of divisor m,—for example, the third and ninth term if $m=9$. For when getting out the third harmonic the ordinate at 15° is multiplied by sin 45°, the ordinate at 45° by sin 135°, and the ordinate at 75° by sin 225°, and in doing this it should be remembered that sin $45^\circ=$ sin $135^\circ= -$ sin 225°. Hence the three ordinate values at 15°, 45°, and 75°, as well as their three supplementals at 165°, 135°, and 105°, since they all have to be multiplied by sin 45° (+ or −) and then added (*i.e.* integrated) together, may just as well be grouped together by adding before being multiplied by sin 45°, so that one multiplication suffices instead of six.

Hence group the numbers to obtain values for use with third and ninth harmonics as follows:—

$$a_1 + a_3 - a_5 = c_1$$
$$a_2 - a_6 = c_2$$
$$b_1 - b_3 - b_5 = d_1$$

The sums $a_1, a_2, \ldots \ldots$, and the differences $b_1, b_2, \ldots \ldots$, are then arranged in a convenient schedule, each number as it is entered being multiplied by the sine of the angle set over against it. The schedule given below shows how the numbers should be entered. Sine terms are entered from the top opposite the angles 0° to 90°, and cosine terms from the bottom, since cos $0^\circ=$ sin 90°. The results for the harmonics will, for reasons stated below, come out in pairs. For example, when the half period is divided into twelve equal parts the harmonics will come out first and eleventh together, third and ninth together, and so forth. Hence in entering the numbers in the schedule they are entered alternately in two columns side by side, in zigzag

order. For the first (and eleventh) order of harmonics the entries fill successively every place opposite the sines.

Sines and Cosines of Angles.	Sine Terms.			Cosine Terms.		
	1st and 11th Harmonic.	3rd and 9th Harmonic.	5th and 7th Harmonic.	1st and 11th Harmonic.	3rd and 9th Harmonic.	5th and 7th Harmonic.
Sin 15° = Cos 75° = 0.262	a_1	...	a_5	b_6	...	b_1
Sin 30° = Cos 60° = 0.500	a_2	...	a_2	b_4	...	b_4
Sin 45° = Cos 45° = 0.707	a_3	c_1	$-a_3$	b_3	d_1	$-b_3$
Sin 60° = Cos 30° = 0.866	a_4	...	$-a_4$	b_2	...	$-b_2$
Sin 75° = Cos 15° = 0.966	a_5	...	a_1	b_1	...	$-b_5$
Sin 90° = Cos 0° = 1.000	a_6	c_2	a_6	...	$-b_4$	...
Total, 1st column . .	...	...	...	...	...	...
Total, 2nd column .	...	...	...	...	...	...
Sum	6 A_1	6 A_3	6 A_5	6 B_1	6 B_3	6 B_5
Difference . . .	6 A_{11}	6 A_9	6 A_7	6 B_{11}	6 B_9	6 B_7

Now, coefficients of the nth order are given by the integrals:—

$$A_n = \frac{2}{\pi}\int_0^\pi y \, . \sin n\theta \, . \, d\theta$$

$$\text{and } B_n = \frac{2}{\pi}\int_0^\pi y \, . \cos n\theta \, . \, d\theta$$

Hence it remains to integrate and take twice the means. The integration here becomes a mere summation of the products made during the preceding process. But since in the case in which twelve ordinates (including the zero) are taken, the eleventh coefficient would be got out by multiplying by the same sines as would yield the first coefficient, only with alternate + and − signs prefixed, the addition for both the first and eleventh harmonics may be effected by finding separately the sums of alternate product numbers, which are therefore set down alternately in two columns. The totals of the first column and of the second column are therefore made separately. The sum of them gives the "integral" for the first harmonic, and the difference the "integral" for the eleventh harmonic, and so forth. Then finally, these sums and differences must be divided through by half the number of ordinates taken so as to give twice the mean. If twelve ordinates (including the zero at the origin) have been taken, division is made by six, but by nine if eighteen ordinates are taken. The summation which

here corresponds to integration is taken over the half-period. But as both half-periods are alike, and as the first process of adding up supplemental ordinates gives the doubles of their respective sine terms the result is the same as if the sum had been divided by two and the summation had been subsequently extended from 0° to 360°.

Analysis of a Periodic Curve up to the Seventeenth Harmonic.—To analyse a periodic curve up to the seventeenth harmonic, the order of procedure would be as set forth below.

Divide the half-period into eighteen parts so as to give the seventeen equidistant ordinates $y_1, y_2, y_3, \ldots . y_{17}$. To determine the sum and difference of the ordinates of supplementary angles, arrange the ordinates in rows, thus:—

	y_1 y_{17}	y_2 y_{16}	y_3 y_{15}	y_4 y_{14}	y_5 y_{13}	y_6 y_{12}	y_7 y_{11}	y_8 y_{10}	y_9 ...
Sums . ..	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
Differences .	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9

Group numbers as below for use in obtaining third, ninth, and fifteenth harmonics.

	For Sine Terms.	For Cosine Terms.
Third and fifteenth	$\begin{cases} a_1 + a_5 - a_7 = c_1 \\ a_2 + a_4 - a_8 = c_2 \\ a_3 - a_9 \qquad = c_3 \end{cases}$	$-b_8 - b_4 + b_2 = d_1$ $-b_7 - b_5 + b_1 = d_2$
Ninth . .	$a_1 - a_3 + a_5 - a_7 + a_9 = c_4$	$b_8 - b_6 + b_4 - b_2 = d_3$

Enter the above numbers as in the table below, each number as it is entered being multiplied by the sine of the angle in the first column. To determine the coefficients $A_1, A_2, \ldots . . A_{17}$ and $B_1, B_2 \ldots . . B_{17}$, find the totals of the first and second columns and then divide the sums and differences by nine.

Example.—As an example of Thompson's method the E.M.F. wave of Figure 11*a* will be analysed as far as the seventeenth harmonic.

The seventeen ordinates—100, 180, 250, 300, 340, 380, 460, 530, 560, 530, 470, 400, 400, 370, 300, 210, and 100—are arranged for determining the sum and difference of the ordinates of supplementary angles.

	100	180	250	300	340	380	460	530	560
	100	210	300	370	400	400	470	530	
Sum	200	390	550	670	740	780	930	1060	560
Difference	0	−30	−50	−70	−60	−20	−10	0	

θ in Degrees.	Sin θ.	Sine Terms.									Cosine Terms.								
		1st.	17th.	3rd.	15th.	5th.	13th.	7th.	11th.	9th.	1st.	17th.	3rd.	15th.	5th.	13th.	7th.	11th.	9th.
10	0.174	a_1			...	$-a_7$		$-a_5$		...	b_8			...	$-b_2$		b_4		...
20	0.342		a_2	c_1			$-a_4$		$-a_8$	...		b_7	d_1			$-b_5$		b_1	...
30	0.500	a_3			...	a_3		$-a_3$		...	b_6			...	b_6		b_6		...
40	0.643		a_4		...		a_8		a_2	...		b_5		...		b_1		$-b_7$	...
50	0.766	a_5			...	a_1		a_7		...	b_4			...	b_8		$-b_2$		...
60	0.866		a_6		c_2		$-a_6$		a_6	...		b_3		d_2		$-b_3$		$-b_3$	...
70	0.940	a_7			...	$-a_5$		a_1		...	b_2			...	$-b_4$		$-b_8$		...
80	0.985		a_8		...		a_2		$-a_4$	...		b_1		...		b_7		b_5	...
90	1.000	a_9		c_3		a_9		$-a_9$		c_4		...	$-b_6$			...		...	d_3
Total, 1st column .			...		...		...		...	...		...		...		...		...	...
Total, 2nd column .			...		...		...		...	...		...		...		...		...	...
Sum . . .		9 A_1		9 A_3		9 A_5		9 A_7		9 A_9	9 B_1		9 B_3		9 B_5		9 B_7		9 B_9
Difference . .			9 A_{17}		9 A_{15}		9 A_{13}		9 A_{11}	...		9 B_{17}		9 B_{15}		9 B_{13}		9 B_{11}	...

Grouping for third, ninth, and fifteenth harmonics:—

$$
\begin{aligned}
c_4 &= 200 - 550 + 740 - 930 + 560 = 20 \\
c_1 &= 200 + 740 - 930 \qquad = 10 \\
c_2 &= 390 + 670 - 1060 \qquad = 0 \\
c_3 &= 550 - 560 \qquad = -10 \\
d_1 &= -0 + 70 - 30 \qquad = 40 \\
d_2 &= 10 + 60 + 0 \qquad = 70 \\
d &= 0 + 20 - 70 + 30 \qquad = -20
\end{aligned}
$$

Tabulate thus:—

FOR SINE TERMS

	1st.	17th.	3rd.	15th.	5th.	13th.	7th.	11th.	9th.
0.174 . . .	200			...	-930		-740		...
0.342 . . .		390	10			-670		-1060	...
0.500 . . .	550			...	550		-550		...
0.643 . . .		670		...		1060		390	...
0.766 . . .	740			...	200		930		...
0.866 . . .		780		0		-780		780	...
0.940 . . .	930			...	-740		200		...
0.985 . . .		1060		...		390		-670	...
1.000 . . .	560		-10		560		-560		20
Multiplying by Sines—									
0.174 . . .	35			...	-160		-129		...
0.342 . . .		135	3			-230		-360	...
0.500 . . .	275			...	275		-275		...
0.643 . . .		430		...		680		250	...
0.766 . . .	570			...	155		710		...
0.866 . . .		675		0		-675		675	...
0.940 . . .	870			...	-690		188		...
0.985 . . .		1040		...		385		-660	...
1.000 . . .	560		-10		560		-560		20
Total, 1st column .	2310		-7		140		-66		20
Total, 2nd column	2280		0		160		-95		...
Sum . . .	4590		-7		300		-161		20
Difference . .	30		-7		-20		-29		...
Divide by 9 . .	$A_1 = 510$ $A_{17} = 3.3$		$A_3 = -0.8$ $A_{15} = -0.8$		$A_5 = 33.3$ $A_{13} = -2.3$		$A_7 = -17.9$ $A_{11} = 3.2$		$A_9 = 2.2$ ...

FOR COSINE TERMS

	1st.	17th.	3rd.	15th.	5th.	13th.	7th.	11th.	9th.
0.174 . . .	0		...		30		-70		...
0.342 . . .		-10	40			60		0	...
0.500 . . .	-20		...		-20		-20		...
0.643 . . .		-60	...			0		10	...
0.766 . . .	-70		...		0		30		...
0.866 . . .		-50		70		50		50	...
0.940 . . .	-30		...		70		0		...
0.985 . . .		0	...			-10		-60	...
1.000 . . .	...		20		...		...		-20
Multiply by Sines—									
0.174 . . .	0		...		5		-12		...
0.342 . . .		-34	14			20		0	...
0.500 . . .	-10		...		-10		-10		...
0.643 . . .		39	...			0		6	...
0.766 . . .	-54		...		0		23		...
0.866 . . .		-43	...			43		43	...
0.940 . . .	-28		...		65		0		...
0.985 . . .		0	...			-10		-59	...
1.000 . . .	...		20		...		...		-20
Total, 1st column .	-92		34		60		1		...
Total, 2nd column	-38		60		53		-10		-20
Sum . . .	-130		94		113		-9		...
Difference . .	-45		-34		7		11		-20
Divide by 9 . .	$B_1 = -14.4$	$B_{17} = -5$	$B_3 = 10.4$	$B_{15} = -3.8$	$B_5 = 12.6$	$B_{13} = 0.8$	$B_7 = -0.1$	$B_{11} = 1.2$	$B_9 = -2.2$ ...

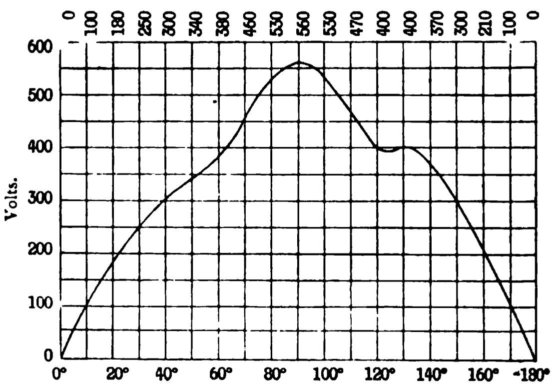


FIG. 11*a*.—Wave form of 3-phase alternator.

The equation to the E.M.F. wave of Figure 11 is therefore

$$e = 510 \sin\theta - 0.8 \sin 3\theta + 33.3 \sin 5\theta + 17.9 \sin 7\theta + 2.2 \sin 9\theta + 3.2 \sin 11\theta - 2.8 \sin 13\theta - 0.2 \sin 15\theta + 3.3 \sin 17\theta - 14.4 \cos\theta + 10.4 \cos 3\theta + 12.6 \cos 5\theta - 0.1 \cos 7\theta - 2.2 \cos 9\theta + 1.2 \cos 11\theta + 0.8 \cos 13\theta - 3.8 \cos 15\theta - 5 \cos 17\theta.$$

Or in its simpler form

$$e = 510 \sin\theta + 10.4 \sin(3\theta + 90) + 36 \sin(5\theta + 22) + 17.9 \sin 7\theta + 3.1 \sin(9\theta - 45) + 3.5 \sin(11\theta + 32) + 2.3 \sin(13\theta + 160) - 3.8 \sin(15\theta - 92) + 6 \sin(17\theta - 57).$$

Analysis of a Periodic Curve up to the Fifth Harmonic.—When the coefficients are required up to the fifth order only, then the method of procedure will be thus:—

Divide the half-period into six equal parts, and measure the ordinates y_1, y_2, y_3, y_4, and y_5. The sine and cosine terms are then separated by arranging the ordinates as under:—

	y_1	y_2	y_3
	y_5	y_4	
Sum	a_1	a_2	a_3
Difference . .	d_1	d_2	

Grouping for third harmonic: $a_1 - a_3 = c_1$.

Entering and multiplying by sines:—

Angle θ.	Sine θ.	Sine Terms. 1st.	Sine Terms. 5th.	Sine Terms. 3rd.	Cosine Terms. 1st.	Cosine Terms. 5th.	Cosine Terms. 3rd.
30°	0.500	a_1		...	b_2		...
60°	0.866		a_2	...		b_1	...
90°	1.000	a_3		c_1	...		$-b_2$
Total, 1st column .		...		...	...		...
Total, 2nd column .		...		...	...		...
Sum		$3A_1$		$3A_3$	$3B_1$		$3B_3$
Difference . .		$3A_5$		...	$3B_5$		...
Result		$y = A_1 \sin\theta + A_3 \sin 3\theta + A_5 \sin 5\theta + B_1 \cos\theta + B_3 \cos 3\theta + B_5 \cos 5\theta$					

Shape of Current Waves when E.M.F. Curve contains Harmonics.—When an E.M.F. of complex wave form is impressed upon a circuit the shape of the current wave in general need not be the same as that of the pressure wave, but depends upon the value of the resistance, self-inductance, and capacity of the circuit. Suppose the E.M.F. expressed by the equation

$$e = 330 \sin \theta + 75 \sin 3\theta$$

be applied to a circuit which has a resistance of 15 ohms and a negligible inductance and capacity. Since each component of the E.M.F. produces its own current and acts as if the other were absent, the resultant current in the circuit will be the algebraic sum of the currents due to the various component E.M.F.'s. As the circuit under consideration is of negligible reactance, the currents will be in phase with their respective E.M.F.'s, and the resultant current

$$\begin{aligned} i &= \frac{330}{15} \sin \theta + \frac{75}{15} \sin 3\theta \\ &= 22 \sin \theta + 5 \sin 3\theta. \end{aligned}$$

Since the ratio of the amplitudes, and also the phases of the two components of the current, are the same as those of the E.M.F. components, it follows that the shape of the current wave will be exactly similar to that of the E.M.F. This conclusion is applicable to all non-inductive circuits, no matter how many harmonics are present.

Next suppose the same E.M.F. be impressed upon a circuit having a resistance of 8.3 ohms and an inductance of 0.05 henries, and that the frequency of the E.M.F. is 50 periods. The instantaneous value of the current is

$$i = \frac{330}{z_1} \sin (\theta - \phi_1) + \frac{75}{z_3} \sin (3\theta - \phi_3)$$

where z_1 = impedance offered to the first harmonic

z_3 = " " third "

ϕ_1 = angle of lag of first harmonic of current behind the impressed E.M.F.

ϕ_3 = " third " " "

Now, the third harmonic has a frequency three times as great as the fundamental, so that

$$z_1 = \sqrt{8.3^2 + 4\pi^2 . 50^2 . 0.05^2} = 17.8 \text{ ohms.}$$

$$z_3 = \sqrt{8.3^2 + 4\pi^2 . 150^2 . 0.05^2} = 48 \text{ ohms.}$$

$$\phi_1 = \tan^{-1} \frac{2\pi . 50 . 0.05}{8.3} = \tan^{-1} 1.87 = 60°.$$

$$\phi_3 = \tan^{-1} \frac{2\pi . 150 . 0.05}{8.3} = \tan^{-1} 5.6 = 80°.$$

Substituting the values in the expression for the current

$$\begin{aligned} i &= \frac{330}{17.8} \sin (\theta - 60) + \frac{75}{48} \sin (3\theta - 80) \\ &= 18.5 \sin (\theta - 60) + 1.5 \sin (3\theta - 80). \end{aligned}$$

The fundamental and third harmonic of the current are indicated by the chain-dotted curves of Figure 12. The resultant current wave obtained by superposing the third harmonic on the fundamental is shown by the full-line curve. It will be noted that the current and impressed E.M.F. waves have somewhat different forms, the E.M.F. wave, though having a depressed peak, giving rise to a current wave which deviates but little from a simple sine curve. The reason for this is that the amplitude of the current waves is cut down at a rate almost proportional to the increase of frequency. Hence the triple frequency harmonic will be damped out nearly three times as much as the fundamental. It is evident, therefore, that one effect of self-induction in a circuit is to smooth down the ripples in the current wave, the higher the frequency of the harmonics the more nearly will the current wave approximate to a sine curve. Hence alternating current generators

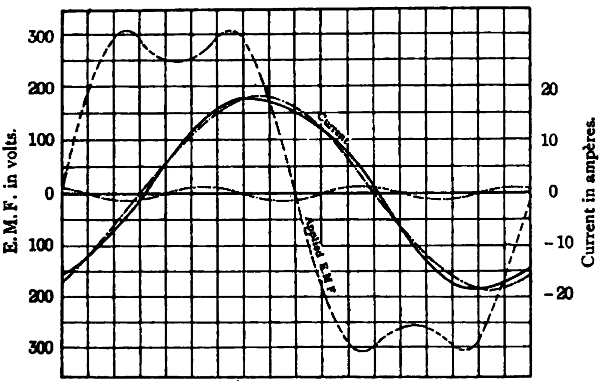


FIG. 12.—Inductive circuit.

whose wave forms of E.M.F. are pure sine curves will be less affected than others by the self-inductance of the circuit to which they supply current.

The effect of the capacity of a circuit on the current wave form will now be examined. Suppose the E.M.F. represented by the equation

$$e = 3300 \sin \theta + 750 \sin 3\theta$$

be applied to an unloaded concentric cable having a capacity of 3 microfarads, the frequency of the E.M.F. being 50 ~. The impedance will now be equal to the reactance of the cable, and the component currents will be in advance of the impressed E.M.F. by almost 90 degrees. The current flowing into the cable is expressed by

$$i = pC\ 3300 \sin (\theta + 90) + 3\,pC\,.\,750 \sin (3\theta + 90)$$

where $p = 2\pi \times 50 = 314$

and $C = 3 \times 10^{-6}$.

Hence

$i = 314.3 \times 10^{-6} \cdot 3300 \sin(\theta + 90) + 3 \times 314.3 \times 10^{-6} \cdot 750 \sin(3\theta + 90)$
$= 3.1 \sin(\theta + 90) + 2.1 \sin(3\theta + 90).$

The component current curves, together with the resultant current wave, are, in Figure 13, plotted on the same reference axes as the impressed E.M.F. The current wave deviates more widely from the simple sine form than does the E.M.F. curve: thus showing that the effect of capacity in a circuit is to produce still greater distortion in the current wave than that which is already present in the E.M.F. curve. In this respect the effect of capacity is the reverse of that produced by

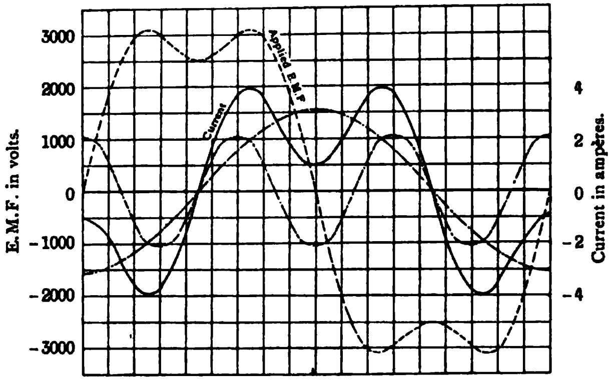


FIG. 13.—Current and E.M.F. wave-forms for a capacity load.

inductance. This is because the capacity current ($=pCE$) increases directly as the frequency, thus bringing the higher harmonics into greater prominence.

Now, when capacity and self-inductance are present the phenomenon of resonance occurs for certain relations between the values of those quantities and the frequency. With a complex wave of pressure a rise of potential due to resonance may occur with any of the harmonics. For example, in a 50 ∼ supply circuit the E.M.F. wave of which contains the fifth and eleventh harmonics, resonance may occur for the three relations

$$(1)\ \frac{1}{2\pi\sqrt{LC}} = 50;\ (2)\ \frac{1}{2\pi\sqrt{LC}} = 250;\ \text{and}\ (3)\ \frac{1}{2\pi\sqrt{LC}} = 550,$$

where L and C respectively denote the self-induction and capacity of the circuit.

Thus with waves of non-sinusoidal form the chances of resonance are increased. In modern systems the capacity of the mains and inductance of the apparatus are generally such that resonance for the harmonics of the ninth order or less is not likely to occur, and even

then the current due to the higher harmonics is kept within reasonable limits, by the losses in the circuit. The effect of resonance with a thirteenth harmonic is clearly shown by the curve of Figure 14, which is the current wave-form from a generator when connected to three unloaded cables, the capacity of the cables and inductance of the alternator having the requisite values for resonance with the thirteenth harmonic.

Effective Value of Complex Quantities.—When odd harmonics are superposed on the fundamental the curve of E.M.F. impressed upon a circuit may be represented analytically by the series

$$e = E_1 \sin(\theta + \phi_1) + E_3 \sin(3\theta + \phi_3) + E_5 \sin(5\theta + \phi_5) + \ldots .$$

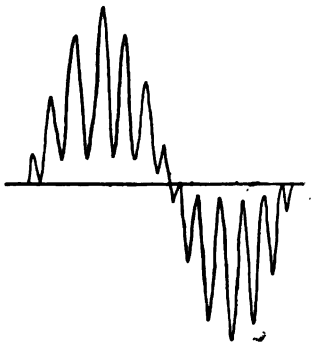


FIG. 14.—Current wave-form showing resonance with thirteenth harmonic.

where E_1, E_3, E_5, etc. are the amplitudes of the fundamental, third, fifth, etc. harmonics. Now, if E denote the effective value of the electromotive force, then

$$E^2 = \frac{1}{\pi}\int_0^\pi \{E_1 \sin(\theta + \phi_1) + E_3 \sin(3\theta + \phi_3) + E_5 \sin(5\theta + \phi_5) + \ldots\}^2 . \, d\theta$$

$$= \frac{1}{\pi}\int_0^\pi \{E_1^2 \sin^2(\theta + \phi_1) + E_3^2 \sin^2(3\theta + \phi_3) + E_5^2 \sin(5\theta + \phi_5) + \ldots$$

$$+ 2E_1E_3 \sin(\theta + \phi_1) . \sin(3\theta + \phi_3) + 2E_1E_5 \sin(\theta + \phi_1) . \sin(5\theta + \phi_5) \ldots\} . d\theta$$

Now $\int_0^\pi \sin^2\theta . d\theta = \frac{\pi}{2}$

and $\int^\pi \sin a\theta . \sin b\theta \, d\theta = 0$, where a is not equal to b and both are whole numbers.

$$\text{Hence } E^2 = \frac{1}{\pi}\left\{E_1^2 . \frac{\pi}{2} + E_3^2 . \frac{\pi}{2} + E_5^2 . \frac{\pi}{2} + \ldots\ldots\right\}$$

$$= \tfrac{1}{2}\{E_1^2 + E_3^2 + E_5^2 + \ldots\ldots\}$$

$$\text{and } E = 0.707\sqrt{E_1^2 + E_3^2 + E_5^2 + \ldots\ldots}$$

A similar expression holds good for a complex current wave.

Example.—Find the effective value of the E.M.F. of an alternator whose wave form is shown in Figure 11.

$$E_1 = 332, \quad E_3 = 80, \quad \text{and } E_5 = 30$$

$$\text{Effective E.M.F.} = 0.707\sqrt{332^2 + 80^2 + 30^2} = 242 \text{ volts.}$$

Note.—Effective value of the fundamental $= 0.707 \times 332 = 235$ volts.

Power and Power-Factor of a Circuit where E.M.F. and Current Waves are distorted.—Suppose the E.M.F. represented by the function

$$e = E_1 \sin\theta + E_3 \sin 3\theta + E_5 \sin 5\theta + \ldots\ldots$$

be applied to a circuit, then each simple harmonic term in the E.M.F. wave will give rise to its own component of current of the corresponding frequency. Let the expression for the current be

$$i = I_1 \sin(\theta + \phi_1) + I_3 \sin(3\theta + \phi_3) + I_5 \sin(5\theta + \phi_5) + \ldots\ldots$$

Then the instantaneous value of the power is

$$w = e\,i = E_1I_1 \sin\theta . \sin(\theta + \phi_1) + E_3I_3 \sin 3\theta . \sin(3\theta + \phi_3) + \ldots\ldots$$
$$+ E_1I_3 \sin\theta . \sin(3\theta + \phi_3) + E_3I_1 \sin 3\theta . \sin(\theta + \phi_1)$$

Integrating between the limits $\theta = 0$ and $\theta = \pi$, the mean value of the power is

$$W = \frac{1}{\pi}\int_0^\pi \{E_1I_1 \sin\theta . \sin(\theta + \phi_1) + E_3I_3 \sin 3\theta . \sin(3\theta + \phi_3) + \ldots\ldots$$
$$+ E_1I_3 \sin\theta . \sin(3\theta + \phi_3) + E_3I_1 \sin 3\theta . \sin(\theta + \phi_1) \ldots\ldots\}d\theta$$

Now, the integral taken between 0 and π of all products of terms of different frequencies is zero, hence

$$W = \frac{1}{\pi}\int_0^\pi \{E_1I_1 \sin\theta . \sin(\theta + \phi_1) + E_3I_3 \sin 3\theta . \sin(3\theta + \phi_3) + \ldots\}d\theta.$$

Now all terms such as $\int_0^\pi \{EI \sin\theta . \sin(\theta + \phi)\}d\theta$

$$= EI\int_0^\pi \{\sin\theta . \sin(\theta + \phi)\} . d\theta$$

$$= EI\int_0^\pi (\sin^2\theta . \cos\phi + \sin\theta . \cos\theta . \sin\phi)d\theta$$

$$= EI\left(\frac{\pi}{2}\cos\phi + 0\right) = EI . \frac{\pi}{2} . \cos\phi$$

Substituting this value in the expression for the mean power

$$W = \frac{1}{\pi}\left\{\frac{E_1I_1\,\pi\cos\phi_1}{2} + \frac{E_3I_3\,\pi\cos\phi_3}{2} + \ldots\ldots\right\}$$

$$= \frac{E_1I_1}{2}\cos\phi_1 + \frac{E_3I_3}{2}\cos\phi_3 + \ldots\ldots$$

That is, the total power is equal to the sum of the powers due to the various simple harmonic components.

The power factor of a circuit is usually defined as the cosine of the angle ϕ by which the current lags behind or leads the E.M.F., ϕ being measured either with respect to the zero or maximum values of the two waves. When dealing with complex wave forms of current and E.M.F. the phase difference considered with respect to the zero values, and also with respect to the maximum values, will in general be different. The above definition of power factor is therefore ambiguous, and the term "angle of lag" when applied to distorted wave-forms must be specially defined. For this purpose each distorted wave may be considered as replaced by an *equivalent sine wave*, *i.e.* a sine wave having the same effective value as the complex curve. The mean product of instantaneous values of the distorted waves of current and E.M.F. will then be the same as that of the equivalent sine waves. If ϕ_e denote the phase angle between the *equivalent sine waves* of current and E.M.F., then cos ϕ_e is the power factor of the circuit. According to this definition no reference whatever is made to the relative positions of either the zero or maximum ordinates of the complex curves.

All measurements of alternating currents and E.M.F.'s, with the single exception of instantaneous readings, yield the equivalent sine wave only, and suppress the higher harmonics. This is because all measuring instruments give either the mean square of the wave, or the mean product of instantaneous values of current and E.M.F., which—by definition—are the same in the equivalent sine wave as in the distorted wave. Hence in all practical applications it is permissible to neglect the higher harmonics altogether, and replace the distorted wave by its equivalent sine wave. There must, however, be kept in mind the existence of the higher harmonics as a possible disturbing factor which may become of importance in those cases where the frequency of the said harmonics is near the frequency of resonance of the circuit.

CHAPTER II

INSULATION

In the design of low-pressure insulation—*i.e.* insulation for pressures not exceeding 1000 volts, the judgment of the designer is assisted only to a small extent by electrical science. The thickness of insulation is more or less defined by the required mechanical strength, and this generally allows a sufficient factor of safety for tests four or five times the normal working pressure. On the other hand, with alternating current machines and apparatus employing the higher electrical pressures in commercial use, the deciding factor is the provision of insulation which has ample strength to withstand the electric strains that are encountered.

The essential properties of an insulating material are: (1) ability to stand the mechanical and electrical stresses due to the voltages used; and (2) such a poor conductivity that only a negligibly small current can flow through it. These properties are respectively termed the "dielectric strength" and the "ohmic or insulation resistance" of the insulator. The other features to be considered in selecting an insulating material are (3) mechanical strength and flexibility, and (4) the ability to resist the influence of moisture, air, oils, acids, etc.

In the early days of electrical engineering the general practice was to consider the ohmic resistance only, and that insulator which by measurement gave the highest insulation resistance was regarded as the best. This, however, is a wrong basis on which to make comparisons, because the insulation resistance depends largely upon the moisture contained, and in a given sample can be varied from a fraction of a megohm to "infinity" simply by putting the sample through a prolonged process of baking. The dielectric or disruptive strength of a material, rather than the insulation resistance, is now recognised as the true measure of its usefulness as an insulator.

High insulation resistance and great dielectric strength are not always found in the same material. For instance, air has an infinitely high insulation resistance, but not much dielectric strength; while glass, with a disruptive strength many times greater, has not nearly such a high ohmic resistance.

Stress on a Dielectric.—In Figure 15, suppose that D represents a dielectric the terminal faces of which are in contact with the

conductors A and B. On connecting the latter to a source of E.M.F. the medium between the conductors will be traversed by Faraday's lines of force, and an electric stress, proportional to the potential difference between A and B, will be exerted throughout the dielectric. The stress or electric intensity at any point is equal to the rate at which the potential diminishes when moving along the line of force passing through that point, and is therefore measured by the potential gradient, which for engineering purposes is generally expressed in volts per millimetre.

Assuming the charges on A and B to have the signs indicated in the diagram, then the positive and negative electrons in the dielectric will, under the influence of the electric stress, be attracted towards A and B respectively—*i.e.* the electric force produces a distortion of the molecular structure of the dielectric. Such a displacement while in progress constitutes an electric current. The displacement of the electrons encounters the reaction due to the internal atomic forces tending to maintain the original structural form, but

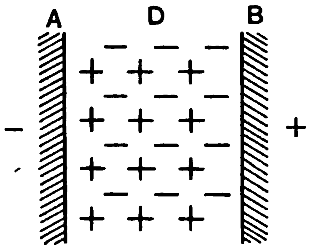


FIG. 15.

when the stress exceeds a certain limit structural rupture takes place and ordinary conduction currents ensue. [Of course, there will always be small currents which pass conductively through all dielectrics when subjected to E.M.Fs.; these currents are, however, so small that for all practical purposes they may be neglected.] The maximum value of the electric stress which an insulating material, in a given physical state, can withstand without breaking down is termed its *dielectric or disruptive strength.* The dielectric strength of an insulator depends chiefly upon its molecular structure, and *in general*, though not always, the greater the density and the more homogeneous a dielectric of a single class is, the higher will be its disruptive strength.

Potential Gradient and Specific Inductive Capacity.—In practice dielectrics are often composed of more than one quality of insulation. For instance, in the case of an armature winding the insulation between the conductors and the core always consists of two parts: (1) the cotton covering round the conductor, and (2) the

slot lining consisting of one or more of the following: micanite, press-spahn, leatheroid, and paper. Now, when a compound dielectric is subjected to electrical stress a different intensity of force is obtained through the various materials. The effect of this is to cause an unequal distribution of pressure over the insulation, the potential gradient in any dielectric being inversely proportional to its specific inductive capacity.

The effect of specific inductive capacity upon the disruptive voltage of a compound dielectric is well illustrated by a classical experiment of R. A. Fessenden.* Referring to Figure 16 (a), two metal plates M, M, were placed 10 mm. apart with an alternating pressure of 10,000 volts between them. As air will stand 15,000 volts per centimetre, this allowed a 50 per cent. factor of safety. Now, when two glass plates G, G, having a specific inductive capacity = 8, and a thickness of 2.5 mm., were introduced as in Figure 16 (b), the voltage divided itself inversely as the specific inductive capacities, so that the pressure between the glass plates became 8890 volts, corresponding to

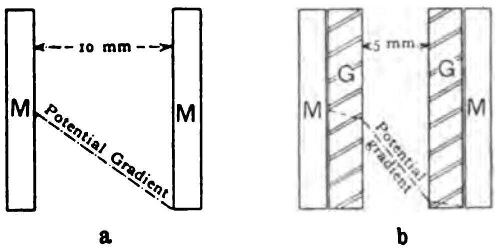


FIG. 16.—Specific inductive capacity and potential gradient.

a potential gradient in the air of 17,800 volts per centimetre. As air will only support about 15,000 volts per centimetre, a spark passed at every reversal of the voltage. This quickly heated the glass and made it conduct, so that the full potential of 10,000 volts existed between the plates G, G, and a regular arc was formed. The introduction of a better dielectric than the air had thus the paradoxical effect of weakening the insulation. This will always be the case with alternating voltages, unless the whole space is filled with the same material—or materials so arranged that each layer of the dielectric has the same specific inductive capacity. The experiment just described illustrates a very important principle, the thorough understanding of which is essential owing to the rapidly increasing refinements required in the application of insulating materials.

When wires have to be insulated the dielectric is applied in the form of concentric tubes. Now since the electric force—*i.e.* the

* *Insulation and Conduction*, Amer. Inst. of Elect. Eng., 1898, vol. xv. p. 119.

number of Faraday's lines in any outer zone surrounding a conductor, must be the same as that through the zone next to the conductor, and since the sectional area determined by the latter zone must inevitably be smaller than the sections at corresponding outer zones, it follows that the electric intensity will be greater in the zone sections next to the conductor. Hence, in order to get a uniform potential gradient right through the insulation, it is necessary to use an insulating material with a high specific inductive capacity inside, and as the radius of the tubes gets bigger the specific inductive capacity of the dielectric should be decreased accordingly. This is what is known as "grading the dielectric," and where there is a considerable difference in radius between the inner and outer layers of insulation it is most important. Similar considerations, which are fully discussed in Chapter VII., have to be taken into account when designing the slot insulation of high-voltage machines.

Test for Dielectric Strength

The thickness of the dielectric required for the insulation of high-voltage circuits is based upon two considerations: (1) the disruptive strength of the various dielectrics used, and (2) the factor of safety permissible. The disruptive strength or puncturing voltage of insulating materials is generally measured by inserting test specimens between metal electrodes and increasing the voltage until rupture takes place.

Within recent years numerous investigations have been made to obtain data regarding the dielectric strength of the various insulating materials, but unfortunately there is a great divergence of the results published by the various authorities. Differences of 150 or even 200 per cent. for materials of the same description are by no means rare; and while it must certainly be admitted that some of the insulating materials vary considerably in quality, others have been more or less standardised, so that the cause of the inconsistency must be looked for in another direction.

When a given sample is tested it will be found that the break-down voltage is mainly influenced by the shape of the electrodes, temperature, presence of moisture, and the length of time pressure is applied. The breakdown voltage is also affected to some extent by the frequency and wave form of the testing voltage, and mechanical pressure on the electrodes.

Shape of Electrodes, Mechanical Pressure, Wave Form, and Frequency.—The influence of the shape of the electrodes on the breakdown voltage has been investigated by C. Kinzbrunner.*

* *Electrician* (1905), vol. lv. p. 811.

He experimented with six brass electrodes of the following dimensions: No. 1, sharply pointed; No. 2, sphere of 5 mm. radius; No. 3, sphere of 10 mm. radius; No. 4, sphere of 20 mm. radius; No. 5, plate of 40 mm. diameter; and No. 6, plate of 100 mm. diameter. The curve in Figure 17 shows how the voltage required to breakdown a sample of press-spahn varies with the radius of the electrodes. For the pointed electrode this radius has been taken as infinitely small, and for flat electrodes as infinitely large. This curve is very useful from the practical point of view, as it shows that in order to obtain the actual disruptive voltage for an insulating material it is essential that it should be tested with flat electrodes. This conclusion is confirmed by several authorities and applies to all solid dielectrics. The size of the electrodes does not appear to have a very great influence upon the disruptive voltage. The usual practice is to employ small

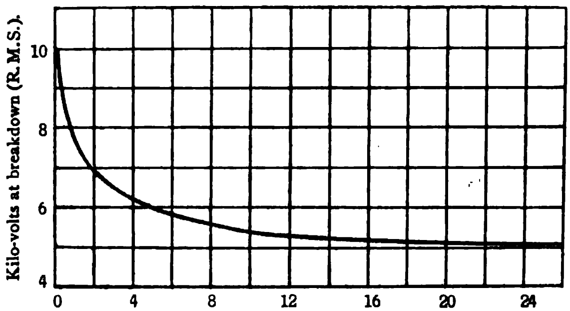


FIG. 17.—Disruptive voltage for 1 mm. press-spahn as a function of the shape of the electrode.

electrodes of from 2.5 to 5 centimetres diameter, the edges being slightly curved in order to ensure a more uniform stress throughout the sample.

For pressures greater than 25 grammes per square centimetre the mechanical pressure on the dielectric does not affect the puncturing voltage, but a standard wave form and frequency are essential. For a given R.M.S. value of pressure a peaked wave will be more liable to pierce a dielectric than a flat wave; hence to enable all tests to be comparable the wave form of the testing E.M.F. should be a sine curve. The results of Kinzbrunner's investigations would indicate that the frequency had very little effect on the disruptive voltage, provided it was below 75 and above 20 cycles per second. E. H. Rayner,* on the other hand, found that the pressure required

* *Journ. of the Inst. of Elect. Engineers* (1905), vol. xxxiv. p. 614.

to break down a specimen fell with the frequency, as is shown by the following results :—

Specimen.	Frequency.	Disruptive Pressure in Volts.
Press-spahn heated for six weeks at 75° C. to 100° C.	56	3750
	36	3300
Oiled cloth (not heated) . . .	50	4580
	37	4370

Whenever possible a frequency of 50 cycles per second should be adopted, as this is probably the most convenient standard.

Influence of Temperature and Moisture.—The temperature of a dielectric at the time the testing voltage is applied has a

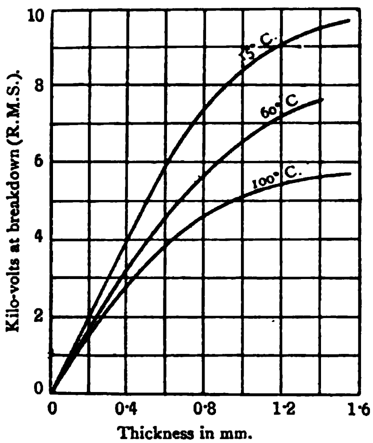


FIG. 18.—Effect of temperature on breakdown voltage of oiled linen.

considerable influence upon the disruptive pressure, the latter decreasing rapidly as the temperature rises. This deterioration, probably due to the disintegration of the dielectric, is shown by the curves in Figure 18, which give the disruptive strength of oiled linen at the temperatures 15°, 60°, and 100° C. When being tested the physical condition of a dielectric should be made to approach, as nearly as possible, the actual conditions met with in practice. Now, the temperature to which the insulation of electrical machines attains may when fully loaded be as high as 60° or 65° C., so that to be of real value to the designer the breakdown voltage should be measured at a temperature of about 65° C. Unfortunately, most of the test results available are for much lower temperatures, generally about 17° C., and this of course detracts considerably from their value.

In the case of hygroscopic dielectrics, *e.g.* leatheroid, paper, linen fabrics, etc., an increase in temperature is at first accompanied by an increase in the puncture resistance; this improvement of insulation is due to the moisture they contain being driven off. When once the dielectric is freed of its moisture any further increase in temperature will be accompanied by a decrease in the dielectric strength. The curves in Figure 19* show how the breakdown voltage of fullerboard is influenced by moisture. Curve A refers to a sample tested just as received from the factory, while Curve B gives the test results made on thoroughly dried samples of the same material. The presence of moisture has thus an appreciable effect on the dielectric strength of insulators, and though it is quite impossible to remove all traces of moisture, care should be taken to attain normal conditions.

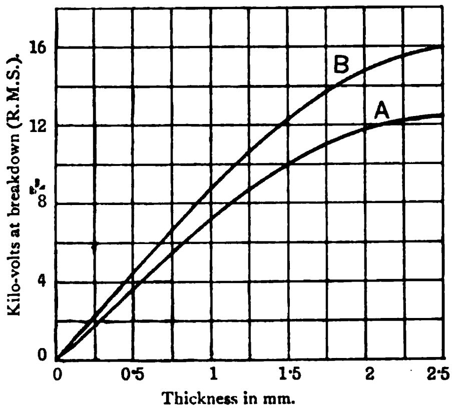


FIG. 19.—Influence of moisture on breakdown voltage of fullerboard.

Hence, before making tests, the specimens should be thoroughly dried in a vacuum oven.

Length of Time Pressure should be Applied. — Many dielectrics, if maintained at a voltage slightly lower than that which ruptures them immediately on the pressure being applied, will break down at the lower voltage if this is maintained for a few minutes. The voltage at breakdown is thus affected by the time of electrification, as shown by the curves in Figure 20. This is of great importance in testing dielectrics, and any material should be subjected to the voltage for at least such a period, after which its time curve becomes asymptotic. In the case of a single layer of varnished paper this time, which might be referred to as the "time factor," is about one minute. The time factor increases with various materials according to their dielectric strength, and for a given material increases with its thickness (see

* Turner and Hobart, *Insulation of Machines*, p. 30.

Figure 20). The explanation seems to be that the energy loss* occurring in the dielectric causes the latter to attain a somewhat higher temperature during the test, so that a corresponding decrease in the dielectric strength results. Hence for a given applied voltage, rupture is more likely to take place at the end of an interval of time equal to the time factor than at the instant of application. In testing the dielectric strength of a material the voltage should therefore be increased by small steps, and maintained constant at each step for definite intervals of from one to three minutes, according to the dielectric strength and thickness of the specimen. The omission to observe this condition is probably the cause of the enormous difference in the values of the dielectric strengths obtained for insulating materials of the same quality. For example, suppose that the time of testing of

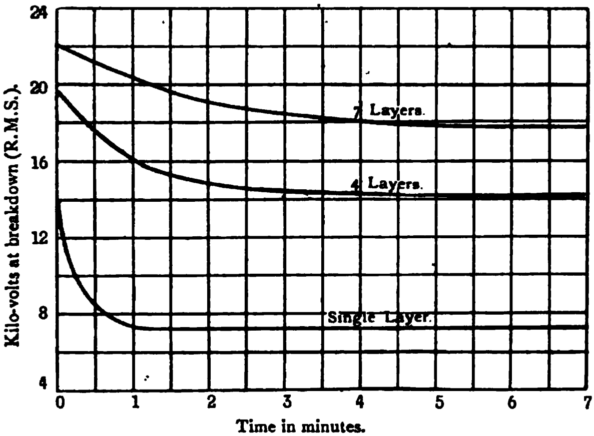


FIG. 20.—Time curves for 0.35 mm. varnished paper, flat electrodes.

the varnished paper of Figure 20 be fixed at half a minute, then for a single layer a voltage would be obtained about 10 per cent. larger than the actual disruptive voltage, while for four layers it would be 25 per cent. too high.

Testing Apparatus.—A suitable form of apparatus for determining the disruptive strength of insulating materials is shown in Figure 21. It consists of a wood box supported on large insulators and divided into two compartments by a horizontal metal shelf S. The sides of the box—except the front to which a glass door is fitted—are lined with asbestos. The electrodes E are placed in the top compartment, the temperature of which can be varied within wide limits by means of glow lamps fitted inside the lower compartment. The leads from the high-tension transformer would be brought through

* See page 46.

two choking coils to the electrodes. The lead connected to the lower electrode would probably be earthed, while the lead of the top electrode would be brought through a porcelain insulator as shown. The weight W serves to load the top electrodes so as to give a total pressure of about 4 kilogrammes.

Variation of Dielectric Strength with Thickness.—With the majority of insulating materials the disruptive strength increases at a less rapid rate than the thickness, as will be seen from the curves in Figure 18. As to the reason for this there appears at present to be no definite information; some authorities say it is due to the difficulty of

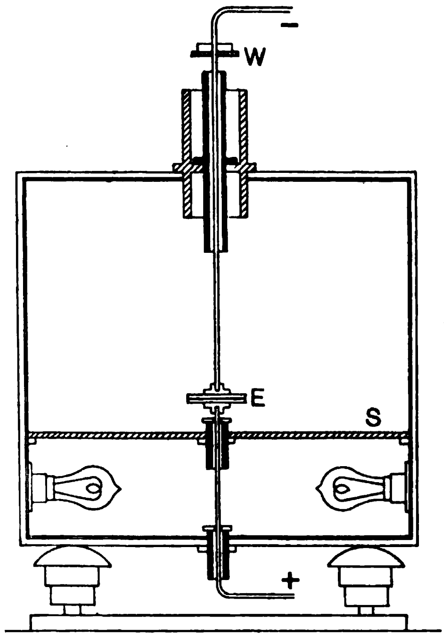


FIG. 21.—Apparatus for testing dielectrics. Scale 1 : 3.

obtaining a uniform composition throughout the thickness of the sheet. That this may be the explanation is supported by the fact that the puncture resistances of porcelain, glass, or mica—which are perhaps the most homogeneous of all the dielectrics—increase almost in proportion to the thickness. In the case of hygroscopic materials such as fibre, leatheroid, etc., this phenomenon seems to be partly due to the difficulty of drying out the inner portions, the insulating strength per millimetre decreasing rapidly with the thickness.

As to what law insulators follow with regard to the increase of puncture resistance with increasing thickness is by no means certain: many experiments have been carried out to determine this, but no

one seems to have arrived at very definite conclusions. According to Baur,* the breakdown voltage as a function of the thickness may be expressed by the equation

$$E = C \cdot \sqrt[3]{t^2}$$

where t = thickness of the dielectric in millimetres.

C = a coefficient which is equal to the voltage necessary to break down a thickness of 1 mm. of the insulation—*i.e.* the "dielectric strength."

In Table II. are given the values of C as determined experimentally by Baur with plate electrodes.

TABLE II.—DIELECTRIC STRENGTH OF INSULATORS AS DETERMINED BY BAUR.

Impregnated jute	2,200 volts.
,, calico	2,200 ,,
Ordinary dry air	3,300 ,,
Empire cloth	12,500 ,,
Hard porcelain	18,000 ,,
Fuller board	19,000 ,,
Paraffin	20,000 ,,
Mica	58,000 ,,

From the more recent experiments of Kinzbrunner † it would appear that the dielectric strength of most insulators, mica and Para rubber excepted, can be expressed by the formula

$$E = C\sqrt{t}$$

where C and t have the same meaning as in the previous formula. The following table gives Kinzbrunner's values for the dielectric strength—*i.e.* the disruptive voltage for 1 mm.—of the respective material, for several of the most usual insulating materials used for the insulation of electrical machinery.

TABLE III.—DIELECTRIC STRENGTH PER MM. OF VARIOUS INSULATING MATERIALS AS DETERMINED BY KINZBRUNNER.

Material.	*Dielectric Strength (Average Values).*
Press-spahn	4,600 volts.
Manila paper	2,200 ,,
Ordinary paper	1,450 ,,
Fibre	2,250 ,,
Varnished paper	10,500 ,,
Impregnated paper	4,200 ,,
Red rope paper	9,400 ,,
Varnished linen	10,700 ,,
Empire cloth	8,400 ,,
Leatheroid	3,050 ,,
Ebonite	28,500 ,,

* *Electrician* (1901), vol. xlvii. p. 759.

† *Ibid.* (1905), vol. lv. pp. 938–941 and 977–988.

Since the disruptive voltage per millimetre decreases with increasing thickness, it would appear that the insulating power of a material could be enhanced by laminating it—*i.e.* by subdividing it into several thin layers. The laminating or subdividing of a material can only be advantageous if the dielectric strength of several layers of a material increases in proportion to the number of layers. The curves of Figure 22 give the dielectric strength as a function of the number of layers for dark brown press-spahn 0.8 and 0.38 mms. thicknesses respectively. It will be observed that these curves are also quadratic, and Kinzbrunner has found that the disruptive strength of n layers is given by

$$E_n = E_1 \sqrt{n}$$

where E_1 denotes the disruptive voltage of one layer of the same material.

The lamination of a material has electrically no effect whatever;

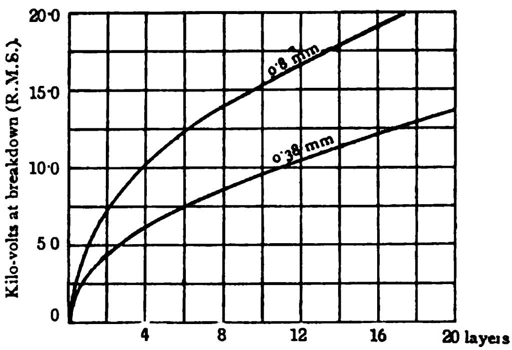


FIG. 22.—Ratio between number of layers and dielectric strength of dark brown press-spahn.

therefore, with regard to the disruptive voltage it does not make the slightest difference whether, for instance, one layer 4 mms. thick be used or four layers of the same material, each layer being 1 mm. thick. On the other hand, it may in some cases be of advantage to subdivide a material into several thin layers, more especially in cases where the material is subjected to mechanical strains, such as, for instance, in bending through a small radius, etc. Thus more especially for slot insulations it would improve the electrical resistance of the insulation if several thin layers were employed instead of one thick layer. With almost all other insulating materials the dielectric strength of several layers increases in a similar manner. Paper is, however, an exception, and gives very interesting results (see Figure 23). With very fine paper the curves are approximately straight lines, but become more bent the thicker the paper is. Finally, with very thick samples the

curves are nearly quadratic. The important deductions to be drawn from the latter curves is that paper should be used in very thin sheets and in as many layers as possible. For example, suppose that a winding has to be insulated with paper and that a space of, say, 0.36 mm. is available for the insulation. By using three sheets of paper, each of about 0.12 mm. thickness, the total dielectric strength of this insulation would be about 1100 volts, while the dielectric strength of 12 sheets of 0.03 mm. thickness would be about 1600 volts—*i.e.* nearly 50 per cent. more.

INSULATING MATERIALS

Mica and Mica Compounds.—Mica is without doubt the most important insulating material used in the construction of electric machinery. Besides possessing extremely high insulating qualities, it can withstand high temperatures without deterioration and is non-hygroscopic. Mica is a mineral consisting in most cases of a double silicate of aluminium and potassium with various other admixtures. In

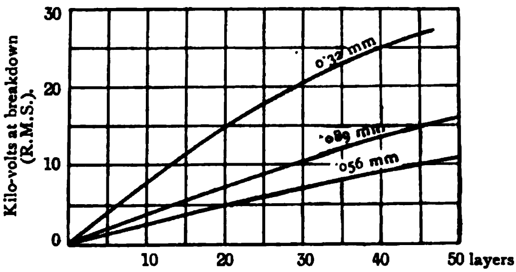


FIG. 23.—Ratio between number of layers and dielectric strength of grey insulating paper.

certain grades the potassium is replaced by sodium or magnesium. The colour depends greatly upon the percentage of the ingredients present; high percentages of magnesium lend a darker colour to the mica than when potassium is present. The presence of iron in large quantities imparts a colour ranging from grey to black. Mica crystallises in a laminated mass and has a remarkably fine and uniform cleavage, so that it can easily be divided up into very thin sheets of uniform thickness.

The curves in Figure 24 show how the disruptive voltage of white Indian and white amber mica varies with thickness. If mica be covered with a film of oil its puncture resistance will be reduced considerably; for instance, a sheet of mica which would resist 10,000 volts for an unlimited time in air would breakdown instantly under oil at a pressure of about 5000 volts. Paraffin and sealing wax produce

the same results. This phenomenon is not because of the difference of the specific inductive capacity, but for quite another reason. The oil under the electrodes has a lower disruptive strength than the mica, and breaks down first: at the point where the breakdown occurs the oil carbonises, becoming a conductor for the time being. This intensifies the electrical stress on the mica just at the point where the oil has broken down, and the mica punctures at a much lower voltage.

The great disadvantage of natural mica is that it is extremely inflexible and cannot be bent—except in the thinnest component laminæ, because in a piece of any thickness the different laminæ adhere immediately to one another. To overcome this disadvantage it has become necessary to employ the artificial forms of mica known

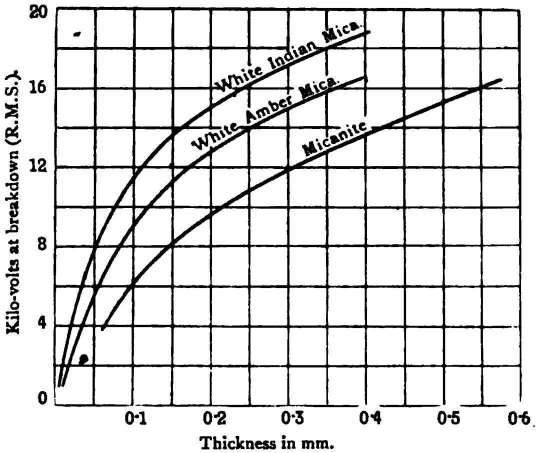


FIG. 24.—Disruptive strength of mica and micanite.

as *micanite* and *mica cloth*, these being largely used in the insulation of transformers and armature coils.

Micanite is prepared from flakes of mica which are re-assembled and cemented together with an insulating gum, *e.g.* shellac, the sheets in the process of manufacture being raised to a temperature of about 100° C. and subjected to considerable mechanical pressure. The cementing material is of such a nature that when heated it permits the different layers to slip slightly over each other, and so allow the mica to conform to any surface to which it may be applied. If allowed to cool in the mould it will retain this form. The disruptive strength of micanite is somewhat less than that of pure mica (see curve in Figure 24), and its breakdown, when the limiting voltage is reached, appears to be due to the chemical decomposition of the cementing gum and not to failure of the mica. Micanite is chiefly used for the

slot insulation of armature coils. For the insulation of transformer or field magnet coils, micanite would be unsuitable owing to brittleness at ordinary temperatures, so that for such purposes mica cloth is generally used. The latter is more flexible and is constructed of composite sheets of mica alternated with sheets of manilla paper or linen specially prepared so as to be moisture proof. Mica cloth so prepared has a disruptive strength ranging from 60 to 70 per cent. that of mica.

Press-spahn.—Press-spahn, in thin sheets, is largely used for the insulation of low-voltage windings. It is flexible, has a uniform thickness, and is homogeneous in texture. Though it has a smooth glossy surface it is somewhat hygroscopic. Before being used it should therefore be thoroughly dried, and to prevent the re-entrance of moisture it is invariably coated with varnish or impregnated with linseed oil. When thin sheets of press-spahn are treated with linseed oil the disruptive strength is improved, but in the case of thick sheets impregnated press-spahn shows a lower disruptive strength per millimetre than when untreated. This is due to the large increase in thickness which results from the process of impregnating, and is shown by the curves in Figure 25. Table IV. gives Holitscher's * results of tests made on samples of 1 mm. press-spahn by seven different firms.

TABLE IV.—HOLITSCHER'S RESULTS FOR DISRUPTIVE STRENGTH OF PRESS-SPAHN—ALL SAMPLES 1 MM. THICK AND TESTED BETWEEN PLATES.

Firm.	Disruptive Voltage.		
	Cold.		Warm.
	Whole.	Creased.	Whole.
A	12,000	8,000	10,000
B	11,000	11,000	11,000
C	22,500	8,500	20,000
D	17,000	8,800	14,090
E	11,000	7,500	9,200
F	13,500	8,800	8,600
G	15,800	9,500	15,500

Vulcanised Fibres.—Red and grey vulcanised fibres are prepared by chemically treating paper fibre and consolidating it under great mechanical pressure. Vulcanised fibre is mechanically strong, but is very hygroscopic. When maintained at a temperature of 50° C.

* *Elektrotechnische Zeitschrift* (1902), p. 117.

or more for any appreciable time it becomes brittle, and is therefore very seldom used in the construction of electrical machinery. Whenever it is necessary to use this material it should be thoroughly painted so as to render it moisture proof. The dielectric strength varies according to the thickness, but good vulcanised fibre should withstand 10,000 volts in thicknesses ranging from 3.0 to 20 millimetres, this puncturing voltage not increasing with the thickness, owing to the increased difficulty of thoroughly drying the inner part of thick sheets.

Leatheroid and Horn Fibre.—Leatheroid and horn fibre are made according to the same process as vulcanised fibre, and possess substantially the same qualities. At ordinary temperatures they are exceedingly tough, horn fibre more so than leatheroid, but when

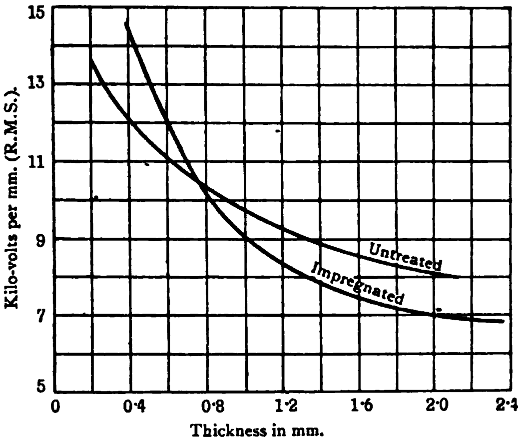


Fig. 25.—Dielectric strength of press-spahn as a function of the thickness.

subjected to prolonged heating at high temperatures become brittle. They should therefore not be relied upon entirely for the insulation of any electrical apparatus subject to vibration. Owing to the difficulty of drying out the inner portions and obtaining uniformity throughout the composition of the sheet an increase in thickness above 2.5 or 3.0 millimetres is not necessarily accompanied by an increase in the breakdown voltage. The dielectric strength of leatheroid and horn fibre has been plotted as a function of the thickness, in Figure 26, from which it will be noted that the disruptive strength of leatheroid is about 50 per cent. greater than horn fibre.

Impregnated Paper.—Paper, owing to its homogeneous structure and even qualities, is extensively used either alone or in conjunction with some other material such as mica. Manila, express, and bond papers are the best so far as disruptive and mechanical strengths are

concerned, although the so-called red rope is probably the most used of all. These four varieties, when coated with good insulating varnishes, are excellent dielectrics having a disruptive strength of from 6,000 to

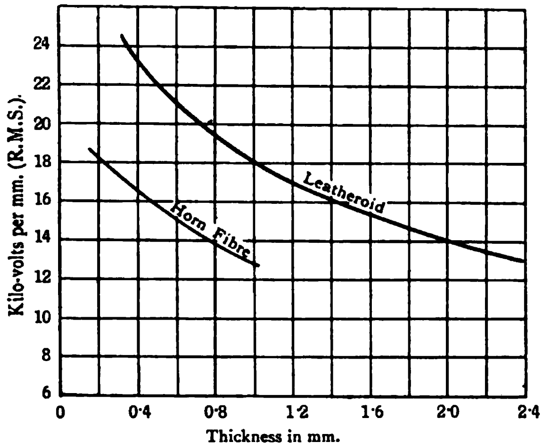


FIG. 26.—Dielectric strength of leatheroid and horn fibre.

10,000 volts per millimetre. On account of their hygroscopic nature these papers must always be treated with some moisture-proof insulating varnish. In fact, the present tendency is to look upon the paper as a medium of applying the insulating varnish, and to depend upon

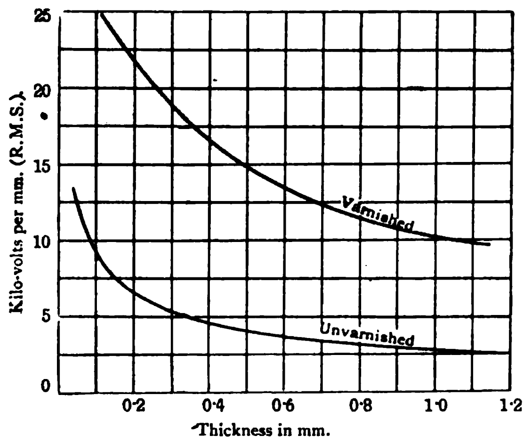


FIG. 27.—Dielectric strength of manila paper.

the latter only for the dielectric strength. The curves in Figure 27 show the marked improvement in the dielectric strength of manila paper when coated with varnish.

Impregnated Cloths.—Cambric, linen, and muslin impregnated with linseed oil or other varnishes are—owing to their flexibility—largely used for the insulation of field magnet and transformer coils. As is the case with impregnated papers, the varnish or oil forms the dielectric proper, the fabric being merely the medium for carrying the varnish or oil. The cloth must be free from chlorine, which is used in bleaching, for if not thoroughly removed it causes fermentation and rotting of the fabric. To test for chlorine, a sample of the fabric should be boiled in distilled water; this water is then put in a test-tube and a few drops of nitrate of silver added. The appearance of a precipitate indicates the presence of chlorides. The curves of Figure 28 give the dielectric strength in volts per millimetre for (1) varnished

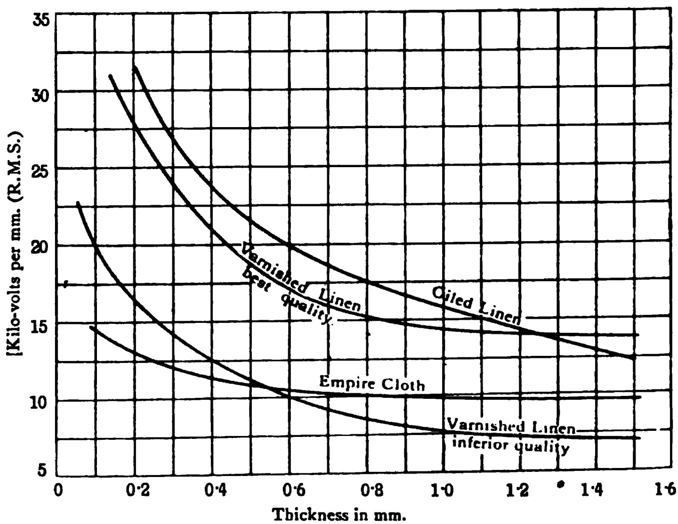


FIG. 28.—Dielectric strengths of impregnated cloths.

linen, best quality, (2) varnished linen, inferior quality, (3) oiled linen, and (4) empire cloth, the latter being the trade name for cambric treated with linseed oil. The disruptive strength of impregnated cloths decreases rapidly with increasing temperature (see curves for oiled linen, Figure 18), and within the usual working range of temperatures for electrical machinery there may be a variation in the disruptive strength by as much as 50 per cent. Again, when freely exposed to air, continued high temperature tends to produce brittleness, due to oxidation, and for this reason it is inadvisable to use cloths impregnated with linseed oil for the insulation of armatures. They are, however, largely used in the construction of transformers, where, owing to immersion in oil, the air is more or less excluded.

Oils.—So far as relates to dynamo-electric machinery, oil is chiefly used in connection with stationary transformers and for impregnating wood and fibrous materials generally. All mineral and vegetable oils, when pure, are good insulators, and the wide differences in the insulating qualities of the various mineral oils is due to the impurities contained: these may include acid, alkali, water, etc. For impregnating purposes linseed oil is the most suitable, on account of its penetrating qualities and its property of expanding on drying, thus filling up the pores and preventing the subsequent re-entrance of moisture.

A mineral oil is generally employed in transformers. Resin and linseed oil were at one time used, but have been discarded owing to trouble from carbonaceous deposits which take place when large quantities are subjected to a temperature of between 60° and 70° C.

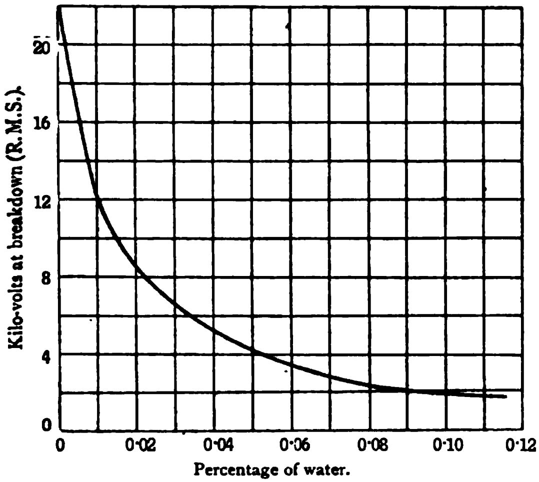


FIG. 29.—Decrease in dielectric strength of mineral oil due to presence of water.

Mineral oil for transformer work should be obtained from the distillation of petroleum, and have a flash-point of not less than 180° C. The oil must be free from moisture, sulphur compounds, and all traces of acid or alkali; and should not show an evaporation of more than 0.2 per cent. when heated to 100° C. for about ten hours. Before use, the oil should be slowly raised to a temperature of about 120° C., so as to drive off moisture and the light volatile oils, the latter of which are always present and are inferior to the heavier mineral oils in insulating properties. The curve in Figure 29 * shows the serious effect of small percentages of water upon the disruptive strength of mineral oils, the addition of 0.1 per cent. of water reducing the disruptive voltage to less than one-tenth its value when dry. The

* Turner and Hobart, *Insulation of Machines*, p. 154.

cause of this enormous decrease in dielectric strength is probably due to the particles of water, which have a much higher specific inductive capacity than the oil itself, arranging themselves in the plane of highest potential stress and thereby causing rupture.

Varnishes.—All fibrous insulating materials used in the construction of electrical machinery should be impregnated with an insulating varnish or compound, the function of which is to fill up the interstices and cover the surface with a skin of highly insulating material. Such treatment should have a twofold effect: first, increase considerably the dielectric strength; and secondly, after the material has been once dried, prevent the re-entrance of moisture into the dielectric and thereby maintain the constancy of the insulation resistance. An ideal varnish, besides being a good insulator, should possess the following properties:—(1) Be quick drying, non-hygroscopic, and unaffected by oils, acids, etc.; (2) have a melting-point of at least 200° C., and be a good conductor of heat; (3) have considerable elasticity and strength, not tending to become powdered when subjected to the combined effects of heat and long-continued vibrations; (4) constant dielectric strength over a wide range of temperature variation; and (5) be chemically stable and free from acids likely to attack copper and form cupric salts, which latter impregnate cotton fabrics and destroy their insulation. It may not be possible to obtain all these embodied in one varnish, but the above are the properties to be sought after.

At one time shellac varnish—*i.e.* gum lac dissolved in alcohol—was widely employed in the insulating of armature and field coils. It, however, becomes brittle through the influences of heat, age, and ordinary service, and is therefore unsuited for general use.

The varnishes generally used for impregnating purposes are those composed mainly of concentrated linseed oil with a small percentage of other ingredients such as turpentine and resin. Linseed oil compounds have an exceedingly high and uniform dielectric strength and, like linseed oil itself, expand on drying, which is of advantage in closely filling up the interstices of the fabric or paper. After applying the varnish the latter should be baked in an oven at from 70° to 80° C., and air admitted at regular intervals. This is necessary in order to oxidise the varnish to a hard dry surface. When baked in a vacuum oven, linseed oil remains soft and sticky, hence the admission of air is essential. Theoretically linseed oil varnishes are objectionable in that they contain acids which act chemically on the copper of the windings, producing a layer of green corrosion over the surface of the copper. In practice, however, very little trouble is experienced from this source, provided the insulation is thoroughly baked before application of the varnish. Another objection to varnishes containing linseed oil is that the latter is dissolved by lubricating oil and compounds

formed which have a deleterious effect upon the material impregnated. Special precautions must therefore be taken to prevent lubricating oils from reaching insulation impregnated with linseed oil varnishes.

Armalac is another varnish sometimes used for impregnating fibrous dielectrics. It consists of black paraffin wax dissolved in petroleum naphtha, and as it is entirely free from resin acids the possibility of the formation of salts of copper is excluded. Armalac has a melting-point exceeding 300° C., and though drying rapidly remains permanently plastic under the influence of heat. This latter property is a distinct advantage, in view of the expansion of armature and field coils as they heat, and their subsequent contraction when the loads are removed. Another advantage claimed for armalac is that it is unaffected by lubricating oils. The test results set forth in Table V. show the effect of the various varnishes upon the dielectric strength of fullerboard and red-rope paper.

TABLE V.—EFFECT OF VARIOUS VARNISHES UPON DIELECTRIC STRENGTH.*

Material.	Breakdown Voltage.			
	0.25 mm. Fullerboard.		0.25 mm. Red-rope Paper.	
	As received.	After drying.	As received.	After drying.
Shellac	2718	4670	1495	1780
Armalac	2880	5090	1555	1670
Sterling varnish	3925	5420	2155	2060
Standard varnish	4110	6240	...	...
Untreated	1490	3660	1340	1275

Tests on Insulating materials.—It is well known that when most insulating materials are subjected to prolonged heating at high temperatures their dielectric and mechanical strength is permanently weakened. With a view to determining the temperature limits consistent with the safe working of electrical machinery, a number of investigations were made—at the request of the Electrical Plant Committee of the Engineering Standards Committee—regarding the effect of heat on the electrical and mechanical properties of such insulating materials as are usually employed in electrical machinery, more especially in the construction of low-voltage armatures and transformers. A number of specimens of various insulating materials were for a period of nine months kept in electrically heated ovens maintained at the temperatures 75°, and 100°, and 125° C.

* G. H. Fletcher, *Electrician* (1909), vol. lxiii. p. 221.

respectively. The alternating pressure required to rupture these specimens was determined both in the case of the specimens heated to the different temperatures, and also in the case of a similar specimen which was not heated at all. As the results of the investigations form an important contribution to the literature on this subject, the results of some of the tests are reproduced in Table VI. The disruptive tests were carried out whenever possible on specimens which were placed between circular electrodes 5 square centimetres area, the upper one being loaded with a total pressure of 2 kilogrammes. In the case of tapes, etc., two rods 6.3 millimetres diameter were used with hemispherical ends, the pressure being 25 grammes. The voltage supply gave a sinusoidal E.M.F. of 50 periods.

The results of these tests show that most insulating materials improve when heated up to 75° C., do not seriously deteriorate by further heating to 100° C.—in some cases they actually improve, but show a rapid deterioration on heating to 125° C. The initial improvement in dielectric strength is, of course, due to the expulsion of the enclosed moisture. The tests also showed that prolonged heating even at 75° C. acts more or less injuriously on the mechanical properties of press-spahn and oiled fabrics. At 100° C. these materials showed signs of deterioration after a few months, and perished on further heating. At 125° C. most of the specimens were much discoloured and deteriorated after a comparatively short time, the only material remaining in a fair condition at the end of nine months being micanite; but even this had suffered to some extent, due to the charring of the shellac adhesive used in its preparation.

Energy Loss in Insulating Materials

When testing the dielectric strength of insulating materials by means of an alternating E.M.F., heat is always developed in these materials, and is especially noticeable when the test is long continued at a point approximating to the breakdown voltage of the material. When the amount of dielectric under test is large, ordinary indicating instruments will show that there is an actual loss of energy. The fact that energy loss exists under such conditions was first mentioned in 1864 by Werner Siemens, who pointed out that the glass wall of a Leyden jar became heated by repeated charging and discharging. As to the cause of the loss, there are essentially two theories. One claims that the loss is due to the presence of some conducting phenomena—perhaps of an electrolytic kind—or a kind of loss taking place in a resistance. The other theory, due originally to Steinmetz, attributes the loss, or at least a portion of it, to a sort of molecular friction by which the electric displacement would be caused to lag behind the intensity of a given electric field. According to the second theory—

TABLE VI.—EFFECT OF TEMPERATURE ON THE DIELECTIC STRENGTH OF INSULATING MATERIALS.

Material.	Thickness in Millimetres.	Temperature in Degrees C.	Average Disruptive Voltage.	Volts per Millimetre.
Press-spahn . . .	0.25	Normal 75 100 125	2843 2640 2778 2888	11,400 10,560 11,100 11,550
Press-spahn . . .	0.6	Normal 75 100 125	5497 5403 5289 5348	9,100 9,000 8,800 8,900
Press-spahn varnished .	0.35	Normal 75 100 125	7696 6243 6787 7600	22,000 17,800 19,400 21,700
Manilla paper . .	0.30	Normal 75 100 125	1847 1712 1800 1613	6,150 5,700 6,000 5,370
Manilla paper varnished	0.355	Normal 75 100 125	3270 4800 5043 8058	9,200 13,500 14,200 22,700
Mica linen . . .	0.23	Normal 75 100 125	2826 3383 3618 3288	12,300 14,700 15,700 14,300
Mica paper . . .	0.125	Normal 75 100 125	2403 2875 3500 2840	19,300 23,000 28,000 22,700
Oiled linen . . .	0.20	Normal 75 100 125	5750 5826 6623 5124	28,750 29,100 33,100 25,600
Varnished canvas . .	0.40	Normal 75 100 125	7025 5833 3014 1598	17,500 14,660 7,550 4,000
Dynamo tape . .	0.18	Normal 75 100 125	... 760 709 896	... 4,220 3,940 5,000
Superfine dynamo tape .	0.11	Normal 75 100 125	485 493 487 Perished	440 4,480 4,400 ...

in all probability the more likely one—the loss would appear as a perfect analogue to that occurring in magnetic materials by magnetic hysteresis, and is therefore often referred to as the loss due to *dielectric hysteresis.*

A determination of the laws which govern the loss in dielectrics when subjected to alternating electric fields turns principally upon the dependence of the loss on the voltage or the intensity of the electric field, and on the frequency. The results of the various investigations differ considerably, especially as regards the influence of the intensity of the field. As to the way in which the loss depends upon the voltage, several authors obtain the result that with a low field intensity the loss is approximately proportional to the square of the voltage, whilst with strong fields it increases more rapidly than the square of the voltage. The numbers 1.5 and 3 may be mentioned as extreme values of the voltage exponent, but numerous figures between these values are to be found. Further, the results of some experiments made within the last few years seem to indicate that the loss cannot be expressed as proportional to a power of the voltage with a constant exponent at all. Amid all these contradictory results it will therefore be apparent that it is exceedingly difficult to form any conclusive opinions as to the relation between the energy loss in dielectrics and the intensity of the electric field.

The most recent contribution to this subject has been made by Dr. Bruno Monasch,* who found that in all the dielectrics tested—these included various kinds of glass, ebonite, press-spahn, mica, and impregnated jute—the square law is obeyed with perfect accuracy so long as the voltage alone is varied. Apparent digressions from the square law result as soon as another quantity by which the loss is influenced—for instance, the temperature—is varied. In the dielectrics glass and ebonite tested in this respect the loss increases appreciably even with a small rise of temperature, whereas dielectric loss in rubber is influenced but slightly with change of temperature.

Although there is considerable doubt as to how this energy loss varies with the voltage, there seems to be a pretty general agreement that the loss is proportional to the capacity, and may be considered as proportional to the frequency through the range of frequency used for power and lighting purposes. The energy loss in a dielectric can therefore be expressed by the formula

$$W = A \,.\, E^x \,.\, C \,.\, \sim$$

where E = R.M.S. value of the voltage.
x = exponent having a value approximately 2.
C = capacity.
A = coefficient which depends upon the quality and temperature of the dielectric.

* *Electrician* (1907), vol. lix. p. 506.

The influence of frequency on the dielectric loss can be observed from the curves in Figure 30, which give the results obtained from a single sample of material when tested at 25, 60, and 133 cycles per second. It will be noted from these curves that the rate of variation of the dielectric loss for this particular set of tests is approximately in proportion to the frequency. The very fact of there being an increase of the loss due to frequency shows that energy is absorbed in taking the dielectric through its cycle of electrification.

The curves of the latter figure also show that there is a large increase in the energy loss of a dielectric due to increase of temperature, the loss increasing very rapidly indeed when the charring temperature of the material is approached. Since the energy loss varies approximately as the square of the voltage when the temperature is maintained constant, it follows therefore that under the usual con-

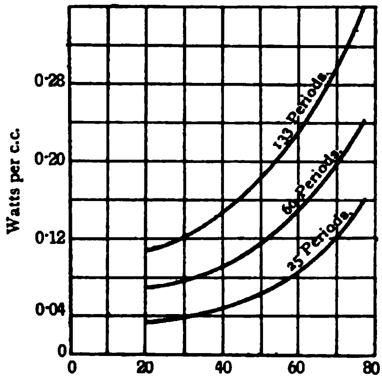


FIG. 30.—Variation of Dielectric loss due to temperature and frequency. Stress = 2900 volts per mm.

ditions of high potential stress it will increase more rapidly than the square of the voltage, because the temperature increases at the same time. The rate of this increase will depend on the facility with which the material can get rid of its heat, and also upon the initial temperature of the material and the temperature of the surrounding medium. As the stress is increased, a point is finally reached where the rate of generation of heat exceeds the rate of cooling, and temperature then rises until the insulation chars and breakdown results. The final breakdown in fibrous materials usually results from the charring of the material and not from mechanical rupture, unless the testing voltage is much above the material's disruptive strength, when the rupture is instantaneous. There may be some attendant chemical action, but this is probably the result and not the cause of the excessive heating.

Figure 31 * gives some interesting curves of rise in temperature in insulating materials when subjected to high potential stress. These curves are plotted from the results of tests on a sample of untreated material which was quite porous and capable of absorbing moisture

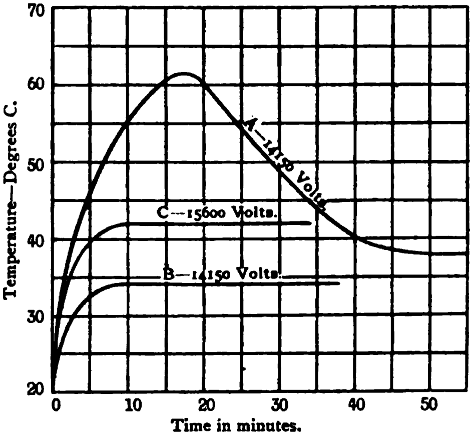


FIG. 31.—Increase of temperature due to dielectric loss.

from the air. During the test the material was well ventilated, so that any moisture could be quickly dissipated. Curve A shows the effect of moisture: the temperature rose very rapidly, reaching a maximum, then fell, and at last became stationary, thus indicating the improvement in insulation and consequent fall of temperature due to drying.

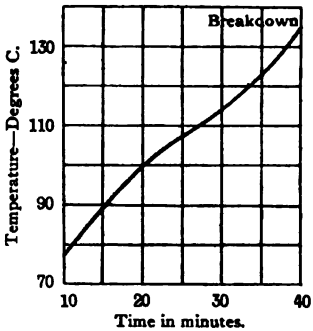


FIG. 32.—Breakdown caused by rise of temperature due to dielectric loss. Applied pressure, 3700 volts.

Curve B shows the effect on the same material after thorough drying, while curve C shows the increased temperature due to a slight increase in the voltage. The curve in Figure 32 * shows the rise in temperature

* From a paper by C. E. Skinner, *Journ. Amer. Inst. of Elect. Engineers* (1902), vol. xix.

of treated material when poorly ventilated, the test being continued until breakdown occurs. The temperature of the surrounding air in this test was kept at 80° C. The curve shows the tendency of the temperature to become constant, but at a point over 100° C. the loss becomes so great that the heat cannot be dissipated as fast as it is generated, hence the change in the direction of the curve and the consequent breakdown. In numerous tests performed by Skinner, the material was found to be badly charred on the interior without breakdown having resulted. This is of extreme practical importance, because it shows that *a long-continued test at high stress may seriously injure the insulation of a machine without this being made apparent by the test.* For want of a better term this is generally referred to as

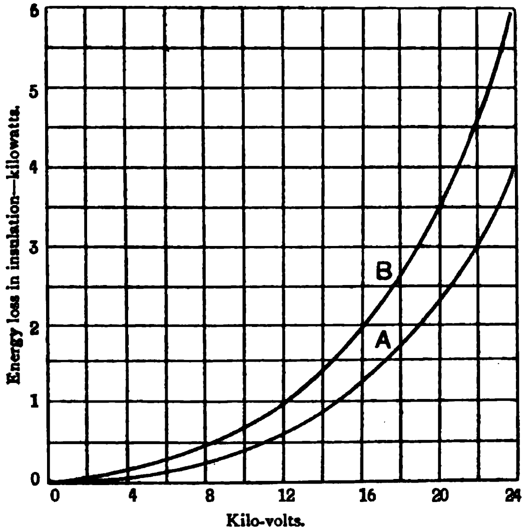


FIG. 33.—Loss in armature insulation of a 5000-K.W., 25~, 11,000 volt, 3-phase star-connected, alternator.

"straining the insulation," the idea being that it is in some way analogous to the straining of a piece of metal beyond its elastic limit.

Skinner has experimentally determined the energy loss in the armature insulation of the 5000 K.W., 3-phase star-connected, 11,000 volt alternators built by the Westinghouse Company, and installed in the Manhattan (U.S.A.) Railway Company's power house. The tests were made on two of these machines after installation, and the results are plotted in the curves A and B of Figure 33, the two curves relating to the results on two different machines. For curve A the temperature was approximately 21°, and 31° C. for curve B. The amount of insulating material under stress in these tests was approximately 320,000 cubic centimetres, this giving a maximum loss of .019

watts per cubic centimetre at 24,000 volts. For a normal stress of 6400 volts between the windings and earth the measured loss was less than 500 watts total for the machine, which is entirely negligible both as regard its effect on the efficiency of the machine and on the rise in temperature. On the other hand, it will be seen that the loss at a stress of 24,000 volts is a very measurable quantity, and the form of the curve indicates that at a point not far above 24,000 volts the loss would be sufficient to cause heating and possible damage to the insulation if the stress were applied for any considerable time.

Insulation of Transformer Windings

The insulation of a transformer consists of two parts: (1) the "external insulation," which protects the high- and low-tension

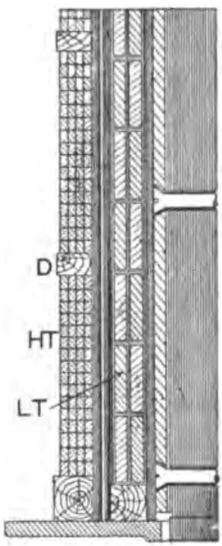


Fig. 34.—Transformer insulation.

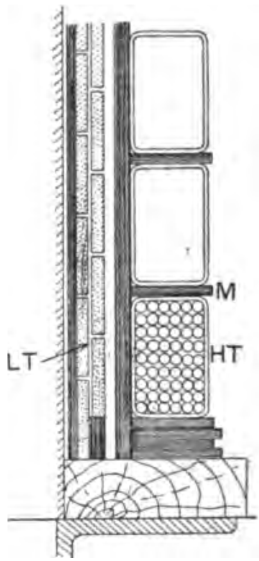


Fig. 35.—Transformer insulation.

windings from each other and earth; and (2) the "internal insulation," which forms the dielectric between adjacent turns.

External Insulation.—With transformers of the core type the windings are generally arranged in concentric cylinders with the low-tension (L.T.) nearest the core, the scheme of insulation being somewhat as shown in Figures 34 and 35. The low-tension conductor, usually of laminated rectangular section and covered with braided cotton or tape to a thickness of about 0.3 mm., is wound over a cylinder of

either press-spahn or manilla paper, the cylinder having a thickness of from 4 to 7 mm. In the case of the high-tension (H.T.) winding the voltage per layer will be comparatively high, and the arrangement of the windings should be such that the maximum voltage between adjacent conductors in neighbouring layers does not exceed about 100 volts. The H.T. winding of each limb is therefore subdivided into a number of coils ranging from 4 to 10, according to the voltage, adjacent coils being separated from each other by wood distance pieces D (Figure 34), or mica rings M (Figure 35). The H.T. coils can be insulated from the L.T. winding either by fibre rods or by a cylinder of micanite slipped on over the inside winding. For voltages above 4000 the latter method is better, but for lower voltages distance pieces are quite sufficient, and from the heating point of view are preferable, as they leave channels between the two windings for the circulation of the cooling medium. The windings are shown separated from the yoke

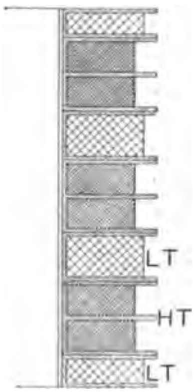


FIG. 36.—Transformer insulation.

by teak flanges about 20 mm. in thickness, and, in order to increase the leakage surface between the H.T. coils and the iron, stepped rings of mica are sometimes inserted between the end-coils and the wood flanges, as is shown in Figure 35.

When the L.T. and H.T. windings are each subdivided into a number of flat coils and interleaved with each other, as in Figure 36, the high-tension coils would generally be wound on suitably shaped formers of press-spahn or manilla paper, whilst the low-tension coils would simply be wrapped over with two or three layers of tape. When completed the individual coils are slipped over a cylinder of micanite or press-spahn, the top and lower coils of the L.T. winding being separated from the yoke by insulating rings.

The scheme of insulation for a shell type transformer is shown in Figure 37, where the H.T. winding is subdivided into 4 flat coils and

alternated with 3 L.T. coils. The insulation between adjacent coils consists of ⊔-shaped pieces of mica or press-spahn inserted at the corners of the coils. When the coils have been assembled they are enveloped with press-spahn, oiled linen, or micanite, so that ultimately the H.T. and L.T. winding are separated from the iron by from 7 to 20 mm. thickness of insulation.

Though no rigid rules can be laid down as to the exact thickness of dielectric required for the external insulation—this being generally settled from actual experience with a particular type and method of construction—the data given in Table VII. may serve as a general

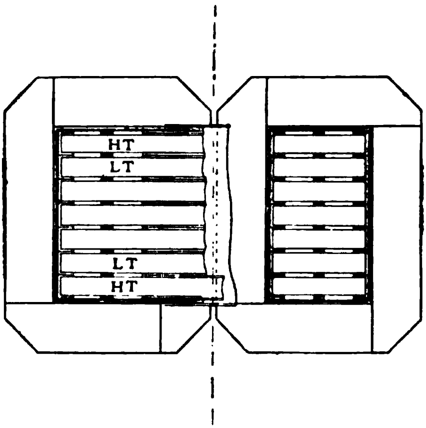


FIG. 37.

guide. A study of the insulation details of the transformer designs given at the end of Chapter V. will, for a range of voltages up to 11,000, give an acquaintance with the best modern practice.

TABLE VII.—THICKNESS OF INSULATION BETWEEN HIGH-TENSION AND LOW-TENSION WINDINGS, OR BETWEEN HIGH-TENSION COILS AND EARTH.

Dielectric.	Voltage per Phase of H.T. Winding.							
	2000	4000	6600	11,000	15,000	20,000	30,000	40,000
Air	8 mm.	10 mm.	12 mm.	15 mm.	...	...	...	...
Oil	5 ,,	7 ,,	10 ,,	13 ,,	...	...	...	...
Micanite	4 ,,	4·5 ,,	5 ,,	7 ,,	9 mm.	10 mm.	12 mm.	14 mm.

Internal Insulation.—The insulation of the low-tension coils is a comparatively simple matter, but in the case of the H.T. winding

special precautions must be taken. The normal voltage between adjacent layers should be reduced to such a limit that the ordinary cotton covering, even if not reinforced by other insulation, will suffice to withstand at least twice the working pressure. The cotton covering on the conductors should, however, be reinforced by inserting sheets of press-spahn, oiled linen, or micanite between adjacent layers, the thickness of this insulation ranging from 1 to 4 mm. for voltages between 2000 and 40,000. When wound, the coils should be thoroughly dried and impregnated with an insulating compound or varnish. The coils are finally lapped over with several layers of linen or tape, and again baked and varnished.

Insulation Breakdowns caused by Concentration of Potential.—An important factor to be taken into account in the design of insulation for high-voltage transformers, is the concentration

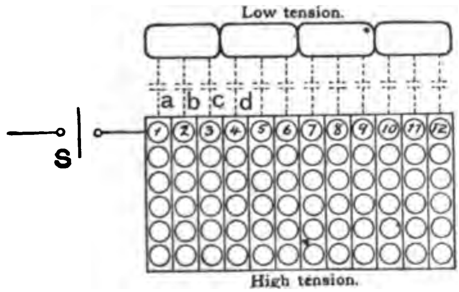


FIG. 38.

of potential upon the outer turns of the winding which occurs at the instant the primary is connected to the supply circuit; for if the end-turns of the high-voltage winding are not specially insulated, rupture of the insulation between adjacent turns of the terminal coils may result. The nature of this phenomenon will be clear from Figure 38, which represents the portion of a high-tension transformer coil connected to the line terminal. The conductors are represented by circles, and the order in which the current passes through the turns of the winding is indicated by the numbered circles. The low-tension winding lies adjacent to the high-tension, and the small condensers *a*, *b*, *c*, etc. indicate the electrostatic capacity of the corresponding wires to the adjacent low-tension coils.

Before either terminal of the transformer is connected to the line, all parts of the high-tension winding will be at zero potential. As the line switch S is being closed there is no change in the potential of the transformer winding until a spark passes between the blade and contact of the switch. Then instantly turn 1 of the winding is brought up to line potential, and turns 2, 3, 4, etc. follow it rapidly, but no turn

can attain its full potential until the hypothetical condenser connected to it is charged. For instance, turn 3 cannot reach its full potential above earth until the condenser *c* connecting it with the L.T. winding is fully charged. Now, the currents to charge *b*, *c*, *d*, *e*, etc. must flow through a considerable amount of inductance, with the result that a short interval of time must elapse before the condensers become fully charged. During this short but definite period the full potential of the line is concentrated upon the first half-dozen turns of the winding, and if the insulation of the winding is insufficient this momentary difference of potential may cause a spark to pass over the surface of the coil, or through the insulation.

Very soon after the closing of the first line switch the whole transformer winding will have assumed the potential of the first line wire; hence just before closing the second line switch the second transformer terminal is at the potential of the first line. As the second switch is closed a spark passes, and the potential of the second terminal of the transformer, which has up to this instant been at the potential of the first line, is suddenly changed to that of the second line—a very abrupt change. Then, as before, during the period required for the necessary charge to penetrate to the inner turns, a very high potential difference is impressed on the outer portion of the coil. The momentary strain on the insulation of the coil is greater when the second switch is closed than the first, for the first transformer terminal experienced an abrupt change only from earth potential to line potential, while the second was changed from the potential of one line to that of the other line, which may be nearly twice the potential from earth.

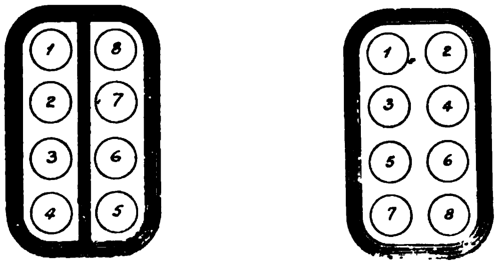
There is another very important difference between the effects of closing the first and second switches. In the case of closing the first, if the momentary strain breaks down the insulation, only sufficient current flows through this break to charge up the inner layers of the transformer coil. This is a very small quantity and can do comparatively little injury to the coil, especially if it be oil-insulated. In the second case the amount of current passing in the static spark is not materially greater than in the first case. But when the insulation between turns is momentarily broken down by this small spark there flows through the break a certain amount of current due to the E.M.F. impressed by the supply upon each turn of the coil. Although the static spark of itself would be but momentary, yet the current supported by the normal voltage of the circuit may be able to maintain the arc and continue indefinitely, destroying the whole coil if not interrupted.

With high-voltage induction motors this concentration of potential may occur in exactly the same way as explained above for a transformer; in fact, high-voltage motors are particularly subject to damage from this cause, on account of the greater difficulty in insulating the individual turns from each other. Transformers are much easier to insulate, and,

in general, it is not necessary to take any special precautions in switching in transformers wound for voltages not exceeding 11,000. The rise of voltage between the various turns of a terminal coil depends upon the suddenness of the discharge and the capacity and inductance of the windings. It should be noted that concentration of potential has no tendency to cause a breakdown to earth, but to produce short circuits between adjacent turns in the terminal coils, and this is one of the most frequent sources of trouble in very high-voltage machines. To diminish the risk of rupture under such conditions, high-voltage transformers must have their end-turns more heavily insulated than the rest of the winding.

Insulation of Stator Windings of Generators and Motors

When spiral coils are used for the windings of alternating current stators, precautions should be taken to ensure that the voltage between



(1) (2)

Fig. 39.—Slot insulation.

adjacent turns is reduced to a minimum. For example, suppose that in a certain 3300-volt alternator each stator coil has to be wound with 8 turns to give an E.M.F. of 160 volts—*i.e.* 20 volts per turn. In winding the coils two different arrangements of conductors are possible. These are shown in Figure 39, the numbers assigned to the conductors indicating the order in which the current traverses the coil. With the first arrangement the full voltage of the coil—*i.e.* 160 volts, exists between the two top conductors, whereas in the second arrangement the voltage between adjacent conductors never exceeds one-quarter the voltage per coil.

Arrangement No. 2 will therefore be the more suitable, as no insulation, in addition to the cotton covering, will be required between adjacent conductors. With arrangement No. 1 a strip of press-spahn 0.5 mm. in thickness should be inserted between the two tiers of conductors.

After winding, all coils should be dried in a vacuum oven, taped

over with several layers of cotten to a thickness of 1 to 1.5 mm., and then, after further drying, thoroughly impregnated with compound or varnish. To minimise the risk of breakdown caused by concentration of potential upon the end turns of a winding, the first and last coils must be more heavily insulated than the others. The conductors of these coils should therefore, in addition to the ordinary cotton covering, be wrapped with several layers of special tape having a high dielectric strength.

Slot Insulation

In low-pressure windings the thickness of slot insulation required for mechanical considerations always gives an ample margin of safety, even if comparatively poor dielectrics such as fibrous materials are used. The disruptive strength of the materials is relatively of second importance compared with their non-hygroscopic and mechanical features, and the permanence of these properties under all conditions, such as dampness, heating, vibration, etc., likely to occur in practice. Horn fibre, leatheroid press-spahn, and manila paper are, on account of their toughness, the materials generally selected for low-pressure slot insulation. Though these can be obtained in sheets of almost any thickness, it is better to use two or more layers of thin material in making up the slot lining to the required thickness, as greater flexibility can then be obtained. The slot lining for a 500-volt generator would have a thickness of about 1.2 mm. If of press-spahn a sheet of this thickness would have a disruptive strength of about 8000 volts per millimetre, so that, for a working pressure of 500 volts this gives a factor of safety of at least 18. For pressures less than 500 volts, mechanical considerations demand much about the same thickness.

The difficulties encountered in the arrangement of insulation for high-voltage windings are entirely electrical. For the purpose of reference the insulation intervening between the conductors and the core will be considered as made up of two parts: (1) The *minor insulation*, represented by the cotton covering round the wire, the function of which is to prevent the short-circuiting of adjacent turns. Like all low-potential insulations, it is mainly designed to meet the structural and mechanical requirements without undue expense. (2) The *major insulation*, represented by the slot lining, and which must have a sufficiently high dielectric strength to withstand continuously the strains due to the total pressure generated, or applied to, the circuits of the armature. The dielectric flux starts from the surface of the outer conductors, and traverses the minor and major insulation in series, stopping at the surfaces of the slot. Since the sectional area determined by the inner zone is considerably less than that determined by the outer zone, it follows that the stress to which

the insulations are subjected will be greater for the minor than for the major insulation. Hence, in a rational design, the dielectric next the conductor surfaces should be of the higher disruptive strength, but unfortunately structural requirements generally make this impracticable. The minor insulation is invariably porous, and may contain air and other gases at normal atmospheric pressure.

The relative potential gradients in the major and minor insulation will be examined by aid of Figure 40, which shows diagrammatically a single conductor, the minor insulation of cotton and air, the adjoining slot lining, and the iron core of the machine. The core is assumed to be at zero potential, and the ordinate CC′ represents the potential of the conductor above the stator frame. The ordinates of the curve AB′C′ will therefore give the potential at any point in the dielectric. The maximum potential gradient in the minor insulation will be at C,

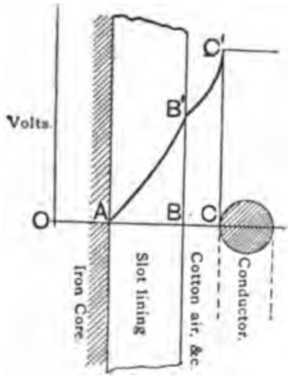


FIG. 40.

next the surface of the conductor—*i.e.* the curve is steepest at this point. If this potential gradient exceeds the limit for air, a brush discharge takes place from the copper to the slot lining, and, with the nitrogen from the atmosphere and moisture from the air, produces nitric acid.

This acid will combine with the gypsum in the cotton and copper of the conductor, forming sulphuric acid and cupric nitrate respectively, the presence of the latter being indicated by a green deposit on the conductor. This action, which is accelerated by dampness, has frequently been the cause of breakdown in the insulation of the armature coils of alternators built for pressures of 6000 volts and upwards.

The principal considerations upon which the maximum potential gradient and shape of the curve AB′C′ depend are: the potential difference between the conductors and iron, the shape of the con-

ductor—with a rectangular conductor, the stress will be largely concentrated at the edges,—thickness of minor insulation, and thickness and nature of the material with which the slot is lined. The electric stress for each layer of insulation will vary inversely as the specific inductive capacity, so that to ensure an approximately uniform potential gradient across the insulating medium separating the conductors from iron, the centre dielectric space should be occupied with materials having the same specific inductive capacity as air. Until recently the invariable practice in high-voltage machines was to build the tubes forming the major insulation entirely of micanite. Now, micanite has a specific inductive capacity of at least five times that of air or cotton, so that, if the insulating tube be of this material, a very abrupt change will occur in the potential slope across the minor insulation compared with that across the mica tube. In high-voltage alternators of 6000 volts and upwards, the electric stress in the cotton and air might be rendered sufficiently high to produce a discharge such as that already referred to. Hence, in the design of high-voltage insulation, the chief problem is to concentrate the stress on the wall AB (Figure 40)—*i.e.* to raise the point B so as to relieve the minor insulation of as much of its stress as possible. Within certain limits this can be done by using, instead of mica, a material having a lower specific inductive capacity. At the present time, however, the selection of such materials is limited, owing to the greater thickness that must be employed to give the requisite factor of safety. A compromise, however, may be effected by using mica and press-spahn sandwiched, thereby reducing the total capacity of the dielectric. This composite insulation has been found generally satisfactory for high-voltage work.

A press-spahn-mica tube 2.5 mm. thick, consisting of seven layers of press-spahn 0.2 mm. thick and six layers of mica, when carefully prepared, should have a disruptive strength of 30,000 R.M.S. volts. The adhesive qualities of the linseed oil with which the press-spahn is impregnated are sufficient, when the tube is warmed up, to be depended upon for cementing adjacent laminæ.

Solid micanite insulation can safely be used for windings up to, say, 4000 or 5000 volts. Very little danger need be feared from discharge, as the potential gradient in the minor insulation is comparatively small even when micanite is used. For higher voltages a composite tube of lower capacity is preferable.

There are other two methods by which the difficulty may be overcome, one of which is fairly self-evident, namely, to impregnate the coil so as to exclude the air altogether. The second method consists in placing a metallic shield between the major and minor insulations. The shield must be electrically connected to one terminal of the coil which it protects, and should be built up with the mica tube so as to exclude air from between the metal and mica.

It would require to be constructed of thin and high resistance material, so as to prevent undue eddy-current loss.

When micanite is used for slot insulation a certain thickness is necessary for mechanical strength and stiffness. It is not good practice to use micanite tubes thinner than 2 millimetres, otherwise they will not stand the mechanical stresses during construction. The factors of safety at the lower voltages will therefore be unnecessarily high. Even where the thickness of insulation is settled purely from considerations of disruptive strength, the factor of safety is less for high voltages than for low. This is because of the difficulties which arise from the excessive space of the active belt of the stator taken up by insulation of great thickness. The thickness of slot insulation required for mechanical and electrical considerations should increase approximately as the square root of the voltage; and the following values may be taken as representative of modern practice:—

Voltage per phase (R.M.S.).	Thickness of Insulation.
1000	2.0 mm.
2000	2.5 „
4000	3.4 „
7000	4.2 „
10,000	5.0 „
15,000	6.5 „

Insulation Tests on Complete Machines and Parts

In order to test the dielectric strength of its insulation, a machine or transformer before it leaves the factory should have a voltage, considerably in excess of the normal pressure, applied to its windings for a definite period, and if no breakdown takes place during that time the insulation may be regarded as satisfactory. The dielectric strength test is applied to the insulation between the windings and core, and also to the insulation between different windings if there are several on the same core. The test should be applied when the windings are warm, say immediately after a full load run of several hours. It has already been shown that if the insulation be subjected to an excessive pressure for some considerable time it may be permanently weakened, and for this reason the time of testing is usually limited to one minute. The magnitude of the testing voltage is settled from the normal working voltage of the winding under test. Since the factor of safety for low-voltage windings is much higher than that to which it is possible to attain with high-voltage windings, the ratio of testing voltage to working voltage should obviously be less for the higher voltages, as otherwise there might be serious danger of

damaging the windings. The data of Table VIII. gives the usual values of the testing voltages adopted by British manufacturers.

TABLE VIII.—INSULATION TESTS.

Part of Machine.	Normal Voltage.	Testing Voltage.
Armatures and Commutators (Copper to Iron).	Up to 230. From 460–600.	1200 2500
Magnet Coils.	Up to 230. From 460–600.	1800 3200
Stators of generators and motors, also transformers (tested from phase to phase, copper to iron, and primary to secondary)	250 500 1,000 2,000 3,000 4,000 5,000 7,000 9,000 11,000 15,000 20,000	1,500 2,000 3,000 5,000 7,000 9,000 11,000 13,000 15,000 17,000 21,000 25,000

CHAPTER III

TRANSFORMERS — FUNDAMENTAL PRINCIPLES — CONSTRUCTION — MAGNETISING CURRENT — VECTOR DIAGRAMS AND REGULATION

THE alternating current transformer, the principle of which is illustrated in Figure 41, consists essentially of a closed magnetic circuit MM of laminated iron, interlinked with which are two electric circuits P and S. The former, known as the *primary*, is connected to a source of alternating current, whilst the *secondary* S has its terminals connected to a load of motors, lamps, or other apparatus to be supplied with power. When a current flows in the primary an alternating magnetic flux is produced nearly all of which is confined to the iron path, and this becomes linked with both windings. There is, however, a small percentage of the primary flux, indicated by Φ_p, which does not link itself with the secondary. This constitutes the primary leakage; but the effect of this, for the present, will be neglected, and the assumption made that the entire flux set up by the primary magnetomotive force is transmitted without loss to the secondary. The alternating magnetic flux will therefore induce in the secondary an alternating E.M.F. of the same frequency. When the switch connecting the winding S with the load is closed, a current flows and energy is transmitted from one electric circuit to the other through the medium of the alternating magnetic field which is interlinked with both windings.

FUNDAMENTAL PRINCIPLES

E.M.F. Equations and Transformation Ratio.—When the number of lines of force linked with a coil is varied, the E.M.F. induced is proportional to the number of turns in series and the time rate of change of the flux. If T denote the number of secondary turns and ϕ the magnetic flux at any instant t, then the instantaneous value of the secondary induced E.M.F. is

$$e_s = T_s \cdot \frac{d\phi}{dt}$$

In order to determine the value of e_s it is necessary to know how ϕ varies with t.

The magnetic flux linked with the secondary is produced by the primary current, and if—as would invariably be the case in practice—it be assumed that the iron of the magnetic circuit is worked below saturation and that hysteresis is negligible, then the flux ϕ will be proportional to the primary current i_p. If it be further assumed that both the applied E.M.F. and the current of the primary vary according to the simple sine law, the magnetic flux will be a periodic sine function expressed by the equation $\phi = \Phi \sin pt$, where Φ denotes the amplitude value of the flux.

The E.M.F. induced in the secondary will therefore be

$$e_s = \mathrm{T}_s \frac{d\Phi \sin pt}{dt} = p\mathrm{T}_s\Phi \cos pt$$

$= p\mathrm{T}_s\Phi \sin (pt - 90)$, *i.e.* it lags 90 degrees behind the magnetic flux and primary current magnetising the core.

The maximum value of the induced electromotive force is $\mathrm{E}_{max} = p\mathrm{T}_s\Phi\ 10^{-8}$ volts, and the corresponding effective or root mean square value

$$\begin{aligned} \mathrm{E}_s &= \frac{p}{\sqrt{2}} \cdot \mathrm{T}_s \cdot \Phi \cdot 10^{-8} \\ &= 4.44 \cdot \mathrm{T}_s \cdot \sim \cdot \Phi\ 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad (1) \end{aligned}$$

The alternating magnetic flux which generates the E.M.F. in the secondary also links itself with the turns of the primary. Consequently, there will be induced in the latter an electromotive force which by Lenz's law opposes the applied E.M.F., and is therefore known as the *counter e.m.f. of the primary.*

The value of the latter may be derived by the same reasoning as has been applied above, and is expressed by

$$\mathrm{E}_p = 4.44\ \mathrm{T}_p \sim \Phi\ 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where T_p is the number of primary turns.

Equations 1 and 2 assume that the same flux Φ links itself with both windings, and are therefore theoretically only applicable to transformers where there is no magnetic leakage. When the secondary is an open circuit and the primary excited from a constant voltage supply, the transformer acts simply as a choking coil, the counter e.m.f. of which is approximately equal to the applied pressure E_p. The actual value to which the former adjusts itself is such that the current taken from the supply is that required to maintain the magnetic flux Φ and make up for the iron losses (*e.g.* hysteresis and eddy currents) corresponding to this flux. In transformers of modern design the no-load primary current is small, ranging from 0.5 to 5 per cent. of that at full-load. The primary IR drop will therefore be negligible, and the ratio of the voltages will be approximately expressed by

$$\frac{\text{Primary voltage}}{\text{Secondary voltage}} = \frac{4.44 \cdot \mathrm{T}_p \cdot \sim \cdot \Phi\ 10^{-8}}{4.44 \cdot \mathrm{T}_s \cdot \sim \cdot \Phi\ 10^{-8}} = \frac{\mathrm{T}_p}{\mathrm{T}_s} = \frac{\text{Primary turns}}{\text{Secondary turns}}$$

The ratio of the number of turns of the primary to the number of turns of the secondary is called the *ratio of transformation*. A transformer having a ratio greater than unity is known as a *step-down transformer*, the power being delivered from the secondary at a lower voltage than the voltage at which it is received. When the ratio of transformation is less than unity the transformer is called a *step-up transformer*. The latter would be employed chiefly for high-tension transmission work where the generator pressure is limited.

Effect of Secondary Current.—Since the voltage induced in the secondary is in the same direction as the counter e.m.f. of the primary, the secondary ampère-turns will magnetise the core in the opposite direction to that of the primary, and so tend to diminish the magnetic flux. Now, if the secondary terminal E.M.F. has to remain approximately constant, the magnetic flux passing through the core must be restored to its original amount. Owing to the self-regulating properties of a transformer, such will always be the case, for any weakening of the main field by the secondary current would at once reduce the primary counter e.m.f., and in consequence of this the applied pressure would send an increased current through the primary. In general, when any change occurs in the load on the secondary, the primary current at once adjusts itself so that the excess of primary over secondary ampère-turns remains nearly constant at a value just sufficient to maintain the core flux at its normal value Φ.

In a loaded transformer having negligible iron losses the primary current can be resolved into two components: (1) a magnetising component which is nearly constant at all loads, and (2) a load component the magnitude of which is such that the ampère-turns due to it are just sufficient to neutralise the opposing ampère-turns of the secondary. Owing to the adoption of a closed magnetic circuit in modern transformers, the magnetising ampère-turns are relatively small; hence, at full-load the primary ampère-turns are approximately equal to the secondary ampère-turns, and the ratio of the currents may be expressed as follows:—

$$\frac{\text{Primary current}}{\text{Secondary current}} = \frac{\text{Secondary turns}}{\text{Primary turns}} = \frac{T_s}{T_p}$$

Now, the ratio of T_s to T_p is also expressed by

$$\frac{T_s}{T_p} = \frac{\text{Secondary voltage}}{\text{Primary voltage}}$$

which when combined with the former gives the relation—

$$\text{Primary current} \times \text{primary E.M.F.} = \text{secondary current} \times \text{secondary E.M.F.}$$

Hence, if a transformer be considered as having a 100 per cent. efficiency, the power input to the primary is equal to the power output of the secondary. This being the case, the low-voltage winding will

carry a heavier current than the high-tension winding, the ratio of the currents being the inverse of the transformation ratio.

Magnetic Leakage.—Up to the present it has been assumed that the transformer has no magnetic leakage, so that the same flux becomes linked with the primary and secondary windings. This condition is not, however, realised in practice, because the two windings must be separately wound and insulated from each other, whereby intervening spaces are produced which admit of the passage of leakage lines.

A transformer having magnetic leakage is shown diagrammatically in Figure 41. Of the total flux generated by the primary ampère-turns, a small proportion, represented by Φ_p, becomes linked with the primary winding alone. These lines vary in phase with the primary current and consequently induce an E.M.F. which will be in quadrature with the current, *i.e.* the primary leakage flux produces the same

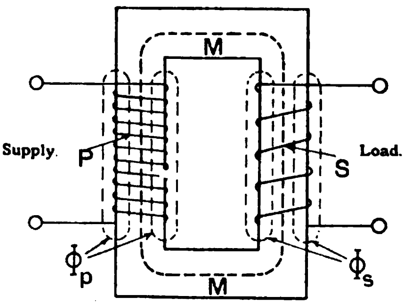


FIG. 41.—Principle of the Transformer.

effect as if an inductive E.M.F. were inserted between the transformer and the supply mains.

The ampère-turns of the secondary also produce, in addition to the flux which directly opposes the main flux, a local or leakage flux Φ_s around the secondary turns. This flux, in passing through the core, acts in opposition to that portion of the primary flux which becomes linked with the turns of the secondary. Strictly speaking, these fluxes do not exist separately, but a resultant flux, due to the resultant magneto-motive force of the primary and secondary windings. The secondary induced E.M.F. is thereby diminished without the counter e.m.f. of the primary being affected, so that in this case also the result is similar to an inductive E.M.F. in series with the winding.

Magnetic leakage has the same effect as that which would be produced by connecting two choking coils, C_p and C_s (Figure 42), in series with the primary and secondary windings of a transformer T, devoid of leakage. If the number of turns on the choking coils be

the same as that on the corresponding transformer windings, then the coils C_p and C_s will become linked with Φ_p and Φ_s lines of force respectively, the inductive voltages induced in the primary and secondary being expressed by the equations

$$v_p = 4.44 \,.\, T_p \,.\, \sim \Phi_p \times 10^{-8} \qquad (3)$$

$$\text{and } v_s = 4.44 \,.\, T_s \,.\, \sim \Phi_s \times 10^{-8} \qquad (4)$$

When the secondary is on open circuit the primary winding receives the magnetising current and produces an alternating magnetic field, which, owing to the low reluctance of the iron circuit compared with the leakage paths, has a negligibly small leakage component. On connecting the secondary to its load the primary current increases so as to counteract the opposing ampère-turns of the secondary, which latter are equivalent to an increase in the reluctance of the main magnetic circuit. As the reluctance of the leakage paths remains constant, the magnitude of the leakage flux will therefore increase in proportion to the primary current. Now, for a given applied pressure

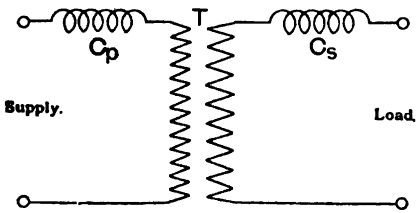


FIG. 42.

the total primary flux is practically constant; hence any increase in the primary leakage is accompanied by a corresponding decrease in the useful flux Φ, with the result that the secondary voltage falls below its open-circuit value. The leakage flux of the secondary has also a path of constant reluctance. Its magnitude will therefore increase with the load, and cause a further drop in the secondary induced E.M.F.

With a constant pressure of supply the secondary terminal voltage will, for an inductive load, be a maximum on open circuit and diminish as the current output from the secondary increases. This drop can be resolved into two components: (1) the IR drop due to the ohmic resistance of the windings, and (2) the inductive drop caused by magnetic leakage. For normal designs the drop due to the first cause is, as a rule, considerably less than that due to the induction of the winding. As close regulation is one of the most important factors to be taken into account when considering the relative merits of various designs, the disposition of copper and iron should be such that leakage is reduced to the minimum consistent with initial cost.

CONSTRUCTION OF TRANSFORMERS

According to the manner in which their electric and magnetic circuits are linked together, transformers may be divided into two types: (1) *core type*, and (2) *shell type*. Figure 41 represents diagrammatically a transformer of the former type, the chief characteristics of which are that the greater portion of the iron is enveloped by the windings, and that the external surfaces of the coils are exposed throughout. With the shell type the windings are partly embedded in the iron laminations (Figure 51), which latter surround the coils like a shell.

Single-phase: Core Type.—Figure 43 illustrates the usual form of construction for small transformers. The iron laminations, ranging

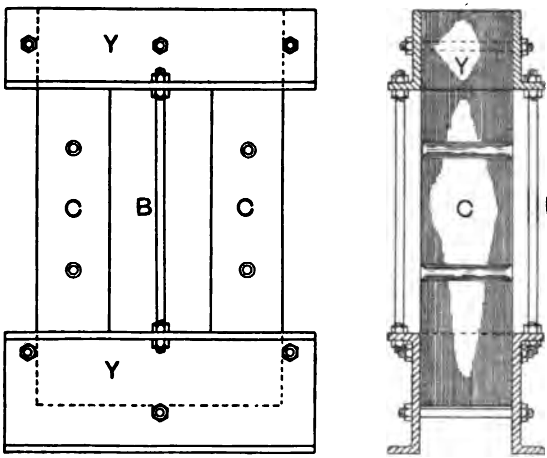


FIG. 43.—Core type Transformer.

in thickness from 0.35 to 0.5 mm., are assembled so as to form two upright cores C, connected by the yokes Y, the two cores and lower yoke being built of ⊔-shaped stampings all of the same width. The laminations are riveted together and bolted between cast-iron clamping plates, all rivets and bolts passing through the core being insulated from it with paper or press-spahn tubes. This latter procedure is necessary in order to guard against the short-circuiting of neighbouring plates. The top yoke is laid across with either butt or interleaved joints and clamped down on the cores by vertical bolts B. In the case of transformers rated for 100 K.V.A. or more, the use of ⊔-shaped stampings would, in the stamping out, entail a large loss of material. To avoid this the standard practice is to assemble the laminations in four blocks, which are put together as in Figure 44.

Some manufacturers wind the coils on circular bobbins, and in order to bring the copper as close to the iron as possible make the

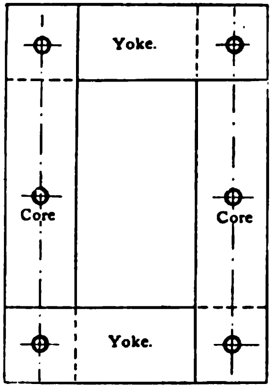


FIG. 44.—Core for large Transformer.

cores of octagonal cross-section (Figure 45). This shape is obtained by building the core of stampings having various widths. The yokes

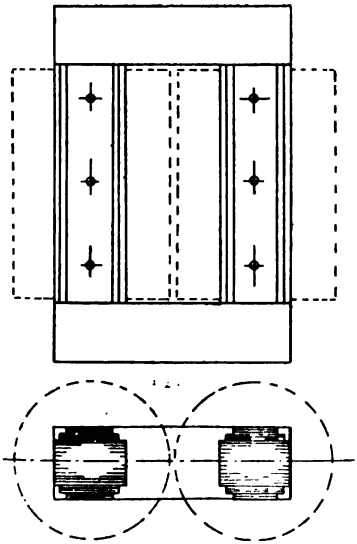


FIG. 45.—Core type Transformer.

are designed with a rectangular form of cross-section, the number of laminations in the core and yoke being equal. The height of the yoke pieces is generally selected so as to give the yokes approximately

the same cross-sectional area as the cores. In large transformers the laminations are separated at intervals with distance pieces D, which form vertical ventilating ducts (Figure 46). These ducts greatly

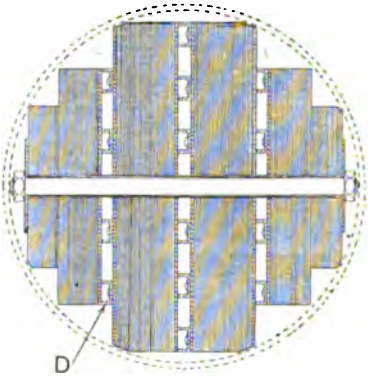


FIG. 46.

facilitate the circulation of the cooling medium, and range in number from one to five according to the size.

Whenever butt joints are employed it is advisable to interpose thin sheets of insulating material, as this prevents the circulation of eddy currents between the cores and their yokes. This procedure, however,

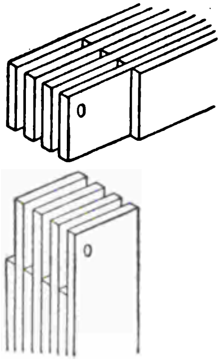


FIG. 47.—Interleaved Joint of core type Transformer.

introduces a short air-gap into the magnetic circuit and so increases the magnitude of the magnetising current. If it be found expedient to reduce the reluctance of the magnetic circuit to a minimum, the joints are then effected by making the yoke and core laminations overlap alternately at the corners, as in Figure 47. This forms an *interleaved* or *imbricated* joint, and the reluctance of the magnetic

circuit will then be about as low as would be the case if the laminations were continuous. After making the joints the laminations are held together at the corners by insulated bolts.

When repairs are necessary an iron circuit with interleaved joints

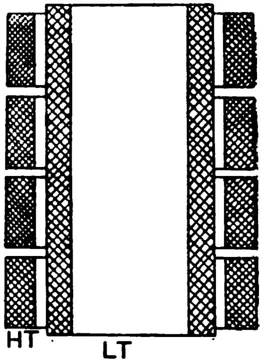


FIG. 48.—Single concentric Winding.

is at a disadvantage, in that the dismantling of the transformer and the subsequent building up of the core involves much more labour than would be necessary if the joints were butted. As a compromise between joints of low reluctance and a construction which facilitates

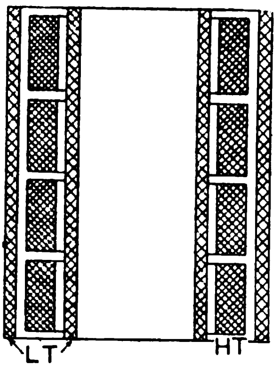


FIG. 49.—Double concentric Winding.

repairs, the cores are in some instances interleaved with the yoke at the bottom, and butt joints made between the top yoke and the cores.

With transformers of the core type three methods of winding, shown in Figures 48 to 50, are possible. The one most frequently adopted is that of Figure 48, where the coils are wound in concentric

cylinders, with the low-tension (L.T.) nearest the core. To diminish the risk of breakdown between adjacent layers, the usual practice is to subdivide the high-tension (H.T.) winding into from four to ten short coils, adjacent coils being separated from each other either by distance pieces or mica rings. In the double concentric cylindrical winding (Figure 49) one-half of the L.T. is inside and the other half outside the H.T. winding. The advantage claimed for this method over the former is that the leakage flux is less, but owing to the diminished copper space factor and increased cost of manufacture this method of winding is only adopted for special designs where a very close regulation is required.

For the third arrangement each winding is wound into a number of flat coils, and the high- and low-tension coils interleaved as in Figure 50. This arrangement makes the insulation of the H.T. coils

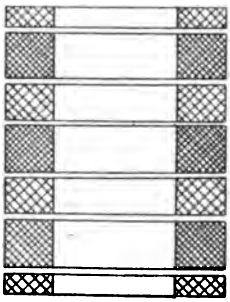
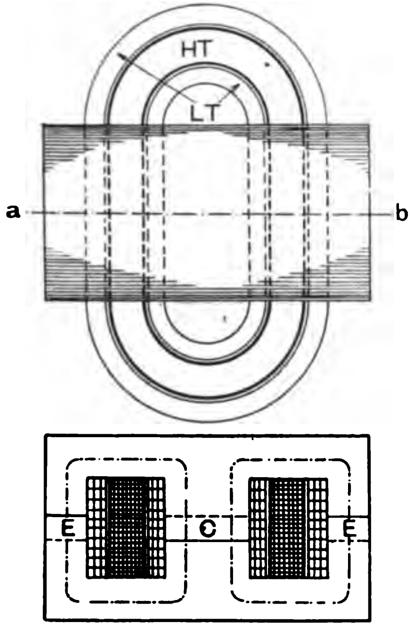


FIG. 50.

from the L.T. more difficult than in the case of cylindrical windings, and it should therefore be limited to voltages not exceeding 6600.

Shell Type with Rectangular Coils.—Figure 51 illustrates the lines upon which shell type transformers are generally constructed. When the laminations are assembled the central part C forms the core proper, whilst the external parts E form the shell. The core part carries the total flux linked with the windings, whilst each shell part carries only one-half the flux. The path taken by the latter is indicated in Figure 51 by the chain-dotted lines. With large transformers the cores and yokes are built up of rectangular stampings (Figure 52) and interleaved with each other. The cooling medium is given access to the inside of the core by leaving a space SS in the central part, and, to ensure exact alignment, the plates are sometimes punched with holes for threading over wooden dowels. The magnetic circuit of small transformers may also be built up on these lines, but the usual practice is to employ E-shaped stampings (Figure 51) in which the core and shell parts are of different lengths. Alternate laminations are

inserted from opposite sides, the joints in each plate being covered by the solid parts of adjacent plates. With this design of stamping



Section on *ab*.

FIG. 51.—Shell type Transformer.

there are only three joints, as against at least six with the former construction.

For very small transformers of 5 K.V.A. or less the type of wind-

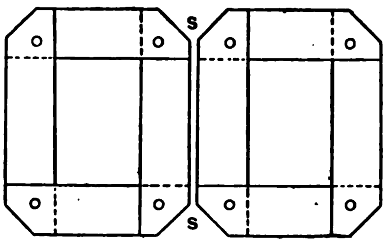


FIG. 52.—Core for Shell Transformer.

ing shown in Figure 51 is generally adopted. The low-tension coil is wound in two sections, and the H.T. inserted between them. The latter, together with the two L.T. sections, would be separately insulated, and previous to assembling the laminations the complete winding

would be wrapped together with varnished tape or linen. In all transformers rated above 5 K.V.A. the windings are made up into a number of flat coils, generally with a single turn per layer, the H.T. and L.T. coils being interleaved as in Figure 53. Since a large portion of the windings extends beyond the core, the ends of the outer coils can

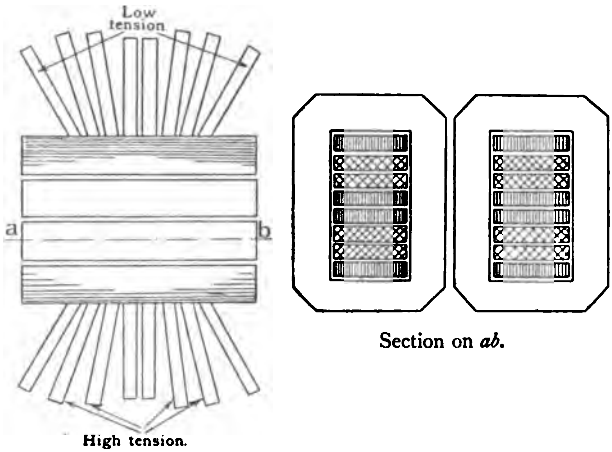


FIG. 53.—Windings of shell type Transformer.

be spread apart as shown, thus giving a much larger cooling surface than would be obtained with the coils compactly assembled.

Comparison of Shell and Core Type.—When designed to comply with the same specification of output, frequency, and efficiency, it will generally be found that, as regards cost, regulation, and heating, there is nothing of fundamental importance to choose between the two types. With very few exceptions the standard practice in this country is to adopt the core type, whereas in America the reverse is the case.

The chief objection to the core type is its long magnetic circuit and proportionally large magnetising current. This type of construction has, however, two distinct advantages: (1) the open position of the coils provides better cooling facilities, and (2) the coils may be withdrawn for repairs without dismantling all the laminations, as would be necessary in the shell type transformers. Shell transformers, owing to the short magnetic circuit, take a very small magnetising current. The mass of copper required is also considerably less than for a core transformer of the same output. On the other hand, only the ends of the coils are accessible, and the cooling surface, owing to the greater portion of the coils being embedded, is somewhat restricted.

Shell Type with Cylindrical Coils.—Figure 54 illustrates

another design of shell transformer manufactured by the British Electric Transformer Company. The coils in this case are wound in concentric cylinders and surround a central core C of laminated iron, the magnetic circuit being completed through another set of laminations S. The stampings are assembled in from 12 to 24 sector-shaped bundles with interleaved joints, and radially arranged so as to form ventilating ducts spaced uniformly over the whole area of the core. The low-tension winding is in two parts, in between which is placed the high-tension. The latter consists of six coils insulated from the L.T. coils as shown. The special features of this construction over the usual

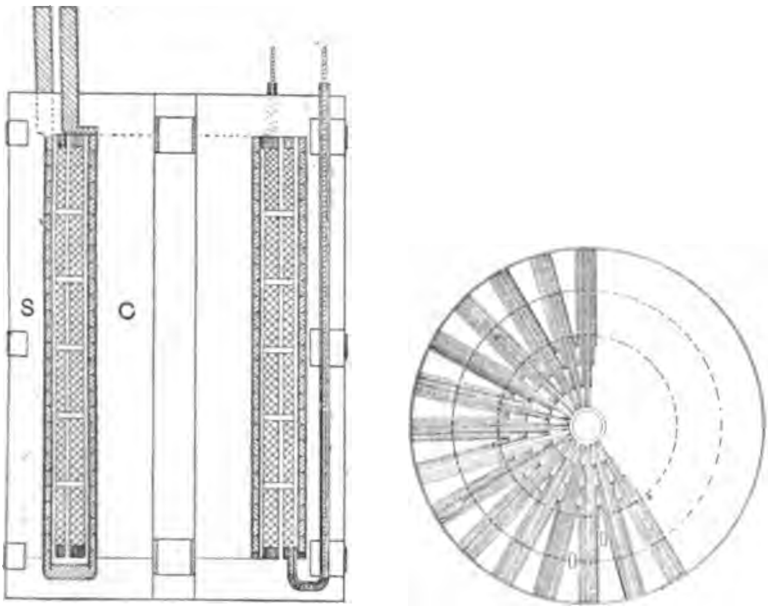


FIG. 54.—Shell type Transformer with circular Coils.

shell type is the large amount of iron and coil surface which is exposed to the cooling medium.

Three-phase Transformer.—The transformation of 3-phase power from one pressure to another can be effected either with three single-phase transformers having their windings connected in star or mesh or with one 3-phase transformer. The prevailing practice is to limit the size of a single 3-phase unit to about 1000 K.V.A. With outputs greater than this, three single-phase transformers will, as a rule, be preferable both as regards initial cost and transport considerations. For special work it may sometimes be necessary to use a 3-phase unit for outputs greater than 1000 K.V.A.; in fact, the standard

continental practice is to employ single units for all 3-phase work, and they have been built for sizes as large as 6000 K.V.A

Three-phase transformers are, from manufacturing considerations, nearly always of the core-type construction, and are built with three cores united by common yokes (Figures 55 and 56). Each core carries the primary and secondary windings belonging to one phase. In Figure 55 the yokes are in the form of laminated iron rings Y, Y, whilst the three cores of octagonal section are spaced 120 degrees apart, the yokes being pressed against the cores by means of suitable clamps. When the primary windings are connected to a 3-phase supply the

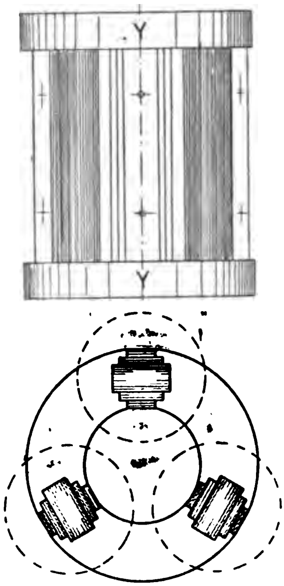


FIG. 55.—Core for 3-phase Transformer.

fluxes in the three cores will vary according to the same law as the respective magnetising currents. A 3-phase magnetic flux will therefore be produced, with a difference of 120 degrees between the flux phase of each core. Assuming a sine wave of magnetising current, the sum of the three fluxes will at any instant be zero, so that at any given instant the flux of one core will have its return path through the other two. The principle of the closed magnetic circuit is therefore realised without the addition of extra cores. With this construction it will be apparent that for the same flux density the yokes will have $\frac{1}{\sqrt{3}}$ the cross-sectional area of the cores.

The construction of magnetic circuit shown in Figure 55, though

giving a perfect symmetry for the three phases, is somewhat expensive, and for commercial reasons has been more or less abandoned in favour of the arrangement shown in Figure 56. This may be considered as the standard construction for 3-phase units. The three cores *c*, *c*, *c*, are in one plane and joined by top and bottom yokes Y, Y, which for equal flux densities will have the same cross-sectional area as the cores. In order to facilitate repairs the usual practice is to have butt joints at the top yoke and interleaved joints at the lower. From the magnetic and electrical standpoint this arrangement with three cores in line is not quite symmetrical, the magnetic circuit of the centre core being somewhat shorter than either of the end cores. In consequence of this it will generally be found that the no-load current

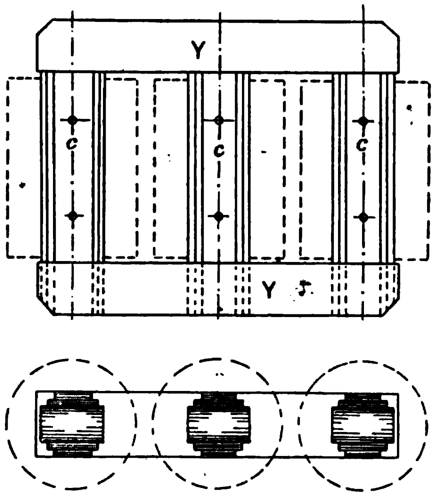


FIG. 56.—Core for 3-phase Transformer.

taken by the primary of the middle core is slightly less than that of the other two phases. This small dissymmetry is, however, of no great practical importance.

Earthed Shields.—Should the insulation between the primary and secondary break down, a leakage current would pass from one winding to the other, and the low-tension circuit might under certain circumstances be raised to a dangerously high potential. To prevent this latter possibility it is often specified that adjacent primary and secondary coils must be entirely separated by an *earthed shield.* The latter, consisting of either a thin brass-sheet or a layer of copper gauze, is embedded in the insulation and connected directly to earth. Should the insulation between the two windings break down, contact will be made between the high- and low-pressure circuits, not directly, but

through the medium of the earthed shield. The potential of the secondary is thus prevented from rising to an abnormally high value.

One disadvantage of earthed shields is that they must, of necessity, weaken the insulation between the windings and earth, so that a breakdown is more likely to take place. The above method of protection also does not prevent a leakage between the terminals or the leading-in wires. A better procedure is therefore to earth some point of the secondary circuit, so that, should a connection take place at any part between the high- and low-tension currents, the earth connection of the latter eliminates any danger of shock which is likely to prove fatal. In order that the potential difference between the secondary mains and earth may be a minimum, the earth connection should preferably be made at the middle point of the winding in a single-phase, and the neutral point of a star-connected 3-phase transformer.

Mechanical Stress on Coils.—In laying out the windings of a transformer so as to ensure the best disposition of insulation, the question of mechanical rigidity must also be considered. Since the magnetising effect of the secondary current opposes that of the primary, there will be a repulsive force proportional to the square of the currents between the two windings. The mechanical stresses, though very small under normal conditions of load, may attain considerable magnitude when the secondary is short-circuited. In modern transformers having very close regulation the current on short-circuit ranges from 50 to 100 times the normal full-load value, corresponding to 2500 to 10000 times the normal stresses. In choosing the most suitable type of transformer, the manufacturer is therefore to a large extent guided by its adaptability for strong mechanical construction.

A disadvantage of the core type with double concentric windings is that considerable difficulty is experienced in obtaining a really good mechanical support for the coils. Experience shows that this arrangement of winding should be limited to sizes of about 150 K.V.A. If adapted for larger units, serious trouble may result from displacement of the windings when a short-circuit occurs.

No-Load Current

When the secondary of a transformer is on open circuit the current taken from the supply mains consists of two parts: (1) the *magnetising component*, which maintains the magnetic flux at the requisite value; and (2) the *energy component*, which balances the loss due to hysteresis and eddy currents, and also the small I^2R loss. The iron losses have exactly the same effect on the primary as if the secondary supplied an equivalent amount of power.

The copper loss at no-load is so small that it may be neglected;

hence, if W_e denote the iron loss in watts and E_p the primary applied E.M.F., then the energy component of the no-load current is

$$I_e = \frac{W_e}{E_p}$$

The magnetising component of the current may be obtained from the fundamental equation

$$\text{Maximum magnetising ampère-turns} = I_m T_p = \frac{0.8\ B_m l}{\mu}$$

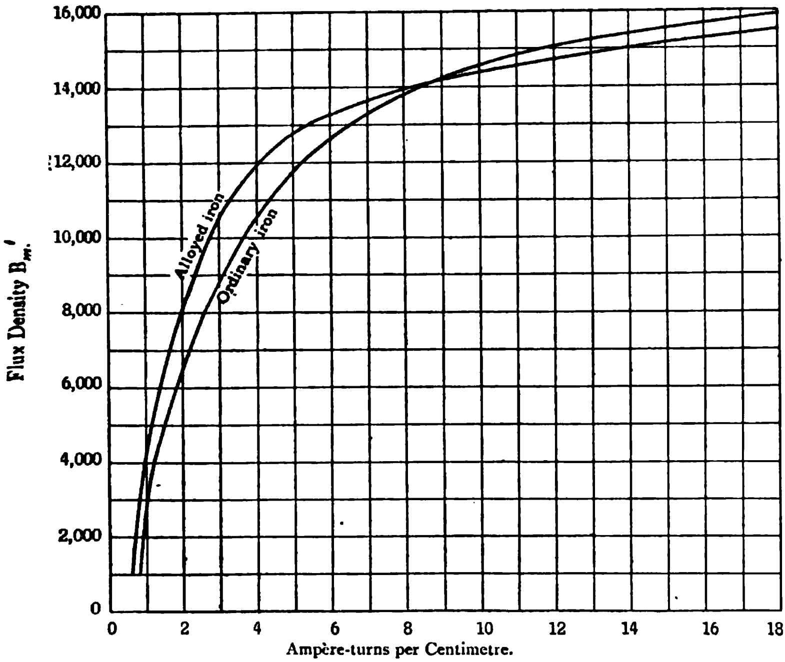


FIG. 57.—Magnetisation curve for ordinary and alloyed Iron.

Where I_m = maximum value of current.
T_p = number of primary turns in series.
B_m = maximum flux density in core in lines per sq. cm.
l = length of the magnetic path in cms.
μ = permeability of the iron.

It is, however, more convenient to have the magnetising current expressed as a R.M.S. value of I_m. Hence the above equation becomes $I_m T_p = \frac{0.8\ B_m l}{\mu} \times \frac{1}{\sqrt{2}} = \frac{0.56\ B_m l}{\mu}$

i.e. magnetising current $= I_m = \frac{0.56\ B_m l}{T_p \cdot \mu}$ . . . . . . (5)

In practice the magnetising ampère-turns are generally derived from saturation curves such as those in Figure 57. These curves show how the ampère-turns required per centimetre length of path vary with the flux density for (1) ordinary transformer iron, and (2) alloyed iron. It will be observed that at low inductions the latter has the higher permeability, but for densities greater than 14,000 the reverse is the case. In order to avoid undue losses and heating, the densities employed in practice seldom exceed 13,000 lines per square centimetre, so that within working limits alloyed iron is superior to ordinary iron both as regards permeability and core loss. The ordinates of the curves in Figure 57 denote the maximum values of flux density, and to obtain the virtual ampère-turns the abscissæ must be divided by 1.41. Should the yoke and core have different cross-sectional areas the ampère-turns for each part must, of course, be determined separately.

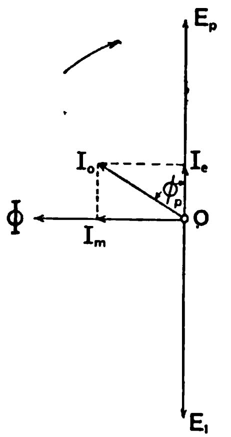


FIG. 58.—No-load vector Diagram.

When the values of the currents I_e and I_m are known the magnitude and phase relation of the no-load current can be obtained from the vector diagram in Figure 58. Let OΦ be the vector representing the core flux, then OI_m marked off along OΦ will represent the magnetising current, which must be co-phasal with OΦ. Since the counter e.m.f. of the primary is in quadrature with the flux, its vector will be in the direction OE_1. Neglecting the small copper drop, the pressure applied to the primary will be 180 degrees in advance of OE_1, and is represented by the vector OE_p. The energy component of the current must be in phase with the latter, and to represent it OI_e is marked off along OE_p. This diagram shows that the magnetising and energy components of the current are in quadrature with each other; hence the no-load current is expressed by

$$OI_o = I_o = \sqrt{I_e^2 + I_m^2}$$

ϕ_p is the angle of lag of the current, and in modern transformers has a value ranging between 45 and 50 degrees. As a result of this large angle of lag an unloaded transformer will have a very low power factor.

Shape of Primary Current Wave as affected by Hysteresis.—Even although the E.M.F. applied to the primary of a transformer be sinusoidal, it does not follow that the current wave will also be such; for, owing to the effect of magnetic hysteresis, the shape of the current curve, when the secondary is only slightly loaded, may be considerably distorted from that of a pure sine curve. Consider the case of an unloaded transformer having a sine wave of E.M.F. applied to its primary. The no-load current wave will consist of two parts: (1) the curve of magnetising current, and (2) the curve of energy current. The latter may be represented by a simple sine wave in phase with the applied E.M.F. To obtain the curve of magnetising current the values of

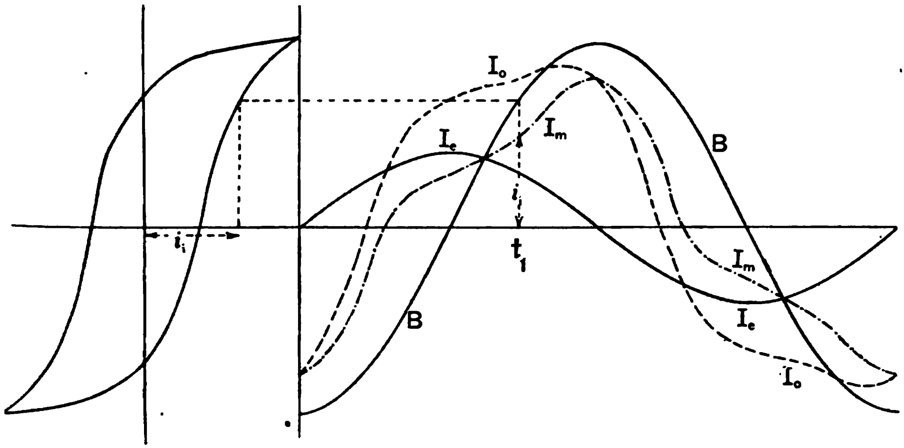


FIG. 59.—Wave-form of no-load current.

the magnetic induction as a function of (1) the exciting current, and (2) time, must be available. The former is given by the hysteresis cycle for the iron, and the latter, for a known wave of applied E.M.F., may be calculated from the equation on page 63.

In Figure 59 there is plotted (1) the hysteresis loop for a given core, and (2) a sine wave BB to represent the flux density in the core which will be in quadrature with the primary E.M.F. The curve of magnetising current is derived as follows. For the instants of time $t_1, t_2, t_3, \ldots . t_n$, observe the corresponding values of flux density B and then find from the hysteresis loop the values of the currents i_1, i_2, i_3, $\ldots . i_n$, corresponding to these values of B. Selecting any convenient scale, mark off these values of i along the corresponding ordinates of the curve BB, then the curve I_m, obtained by joining up the points $i_1, i_2, \ldots . i_n$, is the curve of magnetising current.

The sine curve I_e is the wave of the energy current, and by adding its ordinates to those of I_m a resultant curve I_0 is obtained which shows the no-load current as a function of time.

It will be observed that the effect of hysteresis is to cause the magnetising and no-load currents to be non-sinusoidal in shape. It should also be observed that the magnetising current attains its maximum values at the same instant as the magnetic flux, but passes through its zero value considerably ahead of the latter. With increasing values of maximum flux density the current curve becomes more peaked, the reason for this being that as the iron approaches saturation the maximum value of the magneto-motive force must be raised abnormally high in order to cause even a small increase in the magnetic flux.

When the secondary is loaded the distortion of the wave of primary current rapidly decreases, and becomes almost negligible at about 15 per cent. of full-load; that is, while the distorting component remains the same, the sinusoidal component increases with the load and obscures the distortion. The equation to the magnetising current curve in Figure 59 is

$$i = 0.65 \sin(\theta - 30°) + 0.15 \sin(3\theta - 15°) + 0.03 \sin(5\theta - 20°)$$

Of the harmonics, the third is the most pronounced, its amplitude being approximately equal to 25 per cent. of that of the fundamental. The presence of this large third harmonic in the wave of magnetising current often produces appreciable effects in practice, especially in polyphase work, and will again be referred to.

Transformer Diagrams

The phase relation between the several varying quantities in a transformer when operated under various conditions of load will now be examined by the aid of vector diagrams. Strictly speaking, the vector diagram method of treatment should only be adopted when the quantities under consideration vary according to the simple sine law. Except in the case of the magnetising current this assumption, with apparatus of modern design, is quite legitimate, and even for the case mentioned the error caused by substituting, for the actual curve, an equivalent sine wave will not affect the accuracy to any great extent. For simplicity of treatment it will therefore be assumed that all the quantities vary sinusoidally.

The vector diagrams are generally developed on the assumption that the ratio of transformation is unity, for it will then be possible to use the same voltage and current scales for both primary and secondary. This method is quite valid, since the transformation ratio does not influence the losses and efficiency of a transformer. In the following investigation the secondary turns will be considered as having the

same number as the primary turns, so that the actual secondary currents and pressures must be multiplied by T_s/T_p and T_p/T_s respectively. It will further be assumed that the vectors rotate in a clockwise direction, and that all the diagrams refer to one transformer.

Magnetic Leakage neglected.—Assuming the transformer to be devoid of magnetic leakage, four cases will be examined: (1) secondary on open circuit, (2) secondary current in phase with terminal E.M.F., (3) secondary current lagging, and (4) secondary current leading.

Figure 58 gives the vector diagram for a transformer whose secondary

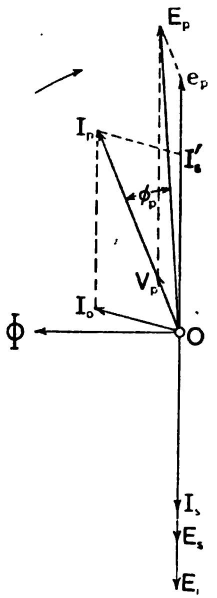


FIG. 60.—Load non-inductive and leakage neglected.

is on open circuit. Since the transformation ratio is unity, OE_1, in quadrature with the flux $O\Phi$, will represent the secondary induced E.M.F. and also the counter e.m.f. of the primary. The copper drop being negligible, the applied primary pressure OE_p will be of the same length as OE_1, but drawn 180 degrees ahead of it. ϕ_p, the angle of lag, will, as already stated, be about 50 degrees.

The vector diagram for a non-inductive load is shown in Figure 60. OE_1 represents the E.M.F. induced in the secondary, and OI_o the no-load current. Since the load is non-inductive, the secondary current must be in phase with the secondary E.M.F., and is represented by OI_s marked off along OE_1. Now, OI_o is the resultant of the primary and secondary ampère-turns, so that producing OI_s upward to OI_s', and

compounding it with the no-load vector OI_o, the line OI_p is obtained, which represents the primary current in magnitude and direction. The pressure applied to the primary has two components. The first balances the counter e.m.f., and is represented by Oe_p equal in magnitude to OE_1, but drawn in the opposite direction. The second component is that required to overcome the ohmic drop, and is represented by OV_p marked off along OI_p. OE_p, the E.M.F. applied to the primary, is obtained by compounding OV_p and Oe_p, and leads, with respect to the primary current, by a small angle ϕ_p. By loading the transformer the primary current has been brought more into phase with OE_p. In

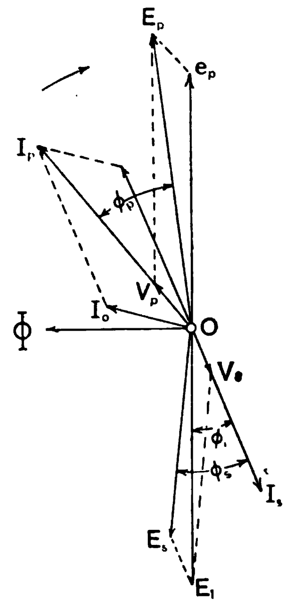


FIG. 61.—Load inductive and leakage neglected.

practice ϕ_p would be so small that cos ϕ_p could, without appreciable error, be considered as equal to unity. On load, the voltage at the terminals of the secondary will not be OE_1, but will be less by the $I_s^2R_s$ drop which is represented by E_1E_s. OE_s therefore gives the terminal voltage of the secondary.

With an inductive load on the secondary the secondary current OI_s will not be in phase with OE_1, but will lag behind it by an angle represented by ϕ_1 (Figure 61). OI_s is of the same length, and is obtained in the same manner as that in the previous diagram. The primary pressure OE_p is the resultant of the ohmic drop OV_p, and the voltage Oe_p overcoming the counter e.m.f. The secondary drop, being in phase with its current, is represented by E_1E_s drawn parallel

to OI_s, ϕ_s being the angle of lag of the latter behind the secondary terminal voltage.

If Figures 60 and 61 be compared, two points of importance should be noted: (1) an inductive load on the secondary reduces the power factor of the primary, the angle of lag of the secondary being transferred, as it were, to the primary; (2) the ratio between the magnitudes of the primary and secondary terminal pressures is so affected that the ratio of transformation is increased with an inductive load to a greater extent than with a non-inductive load of the same magnitude.

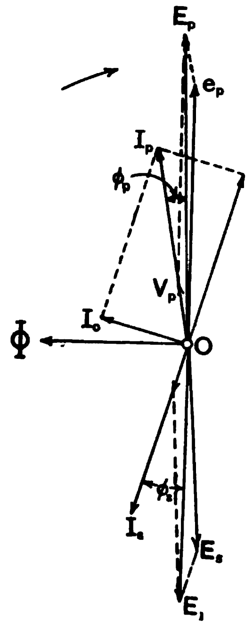


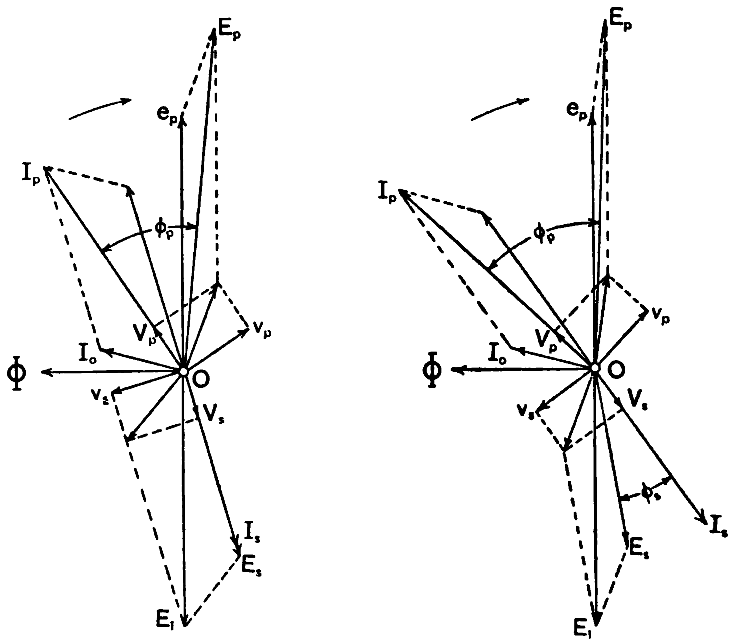
FIG. 62.—Capacity load and leakage neglected.

When the secondary supplies power to a number of over-excited synchronous motors, the secondary current will lead with respect to the secondary E.M.F. For such a condition the construction of a vector diagram would be carried out as in the previous case, except that OI_s will lead with respect to OE_1, as shown in Figure 62. From an inspection of the latter it will be seen that a leading secondary current has the effect of raising the power factor of the primary by reducing the angle of lag ϕ_p.

Magnetic Leakage taken into Account.—In an earlier part of this chapter it was shown that the leakage fluxes in a transformer gave rise to two electromotive forces which were in quadrature with the primary and secondary currents respectively. When these leakage

voltages are taken into account the vector diagram of a transformer whose secondary current is in phase with its E.M.F. will be that shown in Figure 63 (*a*). As with the previous diagrams, $O\Phi$ represents the magnetic flux, OE_1 the secondary induced E.M.F., and OI_s and OI_p the secondary and primary currents respectively.

In reference to the secondary, the component of the induced E.M.F. overcoming the leakage reactance is represented by Ov_s, leading 90 degrees with respect to OI_s; OV_s marked off along the latter represents the ohmic drop. The voltage consumed in secondary impedance is



(*a*) Non-inductive load. (*b*) Inductive load.

FIG. 63.—Leakage taken into account.

the resultant of OV_s and Ov_s which when subtracted vectorially from OE_1 gives OE_s, the secondary terminal pressure,—the latter, of course, being in phase with the current I_s. OE_p is the pressure applied to the primary, and consists of three components: (1) Oe_p balancing the counter e.m.f., (2) OV_p overcoming the ohmic drop, and (3) Ov_p opposing the leakage reactance. The vector sum of the components OV_p and Ov_p is the voltage consumed in primary impedance. ϕ_p, as before, is the angle of lag of the primary current behind the applied E.M.F.

From this diagram it should be noted that, for a load of unity power factor, *i.e.* one in which the secondary current is in phase with

the secondary terminal volts, the transformation ratio remains practically the same as in Figure 61, where magnetic leakage is neglected. The drop in volts is that due to the copper resistance only, the leakage voltage Ov_s being in quadrature with the terminal voltage OE_s, and Ov_p nearly so. With a non-inductive load the effect of magnetic leakage is to increase the phase difference between the primary current and the applied pressure without effecting a reduction in the secondary terminal volts to any appreciable extent.

If the secondary current lags behind the secondary terminal volts by an angle ϕ_s, then the vector diagram will be as Figure 63 (*b*), the lettering of which is the same as in the previous diagram. An inductive load on the secondary has the effect of bringing the primary and secondary currents more completely in opposition of phase. The leakage voltages will each have a larger component in phase with the secondary terminal pressure, with the result that the latter is diminished to a greater extent than when the power factor of the load is unity. It will be clear from the diagram that, for a given load current, the drop will be a maximum for that angle of lag ϕ_s which brings the impedance voltages in line with the induced E.M.F.'s, but should ϕ_s be further increased these vectors will not be in exact opposition of phase, and the voltage drop will diminish.

Regulation

Equivalent Circuits.—In order to facilitate the analytical and graphical treatment, there will be substituted for the magnetic and

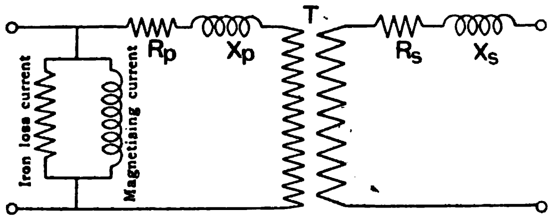
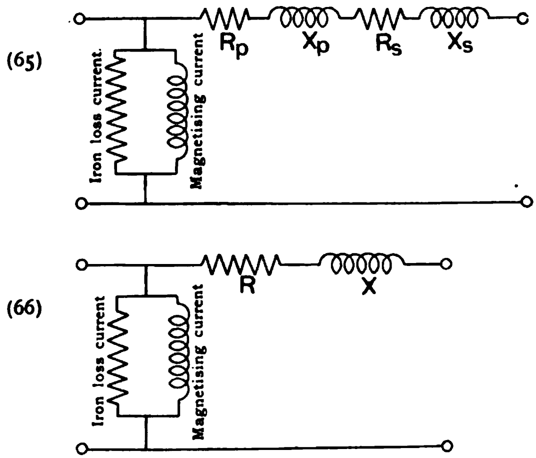


Fig. 64.

electric circuits of a transformer, as represented in Figure 41, the equivalent circuits shown in Figure 64, where T represents a perfect transformer having no leakage, no losses, and requiring no magnetising current. In series with the primary winding are shown two coils R_p and X_p, representing the resistance and leakage reactance of the primary. In the secondary circuit there will also be two similar coils, R_s, and X_s. The core loss current, which remains practically constant for all loads, and the exciting current are represented by the inductive and non-inductive circuits connected in parallel with the primary. Assuming a transformation ratio of unity, the transformer T may be omitted from

Figure 64, and there will only remain the equivalent circuits of Figure 65. The current taken by each of the three circuits will flow independently of the current in the other two circuits, while the total primary current will be the vector sum of the three components. The current taken by the load flows through the local impedance of both primary and secondary coils, and is unaffected by the presence of the currents in the other parallel circuits. The diagrams of equivalent circuits can be still further simplified to that shown in Figure 66, where the primary and



FIGS. 65 and 66.

secondary circuits are combined into one reactance coil X and resistance coil R.

Voltage Drop due to Resistance and Reactance.—When the primary of a transformer is excited from a constant voltage supply, the P.D. at the terminals of the secondary varies with the load by an amount which depends upon the resistance of the two windings and the magnitude of the leakage flux. With a lagging secondary current the voltage drop will be greater than when the current is in phase with the pressure. If, however, the current leads with respect to the terminal E.M.F. by more than a certain angle, the P.D. at the secondary terminals will increase with the load. This latter condition is, however, very rarely met with in practice, a pressure drop being the general rule.

The regulation of a transformer for a load of stated power factor is defined as the percentage drop in the terminal voltage of the secondary when the load is increased from zero to its full-load value, the primary applied E.M.F. being maintained constant. Transformers whose load consists of induction motors or lamps should be designed for as close regulation as commercial considerations will permit. According to the

power factor of the load, the regulation of transformers of present-day design ranges from 1.5 to 5 per cent., the latter being seldom exceeded except in special cases.

For a constant primary applied E.M.F. the voltage drop at the terminals of the secondary may be resolved into two components: (1) the resistance drop, and (2) the drop resulting from magnetic leakage, which need only be considered when the power factor of the load is less than unity. As already shown, the first is in phase, and the second in quadrature with the currents in the respective windings. Though the currents OI_p and OI_s (see Figures 60 to 63) are not in exact phase opposition, for they must be just so far out that the resultant of the ampère-turns due to them is sufficient to magnetise the core to the required degree, yet the phase difference from 180° is, except in

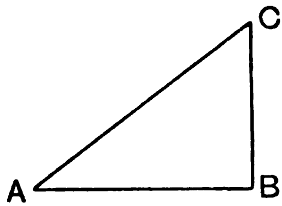


FIG. 67.

the case of the smallest transformers, almost negligible. The voltages V_p and V_s due to the ohmic resistance of the windings and the inductive voltages v_p and v_s can therefore, without introducing an appreciable error, be considered as having a phase difference of 180° with each other. This, together with the diagram of equivalent circuits shown in Figure 66, leads to a convenient method for determining the maximum drop. Referring to Figure 67, let BC represent the ohmic drop in the two windings, and AB the total inductive drop, which will be in quadrature with BC. Then AC, the hypotenuse of the right-angled triangle, is, for a given load, the *maximum* drop in secondary pressure that the transformer can have.

To obtain the combined copper drop, let V_p and V_s denote the resistance drop in the primary and secondary windings respectively, then if T_p/T_s denotes the transformation ratio, a drop of V_p volts in the primary will cause a drop in the secondary of $V_p \times \frac{T_s}{T_p}$. The combined copper drop BC will be given by

$$V_S = V_p \,.\, \frac{T_s}{T_p} + V_s = I_p R_p \,.\, \frac{T_s}{T_p} + I_s R_s$$

where R_p and R_s denote the effective resistances of the primary and secondary winding respectively. The magnitude of the inductive drop

depends upon the arrangement of the windings and the length of the paths taken by the leakage flux, and may be calculated from the formula given below. Let v_p denote the leakage drop of the primary, and v_s that of the secondary, then the drop AB at the secondary terminals due to magnetic leakage $= V_2 = v_p \cdot \frac{T_s}{T_p} + v_s$.

The maximum total drop AC is that E.M.F. which must be applied to the secondary windings in order that the full-load current may flow when the primary terminals are short-circuited. It is the "short-circuit" voltage each component of which, and consequently the whole, is proportional to the current.

Calculation of the Inductive Drop.*—The inductive drop in a transformer is due to the interlinkage of the leakage flux with the turns of the windings, whereby e.m.f.'s of self-induction are set up. If it were possible to determine the leakage flux approximately from the drawings of the transformer, by mapping out the field in proportion to the magnetomotive force acting at any point divided by the magnetic reluctance of the leakage path, then the exact determination of the inductive drop would be an easy matter. Unfortunately, such a mode of procedure could never be carried out so exactly as to give any but qualitative results, the actual value of the flux following any leakage path not being capable of exact calculation. However, by making a general application of the laws of magnetic circuits to different systems of coils, it is possible to see what factors will influence the interlinkage between the leakage flux and the turns composing a winding. The value of the coefficients to be inserted in the mathematical expressions, so as to convert them into definite numerical quantities, *e.g.* volts drop, can only be obtained from experiments carried out on some definite system of coils.

(1) Core Type with Concentric Cylindrical Coils.—Figure 68 represents a cross-section through the two co-axial coils I and II of a core type transformer having a single concentric winding. Leakage lines pass through the space between the two windings, and will confine themselves more or less to the paths indicated in the figure. The flux linked with coil I is confined to a path which is entirely non-magnetic, whereas the flux surrounding coil II passes on one side, through the core of the transformer. Since the magnetomotive force of the two coils will be approximately equal, the leakage lines of coil II will be somewhat greater than those of coil I.

In order to simplify the mathematical expressions, it will be assumed that the currents in the two coils differ in phase by exactly 180 degrees, and give rise to an equal number of ampère-turns, both of these assumptions being approximately fulfilled in practice. It will further be assumed that neither the yoke nor the core has a material influence

* See Kapp, *Transformers*, p. 176.

on the shape of the stray field, and that the latter is symmetrically distributed round the axis of the coils. Since the two fluxes due to the respective coils will, at any instant, have the same direction in the space between the coils, there must be a cylindrical boundary surface, indicated by the line AA_1 (Figure 69), between the two leakage fields. The distance of the boundary surface AA_1 from the inner surface of

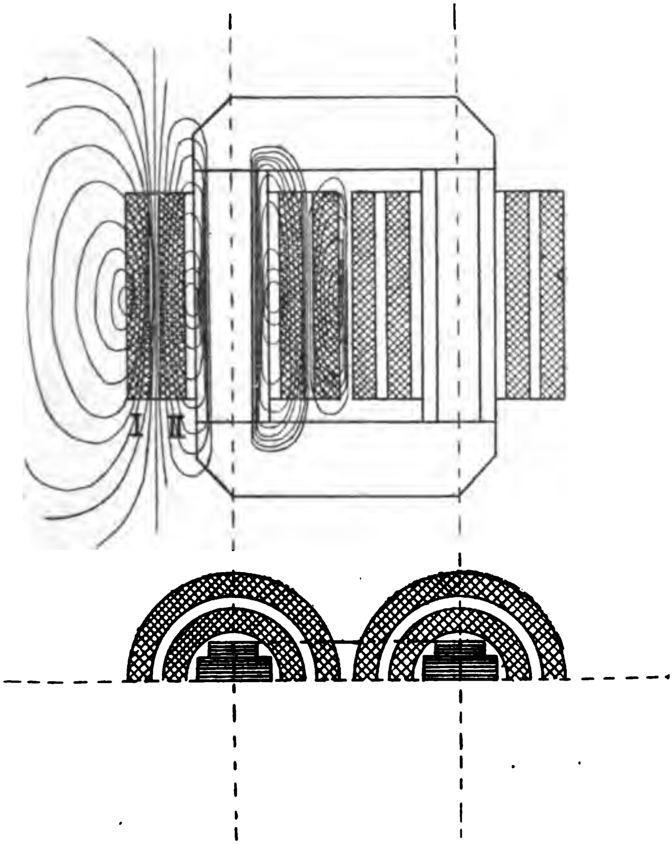


FIG. 68.

the coil I is denoted by Δ', and the distance from the outer surface of the coil II by Δ''. The position of AA_1 cannot be exactly fixed, but from what has been said as to the relative reluctance of the two leakage paths it is obvious that Δ'' must be greater than Δ'.

The magnetomotive force, acting at various points between the edge of the core and the outside of coil I, is shown by the graphs in the lower part of Figure 69. Along the space Δ the magneto-motive force is a maximum, and zero at the inside and outside surfaces

of the coils. Since the flux at any point is proportional to the M.M.F. at that point, the ordinates of the curve also represent, to another scale, the variation of magnetic induction.

If T_2 denote the number of turns in coil II, which is here assumed to be the secondary, T_c the number of turns per centimetre of radial depth of II, and p the perimeter of the boundary surface, then, in a small strip of width dx, and at a distance x from the core, the number of turns $= dT = T_c dx$. The flux linked with these turns is that represented by the shaded area between the ordinates B and B_1 and

$$\cong p\left\{\Delta'' B + (\Delta_2 - x)\frac{(B + B_1)}{2}\right\}$$

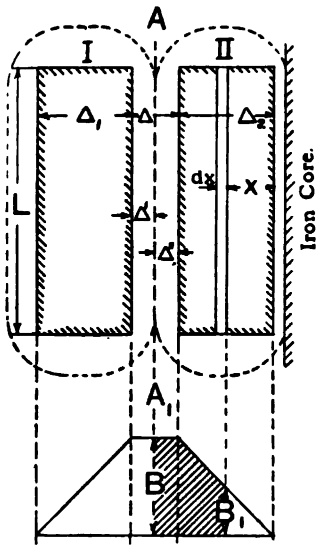


FIG. 69.—Calculation of leakage for core type transformer with single concentric winding.

The corresponding value of the induced E.M.F., expressed in C.G.S. units, is

$$de = 4.44 \sim p\left\{\Delta'' B + (\Delta_2 - x)\frac{(B + B_1)}{2}\right\} T_c dx$$

The total E.M.F. induced in the coil II by the secondary leakage flux is

$$e = 4.44 \sim T_c p \int_0^{\Delta_2}\left\{\Delta'' B + (\Delta_2 - x)\left(\frac{B + B_1}{2}\right)\right\} dx$$

$$= 4.44 \sim T_c p\left\{\Delta'' B \times \Delta_2 + \int_0^{\Delta_2}(\Delta_2 - x)\left(\frac{B + B_1}{2}\right)\right\} dx$$

Since $\frac{B_1}{x}=\frac{B}{\Delta_2}$, *i.e.* $B_1=\frac{x}{\Delta_2}B$

$$\frac{B+B_1}{2}=\frac{B}{\Delta_2}\left(\frac{_2+x}{2}\right)$$

and $(\Delta_2-x)\left(\frac{B+B_1}{2}\right)=\frac{B}{2\Delta_2}(\Delta_2{}^2-x^2)$

$$\text{Hence}\int_0^{\Delta_2}(\Delta_2-x)\left(\frac{B+B_1}{2}\right)dx=\frac{B}{2\Delta_2}\int_0^{\Delta_2}(\Delta_2{}^2-x^2)dx$$

$$=\frac{B}{2\Delta_2}\left(\Delta_2{}^3-\frac{\Delta_2{}^3}{3}\right)=\frac{B\Delta_2{}^2}{3}$$

Since $T_t\Delta_2=T_2=$ the total number of turns in coil II, and B is proportional to $\frac{I_{s\,max}\ T_2}{L}$ which $=\frac{AT_2}{L}$ where AT_2 is the amplitude value of the ampère-turns for II, the secondary leakage E.M.F. may be expressed thus—

$$v_s=4.44\sim T_2 p K_2\frac{AT_2}{L}\left(\Delta''+\frac{\Delta_2}{3}\right)$$

where K_2 is a coefficient the value of which must be determined experimentally. By similar reasoning it can be shown that for coil I the e.m.f. of self-induction is

$$v_p=4.44\sim T_1 p K_1\frac{AT_1}{L}\left(\Delta'+\frac{\Delta_1}{3}\right)$$

Now, the total induced E.M.F. in coil I due to the main flux Φ

$$=E_p=4.44\ T_1\sim\Phi$$

and in coil II

$$=E_s=4.44\ T_2\sim\Phi$$

The ratio of leakage E.M.F. to useful E.M.F. is therefore

$$\frac{v_s}{E_s}=\frac{K_2AT_2}{\Phi}\left(\Delta''+\frac{\Delta_2}{3}\right)\frac{p}{L}$$

$$\frac{v_p}{E_p}=\frac{K_1AT_1}{\Phi}\left(\Delta'+\frac{\Delta_1}{3}\right)\frac{p}{L}$$

Owing to the lower reluctance of the secondary leakage path as compared with that of the primary, K_2 will be greater than K_1. But as it is only the sum of the ratios that is of importance, a mean value $K=\frac{K_1+K_2}{2}$ may be introduced. Since $AT_1\cong AT_2$, the same fraction appears in both equations, and the terms within the brackets may be added together. Thus Δ'' and Δ' are obtained together in the form of their sum Δ, and it is immaterial that their relative values are not known.

The final expression for the inductive drop, expressed as a percentage, may be written

$$V \text{ per cent.} = \frac{100\, v}{E} = \frac{100\, \mathrm{KAT}}{\Phi}\left(\Delta + \frac{\Delta_1 + \Delta_2}{3}\right)\frac{p}{L} \quad . \quad . \quad . \quad . \quad (6)$$

where v = total inductive drop reduced to one circuit.
E = useful E.M.F. of the same circuit.
AT = effective ampère-turns *per core*.
Φ = maximum value of main flux.
Δ = distance between coils (copper to copper) in cms.
Δ_1 = radial depth of coil I in cms.
Δ_2 = radial depth of coil II in cms.
p = mean perimeter of the windings in cms.
L = axial length of coils in cms.

Professor Kapp * has, as a result of a number of experiments, found that the value of K is approximately unity. Hence the percentage inductive drop for a transformer with concentric cylindrical windings is

$$V \text{ per cent.} = 100\,\frac{AT}{\Phi}\left(\Delta + \frac{\Delta_1 + \Delta_2}{3}\right)\frac{p}{L} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In large high-pressure transformers it may sometimes be necessary to reduce the inductive drop by dividing one winding into two concentric cylinders, and sandwiching the other winding between them. When this is done the drop will be approximately half that given by equation 7, that is,—for a double concentric winding, $K = 0.5$.

(2) **Core Type with Coils arranged in alternate Flat Sections, and Shell Type with Interleaved Windings.**—When the windings are arranged in alternate flat sections, as is nearly always the case in transformers of the shell type, and is sometimes adopted in those of the core type, the paths of the leakage fluxes will be as shown in Figures 70 and 71. The same general principles can again be applied to the calculation of the inductive drop, but it must be remembered that, except for the end section, the zero value of B coincides with the plane which passes through the centre of each coil section. The ampère-turns producing the maximum value of B are therefore only one-half the ampère-turns of the section. Only in the end coils, which lie against the two yokes, will the point of zero flux coincide with the outer boundary of the section. In the case of these two end sections the full ampère-turns of the section will act on the leakage path, so that if all the coils are wound with the same number of turns the two end ones will produce twice as much leakage as the intermediate coils.

In order to take these considerations into account equation (7) will be modified as follows. Let n denote the total number of coils on

* *Electrotechnische Zeitschrift* (1898), No. 15.

each core, then $n-2$ coils will have $\frac{AT}{2}$ ampère-turns acting on the leakage path, and the remaining two coils will act with AT ampère-turns. The average ampère-turns are therefore

$$\frac{1}{n}\left\{(n-2)\frac{AT}{2}+2\,AT\right\}=\frac{AT}{2}\cdot\frac{n+2}{n}$$

Substituting this value for AT in equation (7) and remembering that

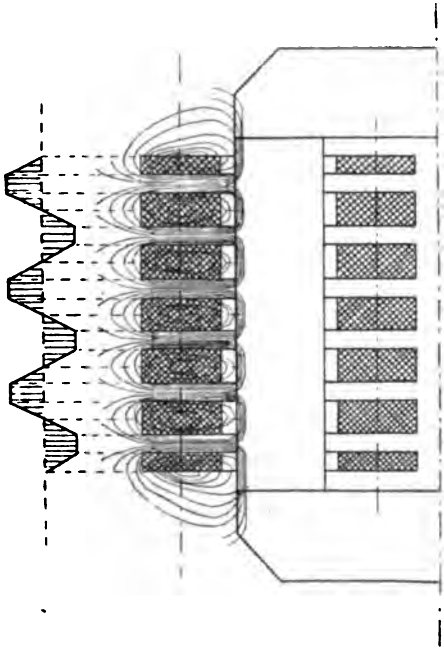


FIG. 70.

only half the values of Δ_1 and Δ_2 must now be taken, the equation for the inductive drop under these new conditions is

$$V \text{ per cent.} = 50\,.\,\frac{n+2}{n}\,.\,\frac{AT}{\Phi}\left(\Delta+\frac{\Delta_1+\Delta_2}{6}\right)\frac{p}{L} \quad . \quad . \quad . \quad . \quad (8)$$

where AT = ampère-turns per section.
L = radial depth of each coil.
p = mean length per turn.

In order that the leakage produced by the end-coils may be the same as that of intermediate ones, the practice is sometimes adopted of winding the end-coils with only one-half the number of turns of the

others. The distribution of B will then be as in Figure 70, and the expression for the inductive drop becomes

$$V \text{ per cent.} = 50\,\frac{n+1}{n} \cdot \frac{AT}{\Phi} \cdot \left(\Delta + \frac{\Delta_1 + \Delta_2}{6}\right)\frac{p}{L} \quad . \quad . \quad . \quad . \quad (9)$$

where Δ_2 and Δ_1 are the depths of the intermediate coils.

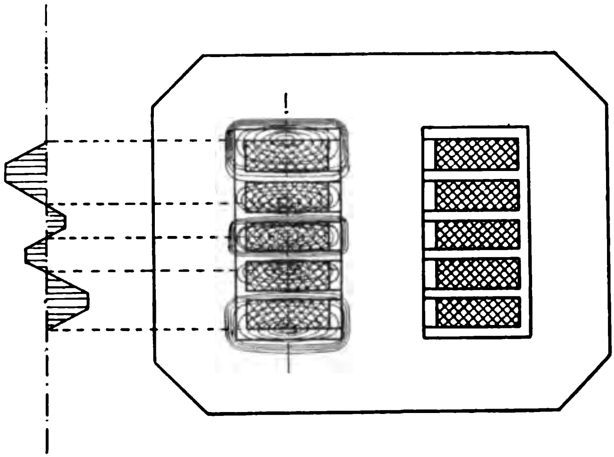


FIG. 71.

The Short-circuit Test.—The regulation triangle (Figure 67) is generally obtained by means of the short-circuit test due to Professor Kapp. The secondary terminals of the transformer are short-circuited by a heavy cable, and a low-voltage alternating current of normal frequency is applied to the primary so as to cause full-load current to circulate in the windings. The applied pressure E divided by the transformation ratio gives the drop in volts at the terminals of the secondary from no-load to full-load; *i.e.*

$$\text{Voltage drop} = V_1 = E \cdot \frac{T_s}{T_p}$$

Further, if I denote the current in the primary when the pressure E is applied, then the equivalent impedance of the transformer expressed in terms of the secondary is

$$Z = \frac{E}{I} \cdot \frac{T_s}{T_p} \text{ ohms}$$

The addition of a wattmeter in the primary circuit obviates the necessity for measuring the resistance drop V_3 by a direct current, since the wattmeter reading divided by the current I gives this quantity directly. Of course to express $\frac{W}{I}$ in terms of the secondary, it must

be multiplied by $\frac{T_s}{T_p}$. Having determined V_1 and V_3, the leakage reactance voltage, as given by the side AB of the triangle in Figure 67, is

$$V_2 = \sqrt{V_1^2 - V_3^2}$$

The short-circuit test is of considerable practical importance, especially in the case of large transformers, as the regulation up to full-load can be determined without actually putting them on load. The only objection to this is that owing to the induction in the core being very low, magnetic leakage is somewhat less than that occurring when normally loaded. When two similar transformers are available, this difficulty can be eliminated and the drop measured with the core normally excited. The connections would be the same as for the Sumpner efficiency test (see Figure 89), where the secondary S_3 of an auxiliary transformer is connected in series with the primary winding of T_1. If V_1 denotes the voltage across the terminals of S_3, when full-load current circulates in the windings, then the voltage drop of each transformer is given by $\frac{V_1}{2}$.

Vector Diagram for Short-circuit Test.—The vector diagram of a transformer with normal full-load current flowing in the

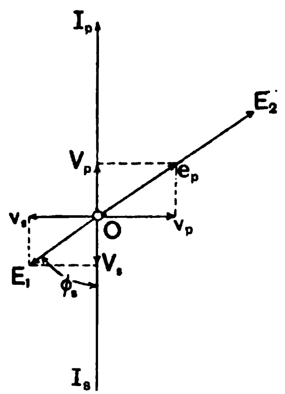


FIG. 72.—Vector diagram for short-circuit test.

short-circuited secondary assumes the form shown in Figure 72. In this case the secondary current OI_s is taken as the line of reference. Owing to the low induction in the core, the magnetising and core loss current will be almost negligible, and the primary current OI_p will be completely in opposition of phase with regard to OI_s. The copper resistance voltages and leakage voltages will be of the same magnitude as before, and have the same lettering as in previous diagrams.

The vector OE_1, obtained by compounding OV_s and Ov_s, represents the secondary induced E.M.F. which leads with respect to OI_s by a large angle ϕ_s. If the secondary were devoid of resistance, ϕ_s would of course be 90 degrees. As regards the primary circuit, the E.M.F.'s balancing the ohmic drop, leakage reactance, and counter e.m.f. are represented by OV_p, Ov_p, and Oe_p respectively. The vector sum of these is given by OE_2, the E.M.F. applied to the primary, which is approximately equal to the sum of the primary and secondary induced E.M.F.'s.

Since the inductive voltages Ov_p and Ov_s are in exact phase opposition, their effect on the terminal voltage will be a maximum. If a transformer be fully loaded under normal conditions and the power factor of the load adjusted to that corresponding to an angle of lag ϕ_s, then the drop will attain its maximum volts OE_1; any variation (either increase or decrease) in the angle ϕ_s will, for full-load current, cause the drop to diminish, as the component of the inductive voltages in phase with the secondary terminal voltage would be less.

Regulation Diagrams.—When the voltage triangle of a transformer corresponding to full-load has been experimentally determined, the voltage drop for any given power factor can be computed by a graphical construction proposed by Kapp. Referring to Figure 73, ABC is the regulation triangle and AI_s the direction of the secondary current. The latter is drawn parallel to the ohmic drop CB, and will be the line of reference. For a load of power factor $\cos \phi_s$, the secondary terminal pressure will be represented by AE_s leading with respect to AI_s by an angle ϕ_s. Assuming that all the reactance is located in the secondary, then the vector sum of AC and AE_s (*i.e.* AE_1) will be the total pressure induced in the secondary at no load.

With a constant supply voltage, AE_1 is also constant, hence when the power factor of the load is varied the locus of the point E_1 will be a circle whose centre is at A. For a given secondary current AC is fixed both as regards magnitude and direction; hence, as the power factor changes the point E_s must also trace out a circle the centre of which is at C. For the power factor to which corresponds the angle of lag ϕ_1, the pressure drop is represented by that part (E_2 E_3) of the radius vector AE_2 which is intercepted between the two circles. Now E_2E_3 approximately $= AB'$, the projection of AC on E_2A produced. Hence the drop for any power factor is given by the component of the total impedance voltage AC which is in phase with the secondary E.M.F.

As the angle of lag ϕ_1 diminishes, so also does that part of the radius vector which is intercepted between the two circles. When $\phi_1 = o$, AE_2 lies along AI_s and the voltage drop is $E_4E_5 = CB$. For a leading current the radius vector will lie to the right of AI_s. The drop gradually diminishes with an increasing angle of lead, vanishes at E_6, the intersection of the two circles, and when the lead of the current

exceeds ϕ_2 degrees the voltage drop will be negative, *i.e.* the terminal pressure rises. When the angle of lag of the current $= \phi_3$ the induced voltage AE_7 is in exact opposition of phase with respect to AC, and consequently the maximum drop at full-load corresponds to a power factor $= \cos \phi_3$.

Another graphical method for determining the voltage drop at any power factor from the characteristic triangle is illustrated in Figure 74. The method is a modification of Kapp's circle diagram. In developing the latter it was shown that the voltage drop is, for a particular power factor, given by the component of the impedance voltage in

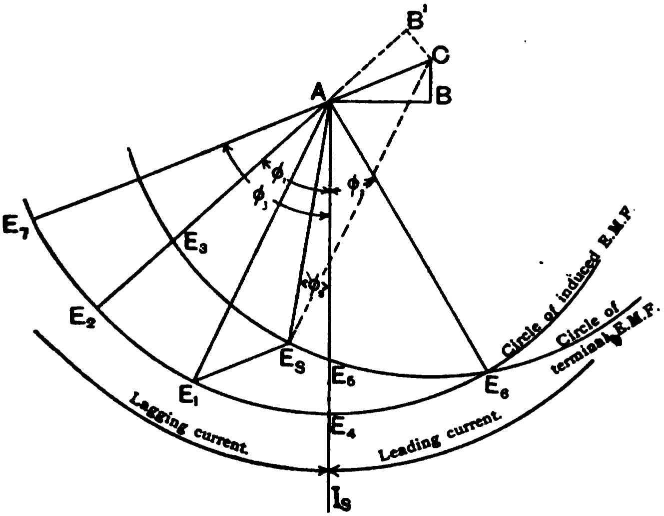


FIG. 73.—Kapp's diagram for transformer regulation.

phase with the terminal E.M.F. Upon this is based the diagram of Figure 74.

The phase of the secondary terminal volts is represented by the line E_1CE_2, along which is measured the variation in secondary volts, a drop in pressure being measured to the right of C and a rise to the left. The characteristic triangle is constructed as follows: Along CE_2 mark off CB to represent the copper resistance drop, and erect from B a perpendicular BA corresponding to the inductive drop. For unity power factor CA, representing the maximum possible drop, will lie along CH, and the drop in terminal volts is given by CB, the projection of CA on the voltage vector E_1CE_2. To obtain the drop corresponding to any power factor $\cos \phi$, the vector CA is moved in a clockwise direction into the position CA′ so as to make an angle of ϕ degrees with the fixed line CH. The component of the short-circuit voltage in phase

with the terminal pressure is then given by CD′, the projection of CA′ on the voltage vector E_1CE_2. Similarly in the case of a leading current of say ϕ_1 degrees, the drop is given by CD″. The maximum drop occurs when the lag of the current is ϕ_2 degrees, and it will be obvious from the diagram that $\phi_2 = ACB$. By drawing lines radiating from C to represent the various positions of the line CA at different power factors, the drop in secondary pressure at full-load for any power factor is then immediately obtained from the figure by inspection.

From this diagram a simple analytic expression can be derived for the secondary drop. Let V_1, V_2, and V_3 denote the equivalent impedance, leakage, and copper resistance voltages = CA, BA, CB,

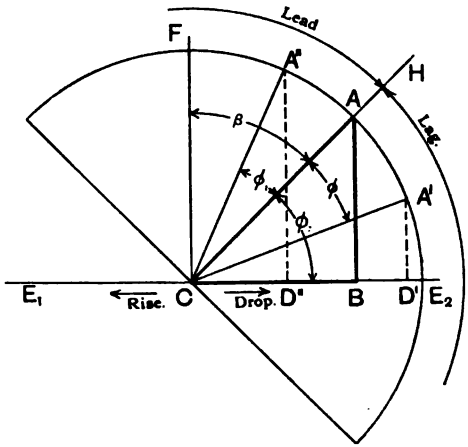


FIG. 74.—Morris and Lister's transformer diagram.

respectively, and β the angle FCH. For any power factor $\cos\phi$ the voltage drop = CD′ = CA′ cos D′CA′ = $V_1 \sin(\phi + \beta)$

$$= V_1 \left\{ \sqrt{1 - \cos^2\phi} \cdot \frac{V_2}{V_1} + \frac{V_3}{V_1} \cdot \cos\phi \right\}$$

$$= V_2 \sqrt{1 - \cos^2\phi} + V_3 \cos\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Example.—From the following data of a 200-K.W. single-phase transformer calculate the regulation corresponding to a power factor of 0.80.

Full-load primary current (I) . .	= 93 ampères.
Wattmeter reading for short-circuit test (W)	= 1350
Primary volts for short circuit test (E) .	= 80
Secondary voltage on open circuit .	= 440
Ratio of transformation $\left(\frac{T_p}{T_s}\right)$	= 5

Equivalent impedance voltage in terms of secondary

$$= V_1 = E \times \frac{T_s}{T_p} = \frac{80}{5} = 16 \text{ volts} = 3.65 \text{ per cent.}$$

Resistance voltage in terms of secondary

$$= V_3 = \frac{W}{I} \cdot \frac{T_s}{T_p} = \frac{1350}{93} \times \frac{1}{5} = 3 \text{ volts} = 0.68 \text{ per cent.}$$

Inductance voltage in terms of secondary

$$= V_2 = \sqrt{16^2 - 3^2} = 15.5 \text{ volts} = 3.5 \text{ per cent.}$$

Regulation for power factor 0.8

$$= 3.5 \sqrt{1 - 0.8^2} + 0.68 \times 0.8 = 2.65 \text{ per cent.}$$

CHAPTER IV

TRANSFORMERS: —LOSSES—EFFICIENCY—HEATING AND COOLING—THREE-PHASE WORKING—SPECIAL TRANSFORMERS

The losses occurring in an alternating current transformer range from 5 per cent. of the rated output in small sizes to about 1 per cent. in those of 2000-K.V.A. capacity or more, and may be classified as follows:—

(1) I^2R or copper loss due to the resistance of the windings.

(2) Iron or core loss due to (*a*) magnetic hysteresis and (*b*) eddy currents in core plates.

Copper Loss.—Let R_p and R_s denote the effective resistances of the primary and secondary windings respectively, then for any values of primary and secondary currents I_p and I the copper loss is

$$W_c = I_p^2 R_p + I_s^2 R_s$$

For a winding having T turns in series the apparent resistance is expressed by

$$R_{app} = \frac{\rho . l . T}{a} \text{ ohms}$$

where ρ = specific resistance of the conductor.
l = mean length per turn.
a = cross-sectional area of conductor.

Taking the specific resistance of copper at 15° C. as 1.7×10^{-6} ohms per cm.,[3] the apparent resistance corresponding to a rise in temperature of T° C. is

$$R_{app} = \frac{1.7 \times 10^{-6} . l . T . (1 + 0.004 T^\circ)}{a} \text{ ohms}$$

The factor 0.004 is the temperature coefficient of resistance for copper in C.G.S. units.

In dealing with alternating current windings the skin effect—*i.e.* the concentration of an alternating current near the surface of the conductor—must be allowed for in calculating the effective resistance. With conductors of small cross-section the increase in resistance due to the non-uniform distribution of the current is almost negligible; but with large conductors the increase may be as much as 30 per

cent. Besides this skin effect there is a further increase in the resistance resulting from the eddy currents induced in the substance of the conductors by the leakage fluxes which close round the windings (see Figure 68). The increase in the resistance due to these causes depends upon (1) the frequency and (2) the size and shape of the conductors, and may be taken into account by multiplying the apparent resistance by a factor k which is greater than unity. The equation to the effective resistance is therefore

$$R_{eff} = \frac{1.7 \times 10^{-6} . k\,l . T . (1 + 0.004\,T^{\circ})}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

The values of the coefficient k must be determined experimentally, and will be somewhat as follows :—

	Small Conductors.	Heavy Conductors.
25~ . . .	1.04	1.08
40~ . . .	1.09	1.11
50~ . . .	1.15	1.20
60~ . . .	1.22	1.30

Iron Loss: Hysteresis.—When iron is placed in an alternating magnetic field—that is, a field produced by a magnetising force which passes through a cycle of values alternating from a $+ve$ to a $-ve$ maximum, but always in the same line of direction—Steinmetz and others have found that the hysteresis loss per cubic centimetre per cycle may be expressed by the empirical formula

$$W_h = \eta\, B_m^x \text{ ergs} = \eta\, B_m^x \times 10^{-7} \text{ joules,}$$

where η = hysteresis constant,

B_m = maximum induction.

The exponent of B_m has a value of about 1.6 for a range of inductions between $B_m = 2000$ and $B_m = 15{,}000$. For very high inductions in the neighbourhood of $B = 21{,}000$ the above empirical formula is no longer applicable, the energy loss attaining a constant value for inductions above $B = 23{,}000$. For practical work it is more convenient to have the loss expressed in terms of the number of cycles of magnetism per second ~ and the volume of the iron. If V denote the cubic centimetres of iron, then the hysteresis loss is expressed by the equation

$$W_h = \eta . \sim . B_m^{1.6}\, V \times 10^{-7} \text{ joules per second, or watts ;}$$

or, since 1 cubic cm. of iron weighs 0.0078 kgs., the loss per kilogramme

$$W_h = 12.8 \times 10^{-6} . \eta . v . B_m^{1.6} \sim \text{watts.}$$

Hence when K_i kilogrammes of iron are subjecied to v magnetic cycles per second the hysteresis loss is given by the equation

$$W_h = 12.8 \times 10^{-6} . \eta . v . B_m^{1.6} \sim K_i \text{ watts.}$$

The value of the hysteresis constant varies with the chemical and physical constitution of the iron, and for transformer plates has a value ranging from 0.001 to 0.002. The lower figure can be obtained with the new "alloyed" irons, but for ordinary transformer plates 0.0015 may be considered an average value. The curve of Figure 75 gives the hysteresis loss per kilogramme at 50 cycles per second as a function of the induction for a sample of iron in which $\eta = 0.0015$.

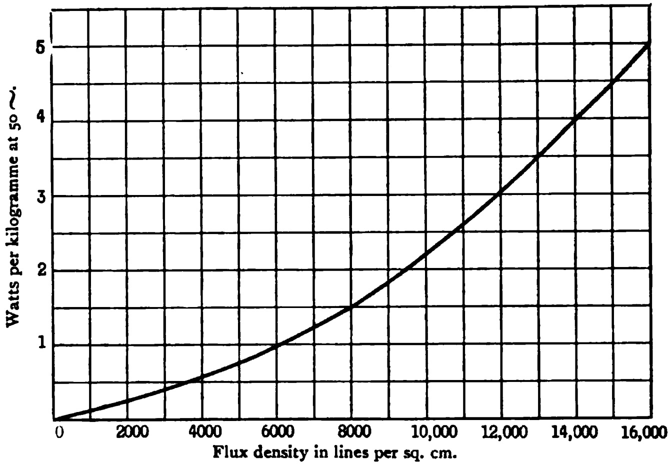


FIG. 75.—Hysteresis loss in ordinary transformer plates at 50 ~.

The loss at any other frequency ~ can be obtained by multiplying the value obtained from the curve by $\frac{\sim}{50}$.

Ageing.—When transformer laminations are subjected to prolonged heating there is a tendency for the hysteresis loss to increase. This deterioration is known as ageing, but as to the real cause producing it there is at present no reliable information. Suffice it to say that some brands of iron exhibit this phenomenon to a very small extent, while in others the deterioration is much more pronounced.

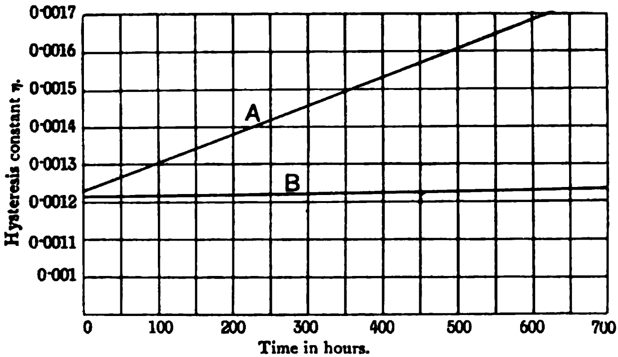
The extent of this deterioration depends upon the composition of the iron and the temperature from which it has been annealed. The increase of hysteresis by ageing is greater with annealed than with unannealed iron. This is clearly shown by the curves in Figure 76.* Two samples of sheet iron were subjected to a temperature of 100° C. for over 600 hours, and the hysteresis constant measured at intervals. Curves A and B are respectively for good annealed and unannealed sheet iron. At the commencement of the test the hysteresis constant of sample A was approximately the same as for sample B, but as the

* *Journ. of the Inst. of Elect. Engineers* (1907), vol. xxxviii. p. 37.

test proceeded it increased rapidly, while the hysteresis constant of B remained approximately constant.

As a general rule the brands of iron used in the construction of electrical machinery begin to age at a temperature of about 85° C.; hence, to avoid the possibility of a large increase in loss through ageing, the best practice is not to allow the iron to attain a temperature higher than 70° C. With this limit of temperature there is now no difficulty in obtaining transformer stampings whose ageing will be within a maximum of 15 per cent.

Again, the hysteresis loss is found to increase when the iron is



A. Annealed iron. B. Unannealed iron.

FIG. 76.—Ageing of sheet iron.

subjected to mechanical pressure, and W. M. Mordey has shown, in one test he performed, that the hysteresis loss increased 21 per cent. when an iron core was subjected to a pressure of 100 kilogrammes per square centimetre. Owing to this injury under pressure it is advisable, in assembling the laminations of a transformer core, to apply as little pressure as possible consistent with good mechanical construction.

Eddy Currents.—A formula for calculating the loss due to the eddy currents which circulate in the core plates may be derived as follows. Suppose Figure 77 to represent the cross-section of a transformer plate at right angles to the magnetic flux, then the alternations in the flux give rise to eddy currents which flow parallel to the boundaries of the plate, as indicated by the paths *ab* and *cd*. The value of the current along the centre line AB is of course zero, and on either side of the latter the currents will flow in opposite directions. Consider the elementary strips *ab* and *cd* whose dimensions in directions parallel to AB and at right angles to the plane of the paper are respectively 1 centimetre. Further, suppose that the strips are of thickness dx and at a distance of x centimetres from AB. The E.M.F. maintaining the

current in this path is that induced round the rectangle *abcd*. Let B_m denote the maximum flux density, then the flux threading this circuit $= B_m \times$ area enclosed by $abcd = B_m\ 2\ x$. This gives rise to an induced e.m.f. $= 4 \,.\, k_1 \,.\, \sim \,.\, B_m\ 2\ x \,.\, 10^{-8}$ volts, where k_1 denotes the form factor* of the e.m.f. wave. Assuming, for the sake of simplicity, that the flux varies according to the simple sine law, then $k_1 = 1.11$, and the E.M.F. induced in the circuit is expressed by

$$E_1 = 4.44 \,.\, \sim \,.\, B_m\ 2x \,.\, 10^{-8} \text{ volts.}$$

Now, if ρ denote the specific resistance of the iron, then the resistance of this circuit is

$$R_1 = \frac{2\rho}{dx}$$

Since the thickness of the plate is always negligible compared with

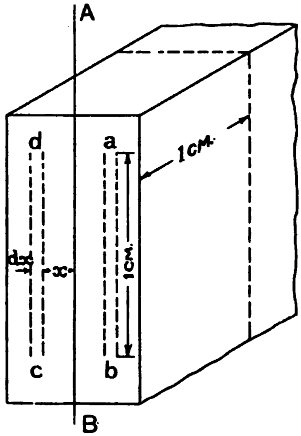


FIG. 77.

the length, the resistance of *ad* and *cb* may be neglected. The watts absorbed by the strips

$$W_1 = \frac{E_1^2}{R_1} = \frac{(4.44 \,.\, \sim \,.\, B_m\, 2x \,.\, 10^{-8})^2 \,.\, dx}{2\rho}$$

$$= \frac{39.6 \,.\, 10^{-16}}{\rho} \sim^2 . \, B_m^2 \,.\, x^2 \,.\, dx$$

If t denote the thickness of the plate in centimetres, then, integrating the latter expression between the limits o and $\frac{t}{2}$, the watts lost in that portion under consideration

$$= \int_0^{\frac{t}{2}} \frac{39.6 \,.\, 10^{-16}}{\rho} \sim^2 . \, B_m^2 \,.\, x^2 \,.\, dx = \frac{39.6 \,.\, 10^{-16}}{\rho} \sim^2 . \, B_m^2 \,.\, \frac{t^3}{24}$$

* See p. 216.

Since the volume of iron in such a strip is t, the loss per cubic centimetre

$$= \frac{1.65 \times 10^{-16}}{\rho} \sim^2 . B_m^{\ 2} . t^2 \text{ watts}$$

For ordinary transformer iron the specific resistance at normal working

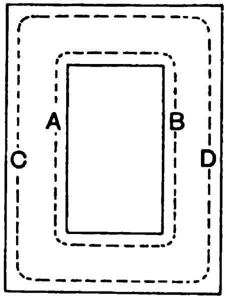


FIG. 78.

temperatures (*i.e.* 60° C.) is 1.2×10^{-5} of an ohm. The eddy current loss per kilogramme will therefore be

$$W = \frac{1.65 \times 10^{-16}}{1.2 \times 10^{-5} \times 0.0078} \sim^2 . B_m^{\ 2} . t^2 \text{ watts}$$

$$= 1.75 \times 10^{-9} \sim^2 . B_m^{\ 2} . t^2 \text{ watts}$$

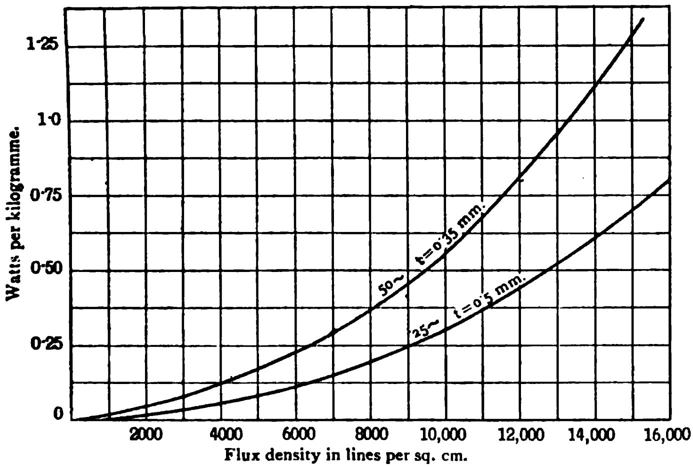


FIG. 79.—Eddy current loss in ordinary transformer iron.

The eddy current loss calculated by this formula is invariably less than that obtained from tests by an amount averaging about 15 per cent. This discrepancy is due to the fact that the formula assumes the flux distribution to be uniform over the cross-section of the core.

This condition is not, however, realised in practice, for, referring to Figure 78, as the length of path AB is less than the length CD, it follows that the flux density at parts A and B will be greater than that at C and D. Now, the eddy current loss being a quadratic function of the flux density, any departure of the latter from a uniform value will give a greater total loss. To meet this contingency the coefficient in the latter formula must therefore be increased by about 15 per cent.; hence if K_i denotes the mass of iron in kilogrammes, the expression for the eddy current loss becomes

$$W_e = 2 \times 10^{-9} \sim^2 . B_m^2 . t^2 . K_i \text{ watts per kilogramme} \quad . \quad (12)$$

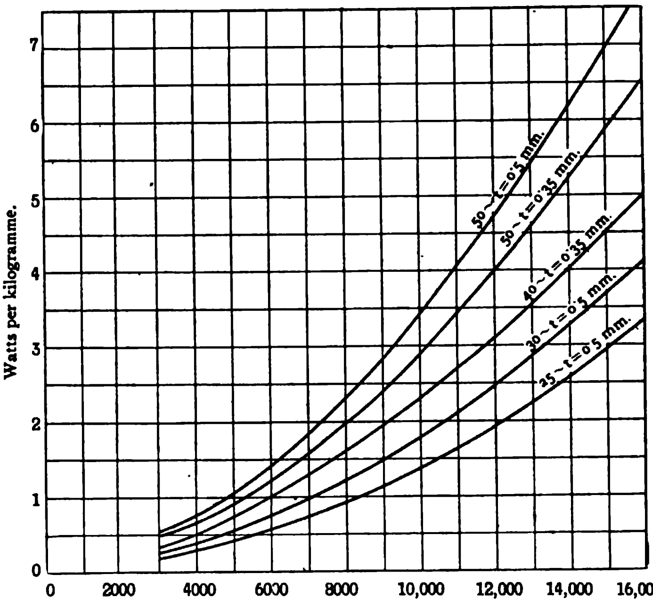


FIG. 80.—Core loss for ordinary transformer iron.

The exact calculation of the eddy current loss is more difficult than that of the hysteresis loss; however, equal exactness is not essential, since the former loss seldom exceeds 25 per cent. of the total core loss, and often constitutes only from 10 to 15 per cent.

Since the eddy current loss is proportional to the square of the thickness of the laminations, it follows that this loss, for a given induction, can be diminished by a corresponding reduction in thickness. Now, for a given flux and gross cross-sectional area the magnetic induction must be increased with the diminishing plate thickness owing to the waste of space taken up by the insulation. The thickness of the latter is independent of the thickness of the plate, and has a

standard value of 0.05 mm. Under these conditions the loss due to hysteresis increases as the thickness of the plate is reduced, while the loss due to eddy currents decreases. There will therefore exist a certain thickness of stamping for which the total core loss is a minimum. This best thickness will depend upon the magnetic properties of the iron, its electrical resistance, and frequency of the magnetic flux. For ordinary transformer iron the usual practice is to employ 0.35 mm. plates for frequencies above 30, and 0.5 mm. plates for lower frequencies.

The curves of Figure 79 give the eddy current loss in ordinary transformer iron for (1) $t=0.35$ mm. and 50 ~, and (2) $t=0.5$ mm. and 25 ~, the ordinates being calculated from equation 12. The curves in Figure 80 show, for the two standard thicknesses of plates, the relation between the flux density in the core and the corresponding total iron loss for a range of frequencies between 25 and 50. For inductions between 8000 and 16,000 lines per square centimetre, the total loss varies according to $B^{1.85}$, whereas for lower inductions the figure is $B^{1.7}$.

Alloyed Iron.—Within recent years a new grade of iron known as "Alloyed Iron" has been introduced by several firms. The loss with this new alloy iron is some 50 per cent. less than that occurring in the ordinary transformer sheets hitherto used, and it has already been adopted to some considerable extent in the construction of transformers. A sample of the iron was analysed in Birmingham University by Professor Turner,* who found it to be of the following composition:—

Carbon	0.03 per cent.
Silicon	3.40 „
Sulphur	0.04 „
Phosphorus	0.01 „
Manganese	0.32 „
Iron (by difference) . . .	96.20 „

Owing to the impurities alloyed with the iron, the permeability at high inductions is somewhat less than that of the older grade of iron. This, however, is not of great practical importance, as the flux-density in the core of a transformer seldom exceeds 13,000 lines per square centimetre.

The hysteresis constant has a value of 0.001, which is 65 per cent. less than that in ordinary grades. The specific resistance of alloyed iron is much greater than the former, and is approximately equal to 2.4×10^{-5} ohm, so that the equation for eddy current loss is

$$W_e = 1 \times 10^{-9} \,.\, \sim^2 \,.\, B_m^2 \,.\, t^2 \,.\, K_t \text{ watts} \quad . \; . \; . \; . \; . \; . \; . \quad (13)$$

The total iron loss per kilogramme obtained experimentally for

* *Journ. of the Inst. of Elect. Engineers* (February 1907), vol. xxxviii.

plates 0.50 mm. in thickness, and at frequencies from 25 to 60 are given by the ordinates of the curve in Figure 81.

To present a comparison of the "ordinary" and "alloyed" grades of iron sheets, Figure 82 has been drawn. The curves are plotted for

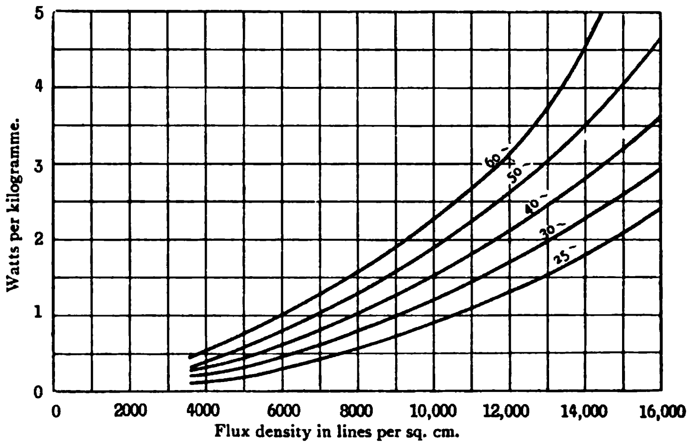


FIG. 81.—Core loss for alloyed iron plates 0.5 mm. thick.

50 periods, and the lower one is that for a modern sheet of special alloy, while the upper curve is for a sheet which was until recently considered as being of first rate quality. Curves showing the loss in both new and old grades of iron as a function of thickness of the plate

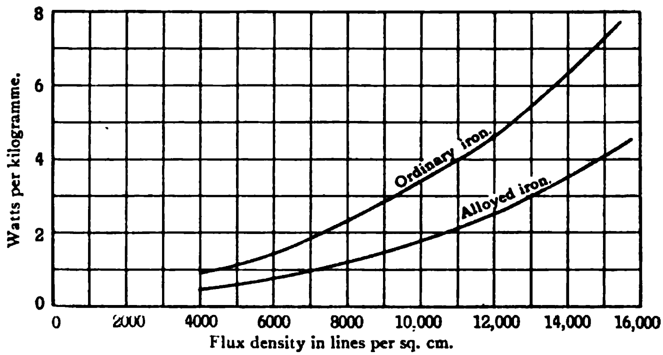


FIG. 82.—Core loss for ordinary and alloyed iron. $t = 0{\cdot}5$ mm. 50 ~.

have been plotted in Figure 83, for B = 10,000, and 50 ~. As compared with ordinary transformer iron, the curve for alloyed iron is flat. The practical importance of this is, of course, very great, as it follows that lamination need not be carried down to anything like the degree necessary with ordinary iron.

With alloyed iron the standard practice for frequencies up to 60~ is to use 0.5 mm. sheets. By increasing the flux density a smaller weight of core will be necessary than that required for a transformer constructed of ordinary iron and having the same loss.

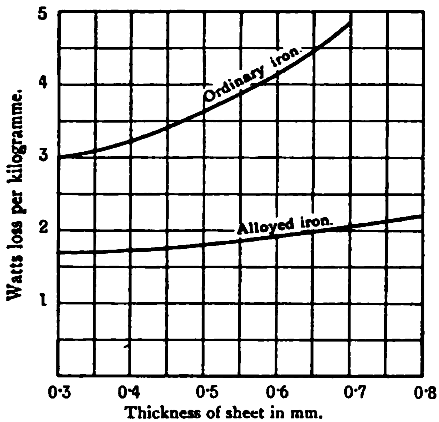


FIG. 83.—Core loss in sheets of various thicknesses for B = 10,000 and 50~.

Core Loss and Shape of E.M.F. Wave.—In deriving equation (12) for the eddy current loss it was assumed that the induced E.M.F. varied as a simple sine law; but if such an assumption is not valid, the formula of eddy current loss per kilogramme must be written in the form

$$W_e = 1.8 \times 10^{-9} \,.\, k_1^2 \,.\, \sim^2 \,.\, B_m^2 \,.\, t^2 \,.\, K_i \text{ watts},$$

where k_1 denotes the form factor of the induced E.M.F. The total iron loss per kilogramme will then be expressed by

$$W_i = K_i \left\{ 12.8 \times 10^{-6} \,.\, \eta \,.\, B_m^{1.6} \,.\, \sim + \, 1.8 \times 10^{-9} \,.\, k_1^2 \,.\, \sim^2 \,.\, B_m^2 \,.\, t^2 \right\} \text{ watts}.$$

Now the primary counter e.m.f., E_p is related to the maximum induction B_m in the core by the equation

$$E_p = 4 \,.\, k_1 \,.\, T_p \,.\, \sim \,.\, A.B_m \,.\, 10^{-8} \text{ volts},$$

where A denotes the net area of cross-section of iron in the core. Substituting the value of $B_m = \dfrac{E_p \,.\, 10^8}{4 \,.\, k_1 \,.\, T_p .\sim. A}$ in the former equation, the total iron loss becomes

$$W_i = K_i \left\{ 12.8 \times 10^6 . \eta . \sim \left(\frac{E_p \times 10^8}{4 \,.\, k_1 \,.\, T_p \,.\sim.\, A} \right)^{1.6} + \frac{1.12 \times 10^6 . t^2 . E_p^2}{T_p^2 \,.\, A^2} \right\} \text{ watts}.$$

This last equation shows that the value of the hysteresis loss depends upon the wave form of the primary E.M.F., whereas the eddy current loss is independent of the form factor. The total core

loss of a transformer is therefore not an absolutely fixed quantity, but, for a given R.M.S. value of applied pressure, varies nearly inversely as the form factor. Now, in connection with the E.M.F. equation for alternators, it is shown that for a given R.M.S. value the form factor of a peaked wave is greater than for the corresponding sine wave, while a flat-topped wave gives a smaller form factor than a pure sine curve. Hence, for minimum iron losses a peaked E.M.F. wave is preferable. A wave having a high form factor would, however, put a greater strain on the insulation than a sine wave of the same effective value. As a compromise between minimum iron loss and minimum insulation stress, the primaries of transformers should be connected to supply mains, the E.M.F. wave form of which deviates but little from a pure sine curve.

Testing of Transformer Iron.—Before being employed for the construction of transformers, consignments of iron plates should be tested for eddy current and hysteresis loss. The usual method of testing is to make up a sample of given weight, surround it with a coil having a known number of turns, and measure the iron loss when the core is excited to a given induction. Several instruments have been devised for this test, but the most reliable one is that due to Professor Epstein of Frankfurt. It is this apparatus which is recommended in the Standardisation Rules on Iron Testing, drawn up by the German Association of Electrical Engineers.

In Epstein's apparatus, shown in Figure 84, the sample laminations are made up into four cores, each having a length of 500 mm. and a cross-section of 30 mm. by about 25 mm. and a mass of 2.5 kilogrammes. In constructing the cores the individual laminations are insulated from one another by tissue paper, and each bundle of laminations bound together with tape. The four cores are fitted together so as to form a square, and are secured in position by wooden clamps placed at each corner. To avoid any additional eddy current loss which might be caused by contact between two sets of plates, a sheet of press-spahn, 1.5 mm. thick, is inserted at each butt joint.

Each core is placed in a fixed magnetising coil C, which consists of 150 turns of copper wire having a cross-sectional area of 14 square mm., and is wound on a press-spahn tube P of internal dimensions 38 by 38 mms., and 435 mm. long. The four coils are connected in series across the terminals, the apparatus having a resistance of about 0.18 of an ohm.

To measure the iron loss of the samples, the instrument is connected to a source of suitable pressure and frequency, a wattmeter, ammeter, and voltmeter being inserted in the circuit. The E.M.F. wave form of the supply circuit should as nearly as possible be a sine curve, and special care must be observed to select a wattmeter which is correct at low power factors. The core loss for various flux densities

is obtained by varying the pressure applied to the terminals of the windings, the corresponding flux density B_m being obtained from the equation $E = 4.44\ T \sim B_m \times A \times 10^{-8}$ volts, where A is the area of cross-section of the iron and E the vector difference between the applied E.M.F. and the IR drop. The latter, however, is so small that the component in phase with the counter e.m.f. may be neglected. As the power to be measured is very small, precautions should be taken to eliminate the losses occurring in the instruments, the voltmeter switch being left open when observations are made by the wattmeter and the ammeter.

The effect of inserting the insulations at the butt joints will be to increase the magnetising current, but there will be no appreciable

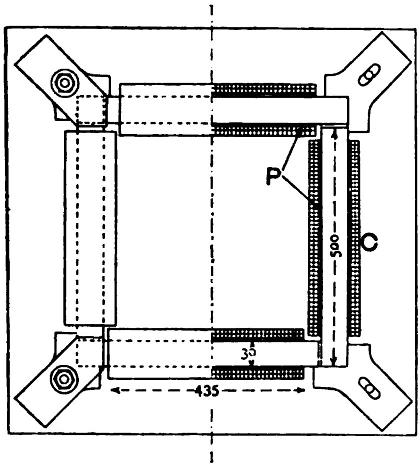


FIG. 84.—Epstein's iron testing apparatus.

difference in the power absorbed. From a knowledge of the mass of iron in the sample being tested, the loss per kilogramme at any stated induction and frequency can be computed from the wattmeter readings and curves plotted, such as are shown in Figures 80 to 83.

In the German rules the watts lost per kilogramme for $B_m = 10{,}000$ lines per square centimetre and $50 \sim$ is termed the "Figure of loss," and since the eddy currents decrease with rising temperature this value must be determined at a definite temperature, which is fixed at 30° C. By the adoption of this standard the relative merits of various grades of iron can be easily compared. For ordinary transformer iron and alloyed iron of 0.5 mm. thickness the figures of loss are 3.4 and 1.8 watts respectively.

Epstein's apparatus may also be employed to discriminate between the losses due to hysteresis and eddy currents at constant induction.

This is done by gradually reducing the frequency and observing the corresponding values of core loss, the flux density being maintained at the desired value by decreasing the voltage so that the ratio of volts to frequency is kept constant. The quotient—watts ÷ frequency—when plotted as a function of the frequency, gives the straight line AB of Figure 85. The hysteresis loss per cycle is represented by the

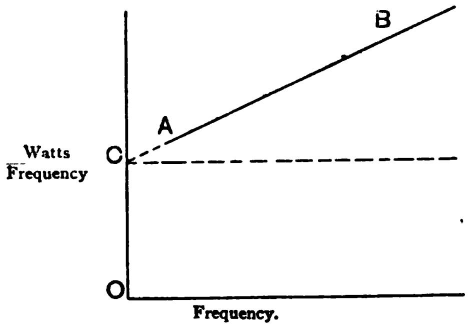


FIG. 85.

ordinate OC, C being the point where AB, produced backwards, cuts the ordinate reference axis.

EFFICIENCY

Let W denote the watts output from the secondary of a transformer, and W_c and W_i the watts expended in the copper and iron losses respectively, then the efficiency expressed as a percentage is given by the equation

$$\text{Per cent. efficiency} = \frac{W}{W + W_c + W_i} \times 100$$

The form of the curve showing the variation of efficiency with the output will depend upon the relative value of the various losses. Owing to the small resistance drop, the counter e.m.f. of the primary is approximately equal to the applied pressure. Now, if, as is normally the case, the latter be maintained constant, the counter e.m.f. must also be approximately constant. The constancy of the latter involves a constancy of the magnetic flux Φ (equation (2), page 64), so that the core loss, which depends on the value of Φ, will remain nearly constant at all loads. Theoretically there will be a slight decrease with increasing load, caused by the increasing resistance drop.

The efficiency, which is zero at no-load, increases with the output as shown in Figure 86, attaining a maximum for that output at which the copper and iron losses are equal. That maximum efficiency corresponds to equality of copper and iron losses may be proved as

follows. Let $E_sI_s \cos\phi$ denote the output from the secondary, I^2R the total copper loss, and C the constant or iron loss. The efficiency is then

$$\eta = \frac{E_sI_s \cos\phi}{E_sI_s \cos\phi + I^2R + C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The efficiency will be a maximum when $\frac{d\eta}{dI_s} = 0$

Differentiating this equation with respect to I_s, and placing the differential coefficient $= 0$,

$$E_s \cos\phi\ (E_sI_s \cos\phi + I^2R + C) = E_sI_s \cos\phi\ (E_s \cos\phi + 2\ IR),$$
$$\text{or } C = I^2R,$$

i.e. the efficiency is a maximum when the sum of the copper losses equals the core loss. Transformers which are operated continuously

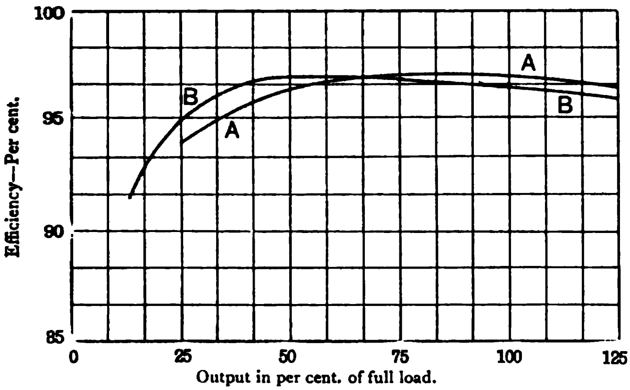


FIG. 86.—Efficiency curves for 50-K.V.A. transformer.

at full-load should therefore be designed so that the iron loss equals the full-load copper loss.

All-day Efficiency.—Transformers for lighting loads usually have their primaries permanently connected to the supply mains, and are generally operated at less than quarter full-load during the greater part of the twenty-four hours. For such work the efficiency at full load is only of secondary importance as compared with the all-day efficiency which is expressed by

$$\eta_a = \frac{\text{K.W.-hours output per 24 hours}}{\text{K.W.-hours output per 24 hours} + \text{K.W.-hours to cover losses}}$$

The copper loss, being proportional to the square of the secondary current, will be small at light loads. The iron losses, on the other hand, are of the same magnitude at all loads, and will have a greater influence on the all-day efficiency than the copper loss. It is therefore desirable to reduce the iron losses to the commercial limit.

As a concrete example, consider two 50-K.W. transformers A and B, having a full-load efficiency of 97 and 96·5 per cent. respectively. Suppose that at full-load the copper and iron losses of transformer A each equal 750 watts, while in transformer B the full-load iron loss and copper loss are 500 and 1250 watts respectively. Further, suppose that the average output per 24 hours for each transformer = 250 kilowatt-hours, equivalent to full-load for 5 hours and no-load for 19 hours. The all-day efficiency of the two transformers will then be as calculated below.

	Transformer A.	Transformer B.
Full-load efficiency	97 per cent.	96.5 per cent.
Total energy output	250 K.W.-hours.	250 K.W.-hours.
Iron loss (24 hours)	18 ,,	12 ,,
Copper loss (5 hours)	3.75 ,,	6.25 ,,
K.W.-hours input	271.75 ,,	268.25 ,,
All-day efficiency. . . .	92 per cent.	93.3 per cent.

The transformer giving the higher full-load efficiency has thus a smaller all-day efficiency. To increase the latter it is necessary to diminish the iron losses. With a fixed frequency this can be attained by adjusting either the area of cross-section of the iron or the value of flux density. As either procedure necessitates a larger number of primary and secondary turns, the copper losses will consequently be increased. There is, however, a limit to the latter set by considerations of regulation—which will not admit of too great a copper loss at full-load, on account of the attendant drop of secondary pressure.

Lighting transformers should be designed to give a high efficiency from 10 per cent. of full-load upwards, the efficiency attaining its maximum value at about one-half full-load. Transformers for power work are nearly always operated at, or near, full-load, and the copper loss should then be approximately equal to the core loss. This will give an efficiency curve which is not quite so flat as that for a lighting transformer. That such is the case will be clear from Figure 86, which gives the efficiency curves for the two transformers A and B mentioned above.

The maximum efficiencies which may be obtained, under ordinary commercial conditions, with transformers of outputs ranging from 1 to 800 K.W., are given by the ordinates of the curves in Figure 124, page 171.

Efficiency Tests.—(1) *Direct Method.*—The direct method of determining the efficiency of a transformer is by connecting the secondary to a suitable load, and measuring by the aid of two watt-

meters the primary input W_p corresponding to a secondary output W_s. The percentage efficiency is then expressed by η per cent. $= 100 \frac{W_s}{W_p}$. With a non-inductive load the secondary output could be measured by an ammeter and a voltmeter, but for obtaining the primary input a wattmeter must always be used.

(2) *Measuring Losses separately.*—If large units be tested in the above manner the energy absorbed would render the tests very expensive. By measuring the losses separately and computing the efficiency from the known output, the power required to carry out the efficiency test will be reduced to a minimum. This method is therefore the one invariably adopted in practice. Again, since a small percentage error in measuring the losses will affect the accuracy to a much smaller extent, it follows that the efficiency by this method can be determined with a greater degree of accuracy than when it is derived from measurements made upon the total input and output.

The *core loss* is determined by connecting the low voltage winding

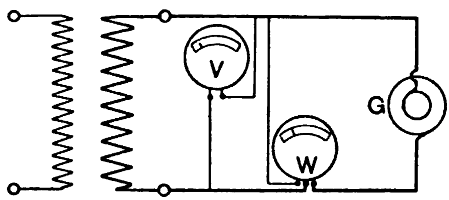


FIG. 87.—Diagram of connections of core loss test.

to a suitable supply G, a voltmeter V, and a wattmeter W being inserted in the circuit as shown in Figure 87. With the high-tension winding on open circuit and the primary E.M.F. adjusted to its normal value E_n, the core loss is given by the wattmeter reading W (neglecting the energy consumed by V). Theoretically this also includes the copper loss due to the no-load current in the low tension winding, but the loss due to this cause is almost negligible.

The *copper loss* obtained by sending a direct current through each winding and observing the drop of potential is generally found to be less than the actual value. This is due to the fact that under normal conditions, eddy currents are induced in the solid copper conductors and the power so consumed must be added to the copper loss. To include this eddy current loss, the best plan is to measure the copper loss as follows. Referring to Figure 88, the secondary terminals are short-circuited by heavy cable X, and the primary connected to a low-voltage supply G of the required frequency, an ammeter A, voltmeter V, and wattmeter W being connected to the circuit. The current in the windings is controlled by the supply voltage, and the copper losses

corresponding to various currents are then obtained from the wattmeter. The measurement of the I^2R loss should, if at all convenient, be made at the end of a full-load run, the temperature then being the same as

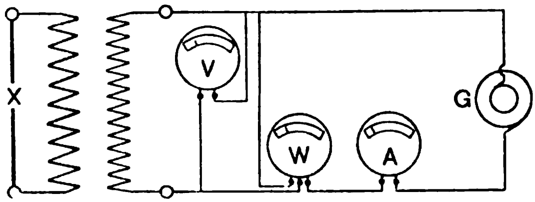


FIG. 88.—Diagram of connections for copper loss test.

that under normal working conditions. Having obtained the core and copper losses as explained above, the efficiency of the transformer at various loads may be computed as set forth below.

EFFICIENCY TEST OF A 200-K.V.A. TRANSFORMER.

Primary volts = 2200. Secondary volts = 440. Core loss = 1040 watts.

Primary Current.	K.W. Input to Primary.	I^2R Watts (from Test).	Core Loss Watts.	K.W. Output from Secondary.	Efficiency per Cent.
110 amps. . .	240	1860	1040	237	98.5
92 ,, . .	202.4	1350	1040	200	98.5
60 ,, . .	132	560	1040	130.4	98.5
40 ,, . .	88	250	1040	86.7	98.0
20 ,, . .	44	62	1040	42.9	97.5
10 ,, . .	22	15	1040	20.9	95.0

Sumpner's Test on two Transformers.—When two similar transformers, built to the same design, are available, the usual practice is to determine the efficiency by a method proposed by Dr. Sumpner. The special feature of this test, which is analogous to the Hopkinson method of testing direct-current machines, is that, as each of the two transformers being tested supplies power to the other, the energy required from the supply circuit is only that necessary to make up the losses in the two transformers, even though they are fully loaded. The method of carrying out the test is as follows.

Referring to Figure 89, the low-tension windings of the two transformers T_1 and T_2 are connected to an alternating current supply G.

The high-tension windings are connected in opposition, so that the pressure generated in each tends to send a reverse current through the other. If the pressure applied to both the low-tension windings be the same, no current will flow in the circuit formed by the high-tension windings. In order that a current may flow through these windings, an auxiliary transformer T_3 is connected in series with the low-tension winding of T_1, in such a manner that it adds to the supply voltage and causes the pressure applied to T_1 to exceed that to T_2. The secondary induced E.M.F.'s will therefore be unequal, and the current circulating through the high-tension coils will be in direct proportion to the secondary voltage of T_3. The voltage of the auxiliary transformer may be varied by a resistance or impedance coil R, connected in series with the primary winding, which is connected to the supply circuit.

When the two low-tension coils are connected direct to the supply,

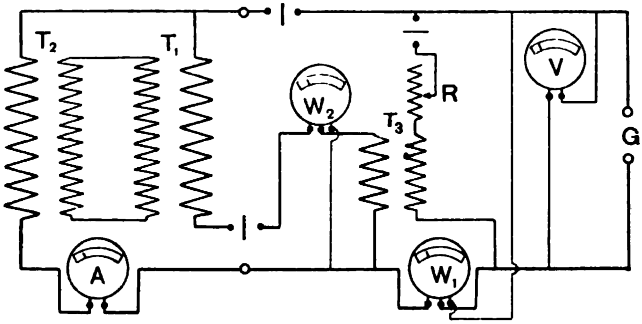


FIG. 89.—Diagram of connections for Sumpner's efficiency test.

the power taken from the latter is equal to the sum of the iron losses in the two transformers, and may be measured by the wattmeter W_1. To load the transformers, the primary of T_3 is connected to the supply, and by adjusting R any desired current may be made to flow in the transformers T_1 and T_2.

The power supplied by the auxiliary transformer T_3 is that corresponding to the copper loss occurring in both the transformers, and is measured on the wattmeter W_2, and should be obtained for values of primary current up to about 25 per cent. overload.

The method of calculating the efficiency of each transformer from the test results is set forth below, half the total losses being allocated to each transformer. The data relates to two 235-K.V.A., 6450/390 volts, 25-cycle single-phase transformers, the design data of which is given at the end of Chapter V.

EFFICIENCY OF TWO 235-K.V.A. TRANSFORMERS BY SUMPNER'S TEST.

Combined iron loss = 3.5 K.W. Iron loss for each transformer = W_i = 1.75 K.W.
Secondary volts = 390 = E_s.

Per cent. of Full-Load.	Secondary Current I_s.	Secondary Output at Unity P.F. $I_sE_s = W$.	Half Total Copper Loss = Copper Loss for each Transformer = W_c.	Total Loss $W_i + W_c$.	Efficiency of each $= \frac{W}{W+W_i+W_c}$.
10 . .	60 amps.	23.5 K.W.	0.05 K.W.	1.80 K.W.	92.0 per cent.
25 . .	150 ,,	58.5 ,,	0.15 ,,	1.90 ,,	96.5 ,,
50 . .	300 ,,	117 ,,	0.50 ,,	2.25 ,,	97.5 ,,
75 . .	450 ,,	175 ,,	1.3 ,,	3.05 ,,	98.0 ,,
100 . .	600 ,,	235 ,,	2.4 ,,	4.15 ,,	98.0 ,,
125 . .	750 ,,	290 ,,	4.0 ,,	5.75 ,,	98.0 ,,

HEATING AND METHODS OF COOLING

When a transformer or other electrical machine is loaded, the energy expended in the iron and copper losses is converted into heat, with the result that there is a rise in temperature of the various parts

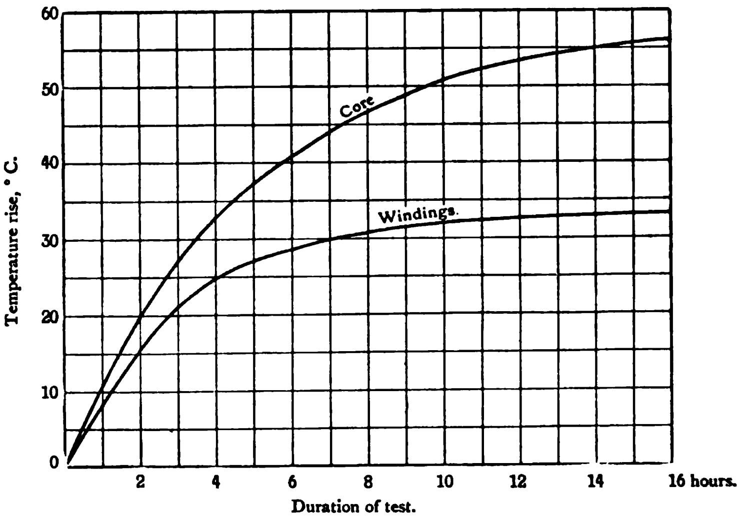


FIG. 90.—Heating curves of 7.5 K.V.A. air-cooled transformer of the core type.

above that of the surrounding air. When the load is first applied there will be very little heat radiated, so that the temperature rises rapidly. As the heating proceeds the transformer begins to give out heat through convection, conduction, and radiation, with the result

that the temperature increases less rapidly than at the initial stage. For a given expenditure of energy the temperature will ultimately attain a steady value, this condition being reached when the rate of generation of heat is equal to the rate of cooling. The steady temperature to which any part of the machine will rise depends upon the magnitude of the losses, the extent and nature of the cooling surfaces provided, and the facilities afforded for cooling. The curves of Figure 90 show for a 7.5 K.V.A. core type transformer the temperature rise of the core and windings as a function of time, and these may be taken as typical for sizes up to about 100 K.V.A.

When the heated body is completely homogeneous and the cooling of its whole outer surfaces is uniform, the curve of temperature rise is an exponential (Fig. 91), the ultimate temperature being approached asymptotically. Now, if T_r° denote the maximum or final temperature

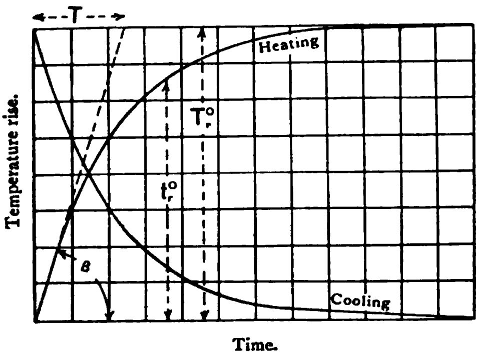


FIG. 91.—Heating and cooling curves.

rise, and t_r° the value of the temperature rise at the end of any time t, then the relation between these quantities may be represented by the equation

$$t_r^\circ = T_r^\circ \left(1 - \epsilon^{-\frac{t}{T}}\right)$$

where $\epsilon = 2.718$ is the base of the Naperian logarithms, T is the time constant of the equation, and is a measure of the time in which the total mass being heated would reach the final temperature if there were no heat lost by conduction, convection, and radiation due to rise of temperature.

By differentiating the above equation with respect to t, and putting $t = 0$

$$\frac{dt_r^\circ}{dt} = \frac{T_r^\circ}{T} = \tan \beta. \quad \text{(Fig. 91.)}$$

Hence at the commencement of heating the slope of the temperature

curve is dependent only on the heat generated and the thermal capacity of the body. From the latter expression it follows that

$$\frac{T}{T_r^\circ} = \frac{\text{Heat capacity of the body}}{\text{Heat generated in the body}}$$

and since, when a steady temperature is reached, all the heating losses are dissipated, the final rise in temperature is expressed by

$$T_r^\circ = \text{const.}\ \frac{\text{Heat generated}}{\text{Cooling surface}} = K \cdot \frac{\text{Watts lost}}{\text{Cooling surface}}$$

where K denotes the heating coefficient.

Since the temperature curve is an exponential function, theoretically it will take an infinite time to attain the maximum value T_r°; but at any time short of this the temperature rise will be a fraction of that maximum, namely, that fraction indicated by the factor $(1 - \epsilon^{-\frac{t}{T}})$. For all practical purposes the machine may be assumed to have reached its final temperature when the actual temperature rise at time t is within from 1.5 to 1.0 per cent. of the maximum rise, this being well within the limit of error that is possible when making the temperature observations. When the temperature rise t_r° is within x per cent. of the final T_r° value, then

$$100\ \frac{T_r^\circ - t_r^\circ}{T_r^\circ}. = x$$

and $\left(1 - \frac{t_r^\circ}{T_r^\circ}\right) = \epsilon^{-\frac{t}{T}} = \frac{x}{100}$. Hence $x = \frac{100}{\epsilon^{\frac{t}{T}}}$

The values of x as calculated from this equation are tabulated below, and it will be observed that after a time 4 T the temperature is within 2 per cent. of its theoretical final value, and may therefore be regarded as constant. The time during which a transformer must be fully loaded, before it will attain its approximate final temperature, ranges from about 12 hours in small sizes of 100 K.V.A. or less to 24 or 48 hours in the largest sizes.

x per cent.	10	5	4	3	2	1	0.5
Time . . .	2.3 T	3 T	3.22 T	3.5 T	3.9 T	4.6 T	5.3 T

Maximum Permissible Temperature.—The maximum temperature to which any part of a transformer may attain must be such that the insulation does not deteriorate, and that no appreciable increase occurs in the core loss due to ageing of the iron. Should the winding be maintained at a temperature of 90° C. or upwards, the insulation rapidly deteriorates both as regards its electrical and

mechanical properties. Cotton and other fibrous materials used for insulating the electrical conductors become charred, and crumble away when subjected to the least mechanical strain, so that, although the insulation may still remain comparatively high, the possibility of a breakdown is considerably increased. Some engineers have suggested the exclusive use of such insulating material as mica or asbestos, which may be subjected to as high a temperature as 120° C. without the slightest deterioration. But suppose the windings were allowed to attain such a high temperature, the core would soon become heated to about the same degree. This would accelerate the ageing of the iron, and cause an appreciable increase in the iron loss.

To reduce to a minimum the deterioration of iron and insulation, the maximum temperature to which any part of a transformer may attain, when working continuously at full-load, should not exceed 80° C. at the most. To be within safe limits, the usual practice in this country is to fix 70° C. as the maximum.

The permissible *temperature rise* will depend on the temperature of the air in the transformer house, which in Britain has an average value of about 20° C. To specify a maximum temperature rise of 50° C. when working at full-load has now become the standard practice. Of course, when transformers are built for use in tropical climates, where the atmospheric temperature may, at certain periods, be close upon 40° C., a much lower temperature rise must be insisted upon, so that under normal working conditions the heating does not exceed the safe limits mentioned above.

Cooling by Natural Draught.—Since there are no rotating parts in a transformer, the cooling facilities are very poor in comparison with the heat generated, and the output is entirely limited to that current which the winding will carry without the permissible temperature rise being exceeded. Small transformers for outputs up to about 20 or 30 K.V.A. offer sufficient cooling surface to dissipate their heat by the natural process of radiation, and, if installed in dry situations may, as a protection against mechanical injury, be enclosed in perforated sheet-metal cases.

Theoretically, transformers cooled by natural draught could be designed for any size. Practically, however, the limits of output for which the natural draught type can be built are very narrow. The reason for this is that, as the size of the transformer increases, the cooling surface per unit volume decreases rapidly, and accordingly also the iron and copper densities which can be allowed without undue heating under operating conditions. Hence, in order to keep down the weight of active material, transformers for larger outputs than 30 K.V.A. must be cooled by some artificial means.

Oil-immersed, Self-cooling.—For outputs up to about 500 K.V.A. adequate cooling is obtained by placing the transformer in a

large iron tank filled with a special insulating oil. As the latter is a better heat conducting medium than air, heat will be transferred from the transformer to the walls of the tank—and from thence radiated to the external air—more easily than would be the case if there were no oil. For this purpose a mineral oil, obtained by the fractional distillation of petroleum, and having a flashing-point of not less than 180° C., should be employed The oil must be free from moisture, sulphur compounds, and all traces of acid or alkali, and should not show an evaporation of more than 0.2 per cent. when heated to 100° C. for eight hours. Vegetable oils should never be used, as a considerable deposit of carbon takes place when they are subjected to the normal temperature of the transformer.

Besides increasing the thermal capacity, the use of oil preserves the insulation, keeping it soft and pliable, and prevents oxidation by air. As an insulator, oil has a disruptive strength several times that of air, and is of particular value for the insulation of exposed surfaces, which in air, under very high voltage strains, act as conductors. The fluidity of the oil is also an advantage, in that, should a disruptive discharge take place, the oil immediately seals up the fault and offers the same insulating strength after the discharge as before. The oil-insulated transformer is therefore used for all high-tension work, the natural draught type being confined to very small sizes and low voltages.

With oil-immersed transformers special attention should be given to the construction of the containing tanks, as they must be entirely impervious to oil. For small sizes it is customary to use cast-iron tanks galvanised or enamelled. For larger sizes the sides are generally of corrugated sheet iron, soldered to sheet iron bottoms and secured in deep grooves to top, bottom, and castings. Another practice is to make the tank sides of plain sheet iron and to solder cooling ribs to the outside. No joints, unless soldered or welded, are safe against hot oil, nor will painting nor any such treatment permanently ensure oil-tightness. For the details of tank construction the reader is referred to the examples of complete transformers given at the end of the next chapter.

Before filling in the oil, special care should be taken to drive off all moisture from the core and windings. This can be accomplished by heating the transformer either in an oven maintained at a temperature of about 60° C., or by sending a current through the windings. In oil-insulated types there is the possibility of the formation of an explosive mixture of oil and air vapour, which in the event of ignition might cause considerable internal pressure; but experience shows that trouble from this case is very remote.

Oil-immersed, Water-cooled.—With very large transformers, from about 500 K.V.A. upwards, the cooling effect obtained from an oil-filled tank would, except in the case of one having abnormal

dimensions, be inadequate. Where there is an abundant supply of cheap water at a low temperature the cooling may be supplemented by water circulation. The oil-immersed, water-cooled type is generally built with a cooling worm fitted in the top part of the tank, and through which the cooling water circulates. A transformer cooled in this manner is illustrated in Figure 92, the cooling worm, according to the output, consisting of from 5 to 50 turns of thin-walled brass or iron piping. The oil, as it becomes heated, rises from the centre parts and, after giving up some of its heat to the water, sinks again to the bottom, passing close to the sides of the tank, to which it gives up the remainder of the heat.

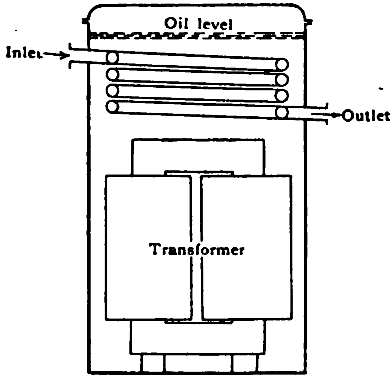


FIG. 92.—Oil-immersed, water-cooled transformer.

The quantity of water required for cooling is derived as follows:—
Let t_1 denote the temperature of the water at inlet, and t_2 that at outlet. Then since one cubic metre of water weighs 10^6 grammes, the quantity of heat dissipated per cubic metre of water

$$= 10^6 (t_2 - t_1) \text{ calories} = 4.2 \times 10^6(t_2 - t_1) \text{ joules.}$$

Since the number of cubic metres required per kilo-joule $= \dfrac{1000}{4.2 \times 10^6(t_2 - t_1)}$, the volume required per second per kilowatt dissipated

$$= \frac{1000}{4.2 \times 10^6(t_2 - t_1)} = \frac{2.4}{(t_2 - t_1) \times 10^4} \text{ cubic metres.}$$

The standard practice is to allow for a 5° rise in the temperature of the cooling water from inlet to outlet; hence, for each kilowatt to be dissipated the quantity of water required is

$$\frac{2.4}{5 \times 10^4} = 4.8 \times 10^{-5} \text{ cubic metres per second}$$
$$= 4.8 \times 10^{-2} \text{ litres per second}$$
$$= 40 \text{ gallons per hour.}$$

Cooled by Air-Blast.—When cheap water is not available, large units are cooled by forcing a current of air at low pressure through

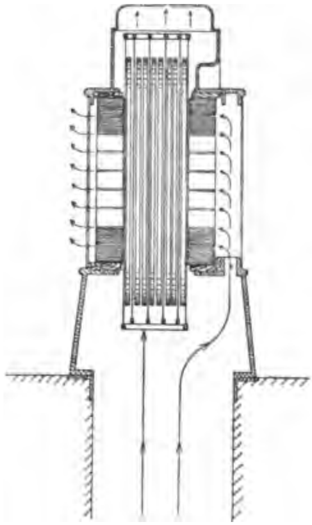


FIG. 93.—Transformers cooled by air blast.

air ducts in the core and coils. This method of cooling has been adopted to a considerable extent in large sub-stations where several

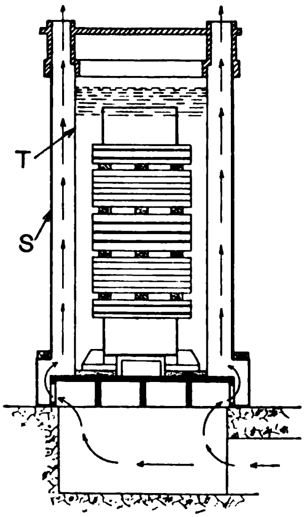


FIG. 94.—Oil-immersed transformer cooled by air blast.

units are banked together. Referring to Figure 93, the transformer shown with the case in section is erected over a chamber through which

air is driven by a fan operated from the low-tension side. The air enters at the base of each unit and passes upwards through air ducts in the core and coils, and finally leaves the case by apertures in the cover. One drawback to cooling by air-blast is that large quantities of dust become deposited in the spaces between the windings. The dust may contain impurities which attack the insulation, and finally cause breakdown.

The air-blast type without oil insulation is the one that has hitherto mostly been used under such circumstance, but its safe application is limited by voltage. A further disadvantage with this type is that it is dependent upon the continuity of the air supply, since if this fails the

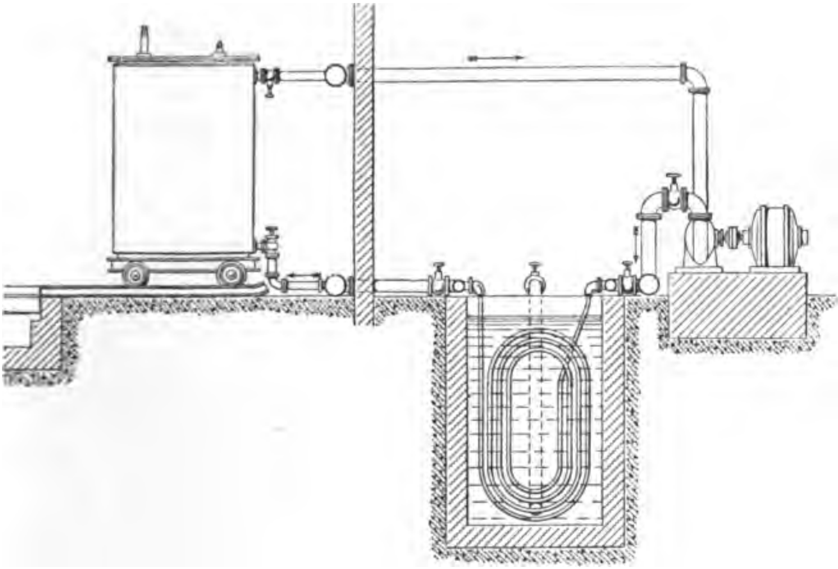


FIG. 95.

transformer would, in a very short time, attain a dangerous temperature. To partially obviate this latter risk, and also to permit of the air-blast being used for high-voltage units, oil-immersed transformers cooled by air-blast have come into use. Referring to Figure 94, the tank T containing the transformer and the oil is surrounded by a cast-iron shell S, erected over a ventilating tunnel in the floor. The blast enters at the bottom and passes to the top by way of the space left between the tank and the outside shell.

The formula for the quantity of air required is derived as for water cooling, page 125, and if K.W. denotes the power in kilowatts to be dissipated by the blast, then for a 10° C. difference in temperature between inlet and outlet the volume of air

$$V = 7.5 \times 10^{-2} \,.\, \text{K.W. cubic metres per second.}$$

An example of a very efficient method of cooling is shown in Figure 95, which represents a 6750-K.V.A. 3-phase transformer, by the Siemens-Schuckert Werke, giving 66,000 volts on the H.T. side. The transformer case is just high enough for the transformer, and no attempt is made to cool the oil in the case, but by means of an oil pump a definite and forced circulation within the case is induced, whilst at the same time the oil is effectively cooled by passing it through an outside worm immersed in water.

Calculation of Temperature Rise.—The final rise in temperature of any part of a transformer will, for a given method of cooling, be proportional to the watts lost per unit area of radiating surface, and may be expressed by an equation of the form

$$T^{\circ} = K\frac{W}{A} \quad \dots \dots \dots \dots \quad (15)$$

where W = watts to be dissipated.
A = cooling surface of heating body in square decimetres.
K = a coefficient.

The value of K will be equal to the temperature rise per watt per square decimetre of cooling surface, and, for a given type of construction and method of cooling, should be determined by experiment.

The temperature of all parts cannot attain to uniform value. There must be a temperature gradient from the inside to the surfaces which are in contact with the cooling medium. The difference between the temperature of the innermost parts and that at the surface will depend upon the thermal conductivity of the materials through which the heat must flow. In the case of the core the flow of heat takes place along a continuous metallic path in a direction parallel to the plane of the plates, with the result that there is only a small increase in temperature from the cooling surface to the centre.

With insulated coils the temperature gradient is more marked; for, owing to alternate layers of insulation, the flow of heat from the central parts is retarded considerably. For a given depth of winding the temperature gradient depends upon the shape of the cross-section of the conductors, and whether the transformer be air or oil cooled. When the coils are wound with circular wire the contact area between adjacent layers is less than that corresponding to a square or rectangular cross-section, and the flow of heat will take place with less facility. When the coils are immersed in oil the latter fills up the interstices, and the heat conductivity from wire to wire is considerably improved. By suitably subdividing the coils, arranging air ducts between them, and immersing the transformers in oil, the heating of the windings can be made more uniform, so that the temperatures of the middle and surface layers do not differ by more than 10 or 12 per cent.

In addition to the temperature gradient throughout the cross-section

of the core and coils, there is also a sudden drop in temperature from the surface of the former to the cooling medium, the difference in temperature depending upon whether the cooling medium is air or oil, and the manner in which it circulates with respect to the heated body. With oil-cooled transformers the heating of the oil is not uniform; for, owing to the heated oil rising to the top, the temperature of the oil at the top of the case will be greater than that at the bottom. However, as it is very difficult to discriminate between the temperature of the various parts, it is generally assumed, in making calculations relating to the heating of a transformer, that the oil has a uniform temperature equal to that at the top. The same conditions hold for transformers cooled by air blast.

From the above statements it will now be clear that the temperature of a transformer varies from point to point, and never attains a uniform value throughout. The value of T°, obtained from the previous equation, will be taken to represent the temperature rise of the hottest part of the heated body above that of the cooling medium which passes along the surface of the former. With natural air cooling, T° will be the difference in temperature between the hottest part of the core or coils and that of the external air. For oil-cooled transformers the temperature rise should be determined, firstly, for the coils and core with respect to the oil; and secondly, for the oil with respect to the external air or circulating water at exit if a cooling worm be used. In the case of air-blast the temperature rise must be reckoned with respect to the exit temperature, which latter generally exceeds that of the external air by about 10° C.

The values of K for different methods of cooling are set forth in Table IX., and these figures may be taken as representative of modern practice.

TABLE IX.—TEMPERATURE RISE PER WATT PER SQUARE DECIMETRE FOR DIFFERENT METHODS OF COOLING, *i.e.* VALUE OF K IN THE EQUATION $T^{\circ} = K\frac{W}{A}$.

	Oil filled.	Air cooled.		
		Natural Draught.	Moderate Blast 7.5×10^{-3} Cubic Metres per Second per K.W.	Strong Blast 15×10^{-3} Cubic Metres per Second per K.W.
Coils: Round wire, d.c.c.	2.0	16	10.0	6.0
Coils: Rectangular wire, d.c.c.	2.0	14	8.5	5.0
Coils: Rectangular wire, bare	1.7	12	7.0	4.0
Iron edge surface	1.7	6.5	3.0	2.0
Oil to air through case	15	...	...	...
Oil to cooling water	2.3	...	...	...

When calculating the cooling surface of a laminated core, only the edge surfaces need be taken into account, for, owing to the intervening insulation between the adjacent plates the flow of heat in a direction normal to the plate is very small compared with that along the plates. In a series of experiments carried out by T. M. Barlow * it was found that in an electrically heated core the ratio of heat conductivities in either direction was as 1 : 100.

For determining the temperature rise of the oil in an oil-cooled transformer above that of the external air or cooling water, the energy transmitted to the oil as heat will be the sum of the iron and copper losses. When no cooling worm is used the volume of the case should be such that the radiating surface (including ribs) must be sufficient to dissipate all the heat without the temperature rise of any part of the transformer exceeding 50° C. above that of the surrounding air. To fulfil this condition the quantity of oil required per kilowatt ranges from 7 kilogrammes in transformers of 10 K.W. or less, to 3.5 kilogrammes in those of 50 K.W. output or more.

Heating Tests.—The object of this test is to ascertain that, when working under normal conditions, the specified temperature rise shall at no time be exceeded. If two similar transformers are available they would be connected up as for Sumpner's efficiency test (see page 119), and full-load current made to circulate in the windings, the magnetic circuit being normally excited. The temperature measurements should, in the first instance, be made by thermometers placed in suitable positions on the core and windings.

In regard to the temperature of the core, there is never a very great difference between the innermost parts and the surface, so that the thermometer readings will give the temperature rise with sufficient accuracy for most commercial work. In the case of the windings, however, there is generally a considerable temperature gradient from the middle to the external layers, and thermometer measurements are therefore quite unreliable. The thermometers placed against the coils should only be used to indicate when a steady temperature is reached. The average temperature rise is then computed from resistance measurements made just before, and immediately after, the test.

When conducting heat tests a great saving in time can be effected by operating the transformers at considerable overload, which is maintained until they have reached the estimated final temperature under normal working conditions. The circulating current is then reduced to its full-load value and kept there until a steady temperature is reached. The final value of the temperature rise when the transformer is normally loaded can thus be correctly estimated. In the case of transformers designed for lighting loads it is quite unnecessary to

* *Journ. of the Inst. of Elect. Engineers* (1908), vol. xxxiv. p. 601.

continue the heat run until a steady temperature is reached. The duration of the test with full-load current should be limited to the number of hours—on an average about five or six—during which the transformer is likely to be operated continuously at full-load.

When only one transformer is available for testing, a continuous run at full-load, which in the case of very large units may extend over several days, would entail a large expenditure of energy. To obviate this the usual practice is to give the transformer a preliminary heating by placing it in an oven maintained at a temperature of about 60° C., or to alternately over-excite the core and send about twice the full-load current through the windings, one circuit, of course, being short-circuited. When the estimated temperature rise has been approximately reached the transformer is normally loaded, and frequent observations made to determine when a steady temperature is reached.

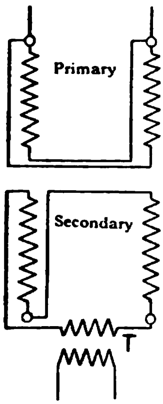


FIG. 96.

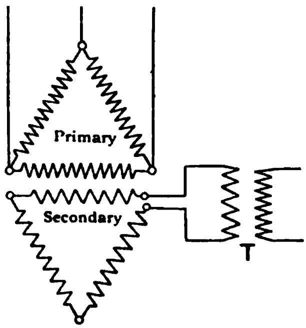


FIG. 97.

A better method of obtaining the normal iron and copper losses with a power expenditure equivalent to the losses only is shown in Figure 96. The primary and secondary are each divided into two equal sections. The secondary sections are connected so that their E.M.F.'s oppose each other, and similarly the two sections of the primary are connected so that their ampère-turns are in opposition. Owing to the paralleling of the two primary sections the primary applied E.M.F. must be reduced to half its normal value, so that the normal induction in the core may be obtained. For regulating the current circulating in the windings a small auxiliary transformer T is connected in series with the secondary, the magnitude of the currents being controlled from the primary of T. To adopt this principle in 3-phase transformers, both primary and secondary must be delta-connected, the auxiliary transformer T being in series with the secondaries (Figure 97).

Transformers for Three-Phase Circuits

Star and Delta Connections compared.—Transformers for 3-phase circuits may have their windings connected in either star or delta. The latter connection is preferable, because, should one phase develop a fault and be automatically cut out of circuit, the supply of 3-phase current will still be maintained, the other two phases taking up the load. Had the windings been star connected, then, under the same circumstance, single-phase current would be delivered, but the motors or rotary converters connected to the secondaries would be unable to cope with anything like their full-load, and, moreover, the supply system would be thrown greatly out of balance.

With the star connection the pressure per phase is about 58 per cent. of the line pressure; hence, on account of the reduction of insulation, the space taken up by the high-voltage windings is somewhat less in comparison with mesh-connected transformers. This is, of course, a distinct advantage—especially when dealing with line pressures of 6600 volts or more. A frequent practice, in the case of step-down transformers, is to connect the primaries in star and the secondaries in mesh.

Primaries connected in Mesh.—It was shown in the previous chapter that, if a transformer be connected to a single-phase circuit having a sine wave of E.M.F., the magnetism in the core will also have a sine-shaped wave, but the current wave will have a third harmonic produced by hysteresis in the iron. This harmonic is always present, though its relative importance diminishes with the load. Now, the voltage waves of 3-phase generators can contain no harmonics having a frequency of three—or multiples of three—times the fundamental.* It therefore follows that, if the primaries of a 3-phase unit be mesh-connected and the curve of applied pressure sinusoidal, the sum of the magnetising currents round the mesh will not be zero. There is, therefore, at all loads a local current circulating in the delta which, since the third harmonic is the more pronounced, is approximately sine-shaped and has a frequency three times that of the supply pressure.

Primaries connected in Star.—*Case* 1.—For star-connected primaries the first case to be considered is that in which the neutral point is connected either *via* earth or through a fourth wire to the star point of the generator winding (see Figure 98). If the voltage curve between each main and the neutral point be sine-shaped, then the magnetising current wave will have the form shown in Figure 59. As the sum of the three currents cannot then be zero, a triple-frequency current, which approximates to a sine wave, will flow between the neutral points of the generator and the transformer.

Case 2.—When the primary windings are connected in star with

* See p. 225.

an insulated common junction, the sum of the three currents flowing into the latter *must* be zero at any instant. The magnetising current waves cannot be shaped like that in Figure 59, since this latter has a large third harmonic. The curve of magnetic induction must therefore be modified so as not to require the triple harmonic of current. This modification of flux wave is equivalent to the introduction of a third

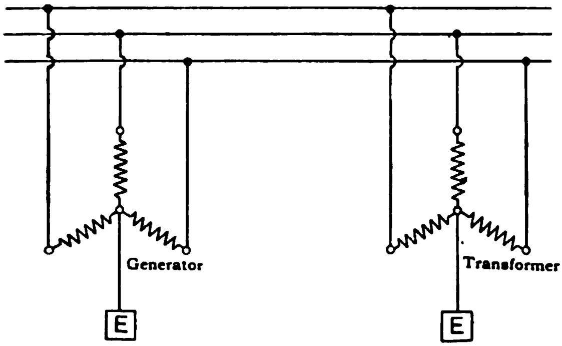


FIG. 98.—3-phase transformers with earthed neutral.

harmonic into the E.M.F. wave taken across the terminals of each phase, which will hence be of a different shape to that across the lines. The introduction of the third harmonic will cause the voltage wave for each phase to become peaked, thus increasing the effective value of the voltage per phase although the line voltages remain the same.

From these statements it at once follows that: (1) if three similar

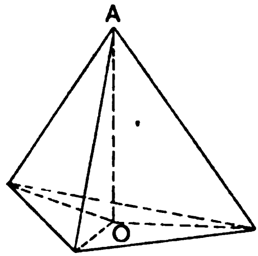


FIG. 99.

transformers—or the three primaries of a 3-phase unit—be star connected to a 3-phase supply having insulated neutrals and a sine wave of E.M.F., then the volts per phase will be greater than $\frac{1}{\sqrt{3}}$ the voltages between the mains, and (2) the core loss of a star-connected 3-phase transformer will be less than that when the same transformer is delta connected to a sine wave supply having $\frac{1}{\sqrt{3}}$ the voltage of the previous case.

In the case of a star-connected 3-phase system, the phase and line voltages cannot in general be represented by the sides of an equilateral triangle and the three lines joining the angular points to the centre. They can, however, be represented by the edges of a tetrahedron (Figure 99). The equilateral triangle, which forms the base, represents

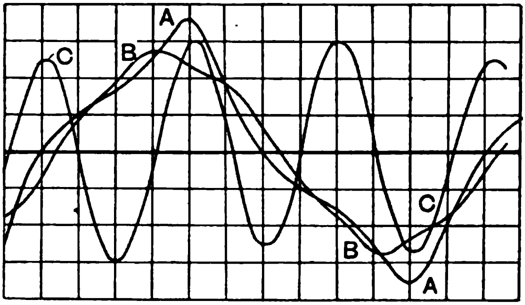


FIG. 100.—Wave forms of exciting current for 3-phase Y-connected transformers.

the three phases, and its centre point O is the neutral point that is obtained by connecting three equal non-inductive resistances to the supply mains. The perpendicular dropped from the apex A to the centre O represents the triple-frequency voltage generated in a load of a transformer. The apex, which is symmetrically placed with regard to

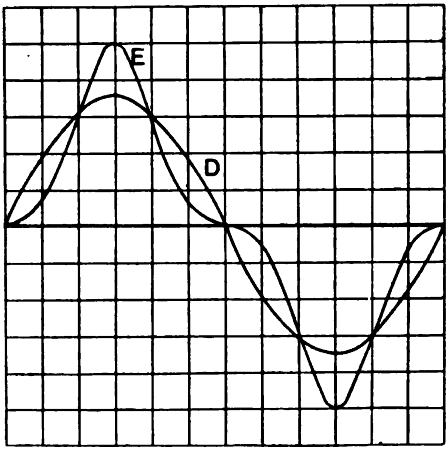


FIG. 101.—Voltage curves for Y-connected 3-phase transformers.

the supply voltage, forms a second neutral point, and differs in potential from the first.

Some interesting curves which verify the above conclusions are given in Figures 100 and 101. A 3-phase star-connected alternator, giving a sine wave of E.M.F., supplied current to the star-connected primaries of three single-phase 10-K.W. transformers. Curve A is the

form of the magnetising current per phase when the neutrals are connected. On separating the neutrals the current wave changes to that shown by curve B, in which it will be found that the third harmonic is absent. The maximum value of the current has been reduced about 25 per cent., so that the maximum value of the flux has also been reduced. Curve C, which is approximately a pure third harmonic, shows the current flowing in the neutral connection. As this current is the sum of

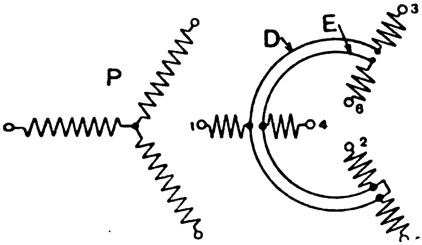


FIG. 102.—3-phase to 6-phase transformation with secondary connected in double star.

the three harmonics in the line wires, it is approximately three times the difference between curves B and A. Curves D and E (Figure 101) are respectively the voltage waves taken across one phase of the generator and the corresponding phase of the transformers, the neutral points being separated. Curve E has a distinct third harmonic, and a peak value 40 per cent. greater than curve D.

Three-Phase to Six-phase Connections.—When rotary con-

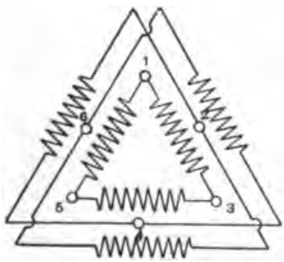


FIG. 103.—3-phase to 6-phase transformation with secondary connected in double delta.

verters are operated from the alternating current side, a 6-phase current is preferable. Now, in most cases the supply will consist of a high-pressure 3-phase current, so that it will be necessary to transform to a low-pressure 6-phase alternating current. This transformation may be carried out as diagrammatically shown in Figure 102, where P represents a star-connected primary supplied with 3-phase current. Each transformer or each core of a 3-phase unit is wound with two similar secondaries. The first set is connected in star by the con-

ductor D. The second set of secondaries is also coupled in star (as at E), but with the connections reversed to those of the first set.

With these connections the E.M.F.'s of any two adjacent phases have a phase difference of 60 degrees, and if the two star points be connected together, 6-phase current will be obtained from the terminals 1, 2, 3, 4, 5, and 6. The voltage between any two adjacent terminals is equal to the voltage of any one phase, and the phase relationship of the various terminal E.M.F.'s can be represented by the sides of a hexagon.

Six-phase current could also be obtained by connecting the secondaries in double mesh (as in Figure 103), the windings of the second set being connected in the reverse fashion to those of the first set.

Three-phase to Two-phase Connections.—When electrical energy has to be transmitted over long distances a 3-phase system is the most economical as regards copper, and is therefore generally used.

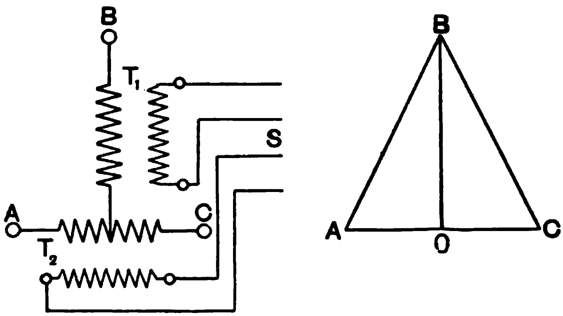


Diagram of connections. Vector diagram.

FIG. 104.—2-phase to 3-phase transformation.

For distributing to a combined load of lamps and motors a 2-phase supply is to be preferred, on account of the better facilities for maintaining the regulation. Hence it may sometimes be necessary to transform from a 3-phase to a 2-phase system, or *vice versa*. Such a transformation, effected by a suitable combination of two single transformers, has been devised by C. F. Scott, and is shown diagrammatically in Figure 104. T_1 and T_2 are the two transformers the primaries of which are connected to a 4-wire 2-phase supply S. One end of the secondary of T_1 is connected to the middle point of the secondary of T_2 so as to leave three terminals A, B, and C for making connection to the secondary circuit. The E.M.F.'s induced in the two secondary windings have a phase difference of 90 degrees, and may be represented by the two vectors OB and AOC set at right angles in such a manner that OA = OC. The voltage between the terminals A and B is represented by the vector, AB, and is the resultant of the two quadrature E.M.F.'s OB and OA. Similarly, the vector BC represents the voltage between

the terminals B and C. Now, if the number of turns for the secondary windings be so arranged that $OA = \frac{BA}{2}$, then $AB = BC = AC$, and $OB = \sqrt{AB^2 - \left(\frac{AB}{2}\right)^2} = 0.867\ AB = 0.867\ AC$. The secondary of T_1 must therefore be wound for a voltage 0.867 times that of the secondary voltage of T_2. Since the three vectors AB, BC, and CA form the sides of an equilateral triangle and are 120 degrees apart, it follows that a 3-phase current will be obtained from the terminals A, B, and C. The above describes the conversion from 2-phase to 3-phase, but the converse is also true. For, by connecting the terminals A, B, and C to a 3-phase supply, 2-phase current will be available at S.

Transformers in Parallel.—In the case of large supply systems it may often be expedient to connect a number of transformers in parallel; the transformers are then said to be "banked." The following are the requirements for satisfactory operation in parallel.

1. Exactly the same voltage ratio at no-load for different temperatures.

2. The same impedance voltage, *i.e.* the voltage required to circulate the full-load current through the secondary winding, on a short-circuit test.

Provided the above conditions are fulfilled, the paralleling of single-phase transformers on both sides presents no difficulties. For, if they are paralleled on the primary side, their secondary voltages may always be rendered co-phasal with respect to the external circuit by a proper mode of connection,—there being, in fact, only two possible cases, one being that of phase conjunction and the other that of phase opposition. By simply reversing the connections of the secondary it is always possible to pass from one case to the other.

This, however, is not the case with 3-phase transformers, for with different modes of connection of the phases, parallel operation of the windings from both sides may become impossible. To illustrate this, the usual methods of making connections, along with the corresponding vector diagrams, are shown in Figure 105. The small arrows in the diagram of connections indicate the direction of the E.M.F. which is regarded as positive, and the lettering at the arrow head of each vector indicates to which part of the winding the vector corresponds. From the vector diagrams it will be obvious that the voltages corresponding to the various connections are out of phase with each other. In dealing with the parallel operation of transformers on both sides it is convenient to consider the phase relation between the primary and the secondary voltages. The smallest angle by which a vector of secondary voltage is in advance of a vector of primary voltage may be defined as the network angle of the transformer. Three-phase transformers which have to be operated in parallel must have equal network angles, otherwise large circulating currents will be set up between the various phases.

The following table gives the network angles corresponding to all possible combinations of primary and secondary connections:—

TABLE X.—NETWORK ANGLES FOR THREE-PHASE TRANSFORMERS.

Network Angle.	Zero.				30°				60°		90°			
Primaries connected as at . . .	A	B	C	D	A	B	C	D	A	C	A	B	C	D
Secondaries connected as at . .	A	B	C	D	C	D	B	A	B	D	D	C	A	B

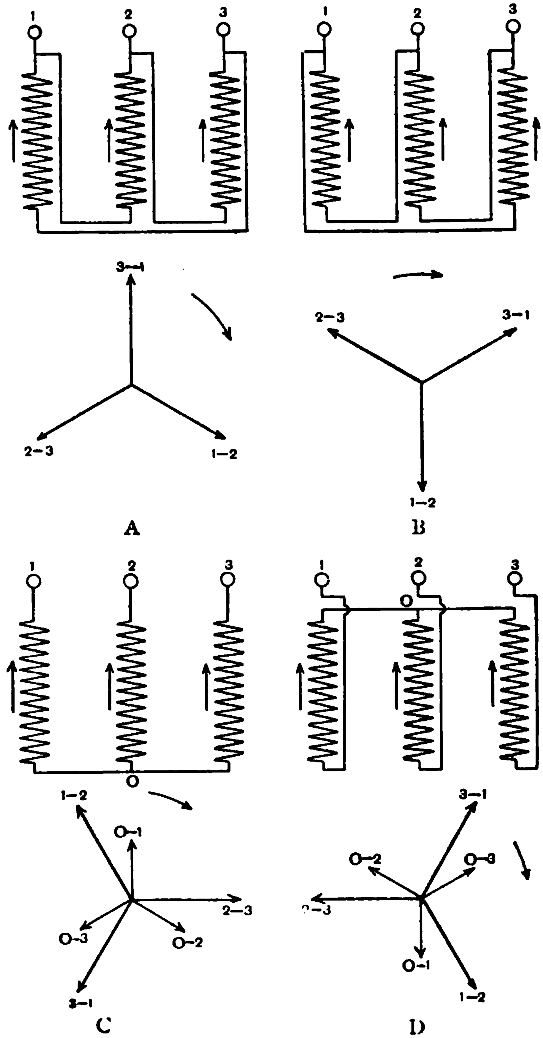


FIG. 105.

Special Forms of Transformers

Auto-Transformer.—Where the voltage of the high-tension side does not exceed about 600 volts and the transformation ratio is small, the secondary winding may be superposed on some of the primary turns, as shown in Figure 106. Such an arrangement is known as an *auto-transformer*. The ends of the turns AB form the primary terminals, while the secondary terminals are represented by the primary terminal A and another one C connected to an intermediate point of the winding.

When the secondary is on open circuit the no-load current flows through both sections, and the magnetising component produces a magnetic flux in the core, which in turn induces a counter e.m.f. in both sections of the winding. On connecting to the load L the secondary current will flow through the turns AC in the reverse direction to the primary current. These turns, which are common to both windings, will therefore be traversed by a current equal to the difference

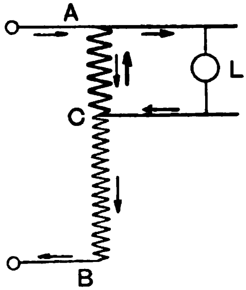


Fig. 106.—Principle of the auto-transformer.

between the primary and the secondary currents. An auto-transformer can therefore be smaller and cheaper than an ordinary transformer with separate windings.

Referring to the figure, let T_1 denote the number of turns between A and B, and T_2 the number between A and C, then the transformation ratio $= r = \frac{T_1}{T_2}$. Further, if I_1 denote the current taken from the supply mains, and I_2 the corresponding secondary current, then the current in the turns AC common to both circuits $= I_2 - I_1 = rI_1 - I_1 = (r-1)I_1$. Assuming equal current densities in each section of the winding, $a_2 = a_1(r-1)$, where a_1 and a_2 denote the cross-sectional areas of the wire for the turns AC and BC respectively. Now, for a mean length per turn $= l$, the total volume of copper is

$$
\begin{aligned}
V_a &= l\,\{a_1(T_1 - T_2) + a_2T_2\} \\
&= l\left\{a_1\left(T_1 - \frac{T_1}{r}\right) + a_1(r-1)\frac{T_1}{r}\right\} \\
&= 2\,la_1T_1 \cdot \frac{r-1}{r}
\end{aligned}
$$

For a transformer with separate windings having the same current density and mean length per turn the volume of copper

$$V_b = l(a_1T_1 + a_2T_2) = l\left(a_1T_1 + \frac{a_1rT_1}{r}\right)$$
$$= 2\ la_1T_1$$
$$\text{Hence } V_a = V_b \times \frac{r-1}{r}$$

In transformers of the same type, but of different sizes, the ratio of the volume of iron to that of copper is approximately constant, so that the following ratio is obtained,

$$\frac{\text{Weight of active material required for auto-transformer}}{\text{Weight of active material required for transformers of ordinary construction}}$$
$$=\frac{r-1}{r}$$

Thus for a transformation ratio of 2, the weight of active material for an auto-transformer will be 50 per cent. less than that required for a transformer of ordinary construction. For a ratio of 3, the saving in weight is only 33 per cent. The above expression shows that for large ratios of transformation the saving in active material effected by the use of auto-transformers is very small. The primary pressure should not exceed about 600 volts, because for voltages greater than this it is necessary, on account of safety considerations, to keep the two circuits entirely independent. The use of the auto-transformer is therefore restricted to low pressures and small transformation ratios.

The chief application of the auto-transformer is as a starting device for induction motors. By means of a suitable switch the motor is first connected across the secondary terminals, from which it receives a supply of current at a fraction of the line pressure. When the motor attains the speed normal to the secondary pressure, the position of the switch is altered so as to connect the motor direct to the supply. The auto-transformer has also a useful field of application in supplying current to lamps which are operated at a somewhat lower voltage than that of the supply mains. For instance, if a group of 125-volt metallic filament lamps has to be supplied with current from 250-volt mains, then an auto-transformer could be used. Its weight and cost would then be approximately the same as an ordinary transformer of half the output.

Boosting or Regulating Transformers.—In the supply of alternating currents it is often necessary to provide arrangements whereby the terminal pressure of the load can be varied within certain limits even although the generator pressure is constant. For instance, when two or more transmission lines of different lengths are fed with an alternating current from one generating centre, it may be necessary to increase the pressure of the longest line to compensate for the greater

drop. This is effected by means of special auxiliary transformers known as "boosters."

The principle of the booster is illustrated in Figure 107, where G is the generating station bus bars, T a transformer the secondary of which is connected to a load L, and F the feeders between G and the primary of T. Suppose that at full-load the drop in the feeder is 50 volts and that the bus bar pressure is constant, then, to maintain a constant terminal voltage on the primary of T, the secondary winding of the auxiliary or boosting transformer B must be connected in series with one of the feeders F. One end of the secondary is connected to the bus bars as shown, and the other end, together with a number of intermediate points, is joined up to a multiple way switch S to which one of the feeder cables is connected. The primary of B is permanently connected to the station bus bars, and as the load varies the consequent rise or fall in the feeder voltage drop is compensated for by altering the position

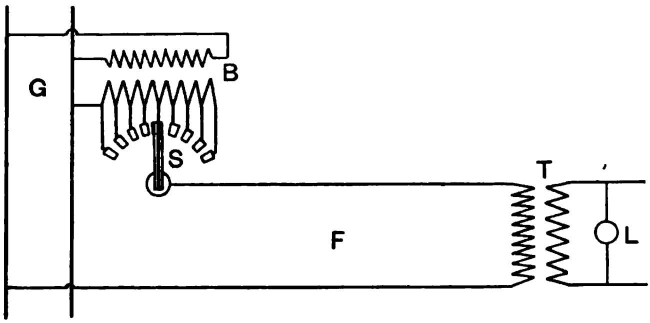


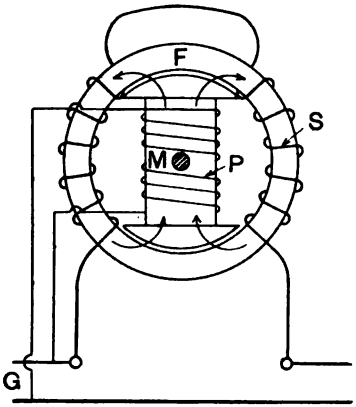
FIG. 107.—Transformer used as a booster.

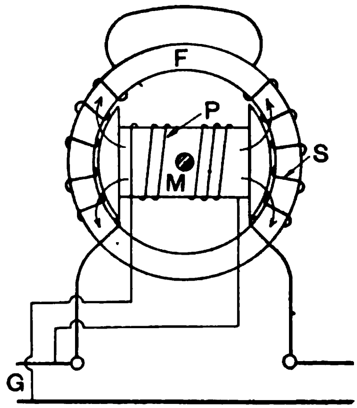
of the switch S so as to insert more or less secondary turns in series with F. At full-load all the secondary turns would be in series with F, whereas at half-load only half the secondary turns would be necessary to make up for the 25 volts drop and to give the required pressure at T. If E denote the boosting volts required for a full-load current of I ampères, then the booster would be designed as an ordinary low-voltage transformer to give an output of $\frac{E \times I}{1000}$ K.V.A. The number of tappings taken from the secondary to the regulating switch depends upon the degree of control required.

The regulating switch S requires to be of special design, so that the voltage may be controlled without interrupting the supply or short-circuiting of any section of the secondary. The contact arm of the switch is therefore in two parts separated from each other by an insulating partition and connected together through a small choking coil. When passing from one stud to the next the turns between the

studs are in series with the choking coil, and the short-circuit current is limited to its normal value.

Another form of booster, chiefly employed in obtaining variable voltages for testing purposes, is shown diagrammatically in Figure 108. It consists of two parts—one fixed and the other movable. The cylindrical fixed part F is constructed of sheet-iron laminations and carries the secondary winding S, which is connected in series with the circuit whose voltage is to be controlled. The movable part M, resembling that of a 2-pole shuttle-wound armature, is also built up of laminated iron mounted on a suitable shaft and carries the primary winding P. The latter is permanently connected through slip rings to the bus bars G. The movable core M can be rotated round an axis coincident with the axis of the cylindrical fixed core, its position being





Variable induction booster.

FIG. 108.—Boosting E.M.F. maximum.

FIG. 109.—Boosting E.M.F. zero.

adjusted by a hand wheel and worm gear. When the primary is in a vertical position the flux of the primary passes through the fixed core in such a direction that the E.M.F.'s induced in the individual coils of the secondary act in the same direction. Hence the booster E.M.F. will have its maximum value. When the primary is moved into a horizontal position (see Figure 109) it is obvious that the induced electromotive forces neutralise one another, and the booster will have no effect on the supply voltage. Between these two positions any desired boosting effect from zero to the maximum can be obtained. By rotating the primary from the position shown in Figure 109 through an angle of more than 90 degrees the direction of the boosting E.M.F. will be reversed, so that the apparatus may also be used as a negative booster. It will be apparent that this arrangement gives a much finer regulation than is possible with the previous arrangement, the voltage being adjusted not in definite steps but as gradually as may be desired.

CHAPTER V

DESIGN OF TRANSFORMERS

TRANSFORMERS are usually designed to a specification of: (1) output at a given frequency, (2) terminal pressure for primary and secondary, (3) efficiency, (4) voltage drop, and (5) temperature rise. Other essential features should be a small magnetising current and durability of the insulating materials. The voltage drop and temperature rise, respectively, can be kept within the prescribed limits by suitably sandwiching the primary and secondary windings, and by providing the requisite cooling facilities. The problem before the designer therefore consists in constructing a transformer for a given efficiency with the minimum expenditure on material and labour.

In designing a transformer a frequent procedure is to assume various inductions, current densities, and dimensions of core; calculate the number of turns for primary and secondary, the losses, and price of active material; and then alter one or more of the assumed values, until, finally, a design is obtained which fulfils the specification with the minimum expenditure on active material. This method is naturally tedious and wasteful of time, and, moreover, it does not give a guarantee that the most economical design has been obtained. However, the problem of transformer design readily lends itself to a simple mathematical solution, and if the procedure * recommended in this chapter be adopted the most economical design will always be derived in the first attempt, the only assumption made being that of efficiency. For a series of standard designs the majority of the formulæ may be plotted as curves, thus reducing the necessary calculations to a minimum.

In working out the design of a transformer the principal unknown quantities to be determined are (1) the current density in the windings per square cm. of cross-section ($=Q$); (2) the maximum flux density in lines per square centimetre ($=B_m$); and (3) the four linear dimensions a, b, c, and d (see Figure 117).

Copper Loss.—If, for the primary winding, I_p denotes the

* Due to Dr. R. Pohl and H. Bohle.

current, and R_p the ohmic resistance, then the loss per kilogramme of copper is

$$W_p = \frac{I_p^2 \,.\, R_p}{8.9 \times 10^{-3} \times a_p \,.\, l_p \,.\, T_p} \text{ watts}$$

where a_p = area of cross-section of conductor in square cms.
l_p = mean length per turn in cms.
T_p = number of turns.

$$\text{Now, } R_p = \frac{2 \times 10^{-6} \times k \times l_p \times T_p}{a_p}$$

where 2×10^{-6} is the specific resistance of copper for a mean working temperature of 60° C., and k is the factor which allows for the additional loss due to eddy currents. Substituting the above value for R_p

$$W_p = \frac{I_p^2 \times 2 \times 10^{-6} \times k \times l_p \times T_p}{8.9 \times 10^{-3} \times a_p^2 \times l_p \times T_p}$$

Now, $\frac{I_p}{a_p} = Q_p$ = current density in the primary winding; hence

$$W_p = 2.25 \times 10^{-4} k Q_p^2$$

The loss per kilogramme for the secondary winding may be derived in a similar manner, and is expressed by

$$W_p = 2.25 \times 10^{-4} \times k Q_s^2$$

where Q_s is the current density of the secondary.

It will be shown below that for minimum copper loss Q_p and Q_s should be equal, hence if K_c denotes the kilogrammes of copper for primary and secondary, the total loss due to the ohmic resistance of the windings is

$$W_c = 2.25 \times 10^{-4} \, k K_c Q^2$$

it being assumed that the primary and secondary are worked at the same current density Q. Combining the numerical constant 2.25×10^{-4} with the factor k, the equation to the total copper loss becomes

$$W_c = C_1 \,.\, K_c \,.\, Q^2 \quad \ldots \quad (16)$$

where the coefficient C_1 has the following approximate values—

2.45×10^{-4} for 25∼
2.50×10^{-4} „ 40∼
2.70×10^{-4} „ 50∼
and 2.92×10^{-4} „ 60∼

Best Distribution of Copper Losses.—The total copper loss may also be expressed by

$$W_c = I_p^2 R_p + I_s^2 R_s$$
$$= I_p^2 \,.\, \frac{\rho L_p}{a_p} + I_s^2 \,.\, \frac{\rho L_s}{a_s}$$

where L_p and L_s denote the total length of conductor in primary and secondary windings respectively, and the other symbols have the

same meaning as before. The condition of minimum copper loss is that

$$\frac{dW_c}{da_p} = 0$$

$$i.e., \frac{d}{da_p}\left(I_p^2 \cdot \frac{\rho L_p}{a_p} + I_s^2 \cdot \frac{\rho L_s}{a_s}\right) = 0$$

Since the total volume of copper $= V = a_pL_p + a_sL_s$, the value of $a_s = \frac{V - a_pL_p}{L_s}$

$$\text{and } \frac{d}{da_p}\left\{I_p^2 \cdot \frac{\rho L_p}{a_p} + I_s^2 \cdot \frac{\rho L_s^2}{(V - a_pL_p)}\right\} = 0$$

Differentiating this equation with respect to a_p, and arranging terms

$$\frac{I_p}{a_p} = \frac{I_s}{a_s}$$

That is, the copper losses in a transformer are a minimum when the current density in the primary is the same as that in the secondary winding. In the case of high-voltage transformers equal current densities in primary and secondary may not be adhered to, for owing to the lower heat conductivity of the heavily insulated winding it is often expedient to work the high-tension winding at a lower density than the low-tension winding. This will ensure a more uniform temperature rise than would be the case if the current densities expressed were equal.

Iron Loss.—The iron loss in a transformer consists of two parts: (1) the hysteresis loss proportional to $B_m^{1.6}$, and (2) the eddy current

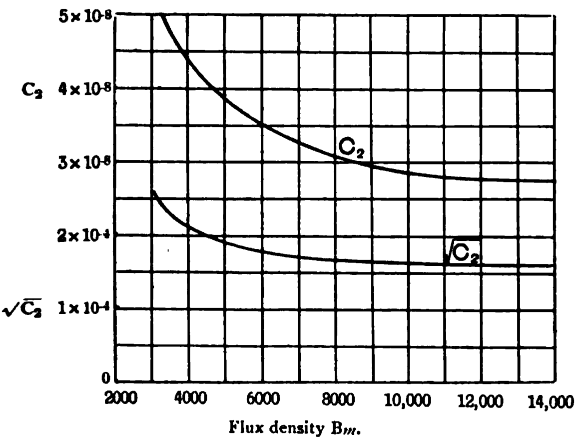


FIG. 110.—Value of C_2 and $\sqrt{C_2}$ for ordinary iron 0.35 mm. thick at 50~.

loss which varies as B_m^2. The loss per kilogramme at 50~ can be expressed by the equation

$$W_i = C_2 \times B_m^2$$

Since the hysteresis component of the core loss varies as $B_m^{1.6}$, the coefficient C_2 cannot be a constant, but decreases with increasing values of flux density. For ordinary iron 0.35 mm. thick, the value of C_2 at 50~ corresponding to flux densities of between 4000 and 14,000 may be obtained from the curves of Figure 110. If K_i denote the mass of iron in kilogrammes, then the core loss is expressed by

$$W_i = C_2 \,.\, K_i \,.\, B_m^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

Ratio of Copper Loss to Iron Loss.—Let p_c and p_i, expressed as a fraction of the total loss W_t, denote the copper and iron losses respectively, then

$$W_c = C_1 \,.\, K_c \,.\, Q^2 = p_c \,.\, W_t$$
$$\text{and } W_i = C_2 \,.\, K_i \,.\, B^2_m = p_i \,.\, W_t$$

On multiplying and extracting the square root the following expression is obtained for the sum of the copper and iron losses.

$$W_t = \sqrt{C_1 C_2 \times K_c \,.\, K_i \times \frac{Q^2 B_m^2}{p_c \,.\, p_i}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (18)$$

Now, it was shown in the previous chapter that the total losses are a minimum when those of the iron are equal to those of the copper,—*i.e.* when $p_c = p_i$. The proportion in which the total loss is divided between the copper and iron depends upon the load factor. Transformers, having an induction motor or rotary converter load, are generally operated continuously at full-load; the best design will therefore be obtained when there is equality of iron and copper losses. Transformers for lighting work are only occasionally fully loaded, and in such cases it is advisable to make the copper and iron losses as nearly equal as possible for the average load, in so far as the drop and temperature rise at full-load will permit. Experience shows that in view of the latter considerations it is not advisable to allow the copper loss to exceed 60 per cent. of the total.

For any transformer, the load conditions of which are known, the distribution of the losses are thus fixed. For power transformers the design is worked out for $p_i = p_c = 0.5$; for lighting transformers, p_i is reduced and p_c increased, the limit to these values being 0.4 and 0.6 respectively.

Best Ratio of Iron to Copper.—The most important factor affecting the cost of a transformer to fulfil a given specification is the ratio

$$\frac{\text{Weight of iron}}{\text{Weight of copper}} = \frac{K_i}{K_c} = C_0$$

Let P_c and P_i denote the price of the copper and iron respectively, p'_c and p'_i the fractional copper and iron prices such that $p'_c + p'_i = 1$; and S_c and S_i the price per kilogramme of insulated copper and stamped iron sheets respectively; then

$$P_c = S_c \,.\, K_c = p'_c \,.\, P_t$$
$$\text{and } P_i = S_i \,.\, K_i = p'_i \,.\, P_t$$

where P_t is the total cost of active material. By multiplying and extracting the square root

$$P_t = \sqrt{\frac{K_c \,.\, K_i \times S_c \,.\, S_i}{p'_i \,.\, p'_c}}$$

Consider now a transformer the iron and copper weights of which may be increased or decreased by adding to or reducing the cross-section of iron and number of turns in the winding respectively, without altering either the mean length of the magnetic circuit or the mean length of copper turn. Assuming, further, that the current density and induction are constant, the power output from the transformer may be expressed by

$$W = C_8 \,.\, K_i \,.\, K_c$$

where C_8 is a coefficient. If, in the equation for P_t, there be substituted for $K_i \,.\, K_c$ the value $\frac{W}{C_8}$, then the cost of active material is expressed by the equation

$$P_t = \sqrt{\frac{W \times S_c \times S_i}{C_8}} \times \sqrt{\frac{1}{p'_c \,.\, p'_i}} \quad \cdots \cdots \quad (19)$$

This latter equation shows that for a given output the cost of active material will be a minimum when $p'_c \times p'_i$ is a maximum, or when $p'_c = p'_i$. Hence the most economical transformer is the one for which the cost of copper equals the cost of iron. This holds good for all types of transformers, and its truth is independent of the distribution of the losses. Since for minimum cost $p'_c = p'_i$ or $K_i S_i = K_c S_c$, the ratio C_0 for minimum cost is also expressed by

$$C_0 = \frac{K_i}{K_c} = \frac{S_c}{S_i}$$

For lighting transformers it may often be necessary to make C_0 smaller than $\frac{S_c}{S_i}$, as otherwise the temperature rise of the windings may become excessive. The value of C_0 is then more accurately expressed by

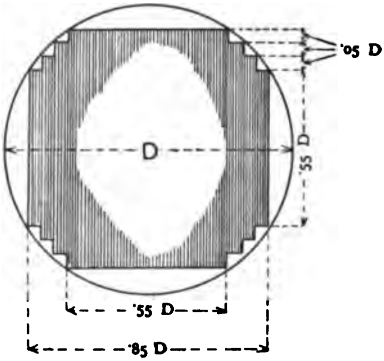
$$C_0 = \frac{S_c}{S_i} \times \frac{p_i}{p_c}$$

This does not influence the cost to any great extent, as the curve which shows the relationship between C_0 and the cost of the transformer is very flat for such values of C_0 as lie between $0.5 \frac{S_c}{S_i}$ and $1.5 \frac{S_c}{S_i}$.

Having determined the formulæ for the various losses, and settled the most favourable relationship between the weights of the copper and the iron, equations can now be derived from which may be calculated the dimensions of transformers of either the shell or core types.

Core Type Transformers

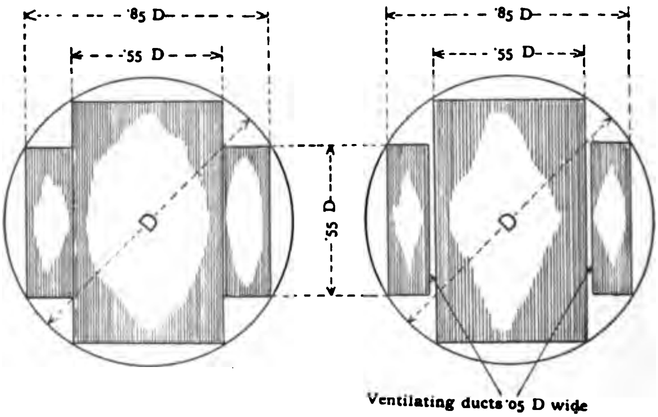
Cross-section of Core.—A core of circular cross-section is the most economical, because for a given cross-sectional area the mean



0·35 mm. plates F_i=0·63. 0·50 mm. plates F_i=0·65.

FIG. 111.—Cross-section of core for transformers up to 50 K.V.A.

length of a copper turn will be a minimum. In practice, however, a perfect circle cannot be obtained, as too many different sizes of



0·35 mm. plates F_i=0·67. 0·50 mm. plates F_i=0·70.

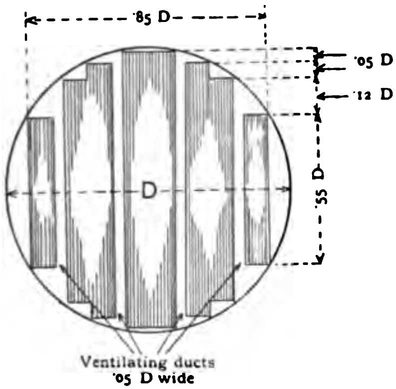
0·35 mm. plates F_i=0·63. 0·50 mm. plates F_i=0·65.

FIGS. 112 and 113.—Cross-section of core for 50–250 K.V.A. transformers.

stampings would be required. From the manufacturer's standpoint the latter are limited to two, three, or four, with the result that the

standard practice is to adopt the shapes of cross-section shown in Figures 111 to 114. Each core is circumscribed by a circle of diameter D, the dimensions of each packet of stampings being expressed in terms of D.

The nett area of cross-section of iron in a core $= A = \frac{\pi D^2}{4} . F_t$ where F_t, the space factor of the core, takes into account the insulation between the plates, ventilating ducts, and the waste of space due to the imperfect shape of the core. The factor F_t has been calculated for each of the cross-sections for thicknesses of plates 0.35 and 0.5 mm. respectively, it being assumed that the insulation has a standard thick-



0·35 mm. plates F_t=0·63. 0·50 mm. plates F_t=0·65.

FIG. 114.—Cross-section of core for large transformers of 250 K.V.A. and upwards.

ness of 0.05 mm. In a circular core-type transformer the unknown dimensions *d* and *c* in Figure 117 are thus replaced by a single one—namely, the diameter D of the circle circumscribing the core.

If, in large transformers, the circular core be adopted the waste of space due to the imperfect shape of the core becomes considerable; hence, in order to obtain a better iron space factor, the more usual practice is to adopt a core of rectangular cross-section for transformers of 200 K.V.A. or more. Some manufacturers have even adopted the rectangular core for all transformers rated from about 75 K.V.A. The ventilating ducts in the core are generally 0.5 cm. wide, and vary in number from one to five according to the size. The corresponding values of the iron space factor F_t are set forth in Table XI.

TABLE XI.—IRON SPACE FACTORS F_i FOR RECTANGULAR CORE-TYPE, VENTILATING DUCTS, 0.5 CM. WIDE.

Size of Transformer.	Number of Ducts.	Space Factor F_i.	
		0.35 mm. Plates.	0.50 mm. Plates.
75–150 K.V.A. . .	1	0.83	0.87
150–250 ,, . .	2	0.78	0.82
250–500 ,, . .	3	0.74	0.77
500–1000 ,, . .	4	0.70	0.73
1000 upwards . . .	5	0.65	0.68

Space Factor of Windings.—The space factor of the coils, expressed by

$$F_c = \frac{\text{Volume occupied by copper}}{\text{Total available winding space}}$$

depends upon the primary and secondary voltages. If F_c be large, a comparatively small transformer is obtained, but the space left for

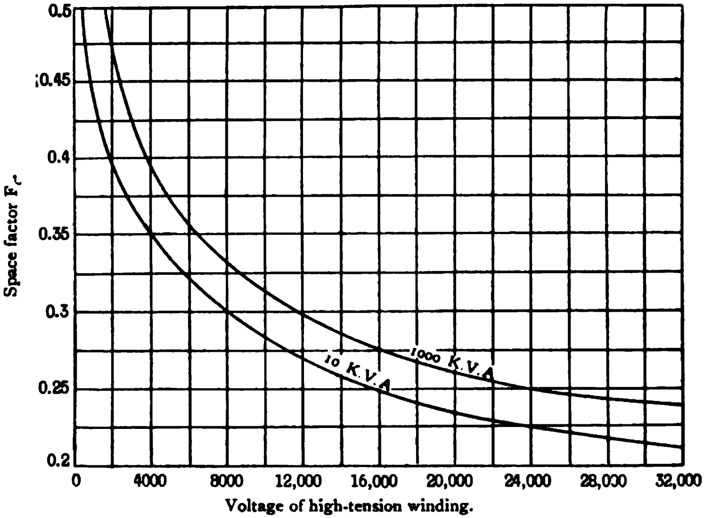


FIG. 115.—Copper space factor for oil-immersed transformers.

properly insulating the coils will probably be insufficient. Figure 115 gives, for oil insulated transformers, the average space factors for outputs ranging from 10 to 1000 K.V.A., and secondary pressures up to 32,000 volts. Since air is not so good a dielectric as oil, natural air-cooled and air-blast transformers must be designed for space factors some 20 per cent. less than those obtained from the above curves.

SINGLE PHASE

(*a*) **Circular Core.**—The output from the secondary of a transformer in K.V.A. is

$$\frac{E_s \,.\, I_s}{1000} = 4.44\ T_s \,.\, \sim \,.\, \Phi \,.\, I_s \times 10^{-11}$$

But $I_s = a_s \times Q$, and $\Phi = B_m \times A$, where $A\left(= F_i \,.\, \frac{\pi D^2}{4}\right)$ is the effective cross-sectional area of the core. Hence

$$\text{K.V.A.} = 4.44\ T_s \sim \times B_m \,.\, A \,.\, a_s \,.\, Q \times 10^{-11}$$

$$\text{or } T_s \,.\, a_s \,.\, QB_m = \frac{\text{K.V.A.} \times 10^{11}}{4.44 \sim .\ A}$$

The space between the cores is taken up partly by primary and

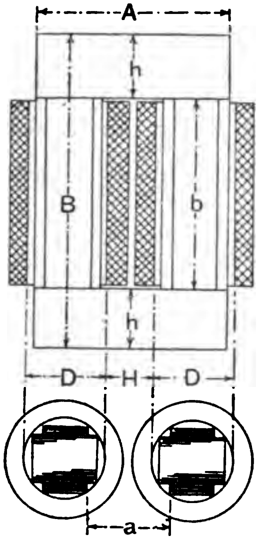


FIG. 116.—Circular core type transformer.

partly by secondary winding. Since the ampère-turns of the latter are approximately equal to those of the primary,

$$(T_s \,.\, a_s + T_p \,.\, a_p)\ QB_m = \frac{2\ \text{K.V.A.} \times 10^{11}}{4.44 \sim .\ A}$$

From Figure 116, which gives the notations for the principal dimensions, it is obvious that

$$a_s T_s + a_p T_p = b \,.\, H \,.\, F_c$$

Substituting this value in the above equation and putting $A = \frac{\pi}{4} D^2 F_i$

$$Q \,.\, B_m \,.\, b \,.\, H \,.\, D^2 = \frac{2\ \text{K.V.A.} \times 10^{11} \times 4}{\pi \times 4.44 \sim \times F_i \,.\, F_c} = \frac{5.7 \times 10^{10}\ \text{K.V.A.}}{\sim \times F_i \,.\, F_c} = a \quad . \quad . \quad (20)$$

b, H, and D being expressed in centimetres.

The weight of iron and copper must next be considered. In practice the height h of the yoke is generally fixed so as to give equal cross-sectional area of iron in the cores and yokes, but for the sake of simplicity it will be assumed that $h \cong D$. This assumption, though not absolutely correct, introduces very little error into the final result.

The weight of iron is

$$K_i = F_i \times \frac{\pi D^2}{4} \times (4\ D + 2\ H + 2\ b) \times 7.8 \times 10^{-3}$$

$$= 12 \times 10^{-3} \,.\, F_i \,.\, D^2 \,.\, (2\ D + H + b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The weight of copper may be similarly expressed by

$$K_c = 2F_c . \{(D+H)^2 - D^2\} \,.\, \frac{\pi}{4} \,.\, b \times 8.9 \times 10^{-3}$$

$$= 28 \times 10^{-3}\ F_c\ (DH + 0.5\ H^2)\ b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Expressions for K_i and K_c can also be obtained from equation 18 when $\frac{K_i}{C_0}$ and $C_0 K_c$ are substituted for K_c and K_i respectively. Thus

$$K_i = \frac{C_0 W_t}{Q \,.\, B_m} \times \sqrt{\frac{p_c \cdot p_i}{C_0 \cdot C_1 \cdot C_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$\text{and } K_c = \frac{W_t}{Q \,.\, B_m} \times \sqrt{\frac{p_c \cdot p_i}{C_0 \cdot C_1 \cdot C_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

When equations 23 and 24 are combined with equations 21 and 22 respectively the following equations are obtained :—

$$Q \,.\, B_m \,.\, D^2 (2\ D + H + b) = \frac{C_0 W_t}{12 \times 10^{-3} F_i} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c \times p_i}{C_0 \cdot C_1}} = \beta \quad . \quad . \quad (25)$$

$$Q \,.\, B_m (D.H + 0.5 H^2) b = \frac{W_t}{28 \times 10^{-3} F_c} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c \times p_i}{C_0 \cdot C_1}} = \gamma \quad . \quad . \quad (26)$$

In Figure 110 (page 145), $\sqrt{C_2}$ has, for ordinary transformer iron 0.35 mm. thick, and at 50~, been plotted as a function of the flux density B_m. The curve shows that, for the usual values of B_m, *i.e.* from 5000 to 12000, $\sqrt{C_2}$ varies only from 1.8×10^{-4} to 1.65×10^{-4}. For alloyed iron the constancy of $\sqrt{C_2}$ is even more marked. This being the case, no great inaccuracy will be introduced if $\sqrt{C_2}$ be treated as a constant equal in value to that for $B_m = 7000$ in the case of ordinary iron, and $B_m = 10{,}000$ for alloyed iron. On this basis the values of $\sqrt{C_2}$ for various frequencies will be as set forth in Table XII.

TABLE XII.—VALUES OF $\sqrt{C_2}$ FOR TRANSFORMERS.

Grade of Iron.	25~	40~	50~	60~
Ordinary transformer, 35 mm.	1.1×10^{-4}	...	1.75×10^{-4}	...
Ordinary transformer, 0.50 mm.	1.15×10^{-4}	...	1.9×10^{-4}	...
Alloyed iron, 0.35 mm.	...	...	1.65×10^{-4}	...
,, 0.50 mm.	...	1.22×10^{-4}	1.38×10^{-4}	1.48×10^{-4}

When working out a new design, of which the efficiency and distribution of the losses are known, and the space factors fixed, the value of α, β, and γ are first determined. Equations 20, 25, and 26 may then be used for a solution of the problem. The unknown quantities Q and B_m appear as the product $Q \cdot B_m$ in all of these equations. Treating this product as a single unknown quantity, the following equations are obtained:—

From equations 20 and 26—

$$Q \cdot B_m = \frac{\alpha}{b \cdot H \cdot D^2} = \frac{\gamma}{(DH + 0{\cdot}5\ H^2)\ b}$$

$$\textit{i.e.},\ H = 2\ D\left(\frac{\gamma}{\alpha} \cdot D - 1\right) \quad \ldots \ldots \quad (27)$$

From equations 20 and 25—

$$\beta = QB_mD^2\ (2\ D + H + b)$$

$$= QB_mD^2\left(2\ D + H + \frac{\alpha}{QB_mHD^2}\right)$$

$$= QB_mD^2\ (2D + H) + \frac{\alpha}{H}$$

$$\textit{i.e.},\ QB_m = \frac{\beta - \frac{\alpha}{H}}{D^2\ (2\ D + H)} \quad \ldots \ldots \quad (28)$$

Finally, from equation 20—

$$b = \frac{\alpha}{QB_mHD^2} \quad \ldots \ldots \quad (29)$$

Bearing in mind that the most economical design will be the one for which the product QB_m is a maximum, the main dimensions of the transformer may be derived as follows.

By means of equations 20, 25, and 26 the factors α, β, and γ are first determined, then from equations 27 and 28 the values of H and the corresponding values of QB_m are calculated for several *assumed* values of D. If the former be plotted as a function of D, the curve (see Figure 119) so obtained gives the most economical value of QB_m and D. The corresponding values of H and b are then calculated by

equations 27 and 29 respectively. The separate values of Q and B_m are expressed by—

$$(\textit{vide}\ \text{equation 16})\quad Q=\sqrt{\frac{W_c}{C_1K_c}}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

K_c being determined from equation 22,

$$\text{and } B_m=\frac{QB_m}{Q}$$

After having settled the principal dimensions of the transformer, the calculation of the flux, the number of primary and secondary turns, the radiating surface, the no-load current and the drop is a simple matter. If it be found that the radiating surface is insufficient, either the efficiency must be increased or a more suitable method of cooling adopted.

(*b*) **Rectangular Core.**—The outline of the magnetic circuit for a transformer having a rectangular core is shown in Figure 117. The

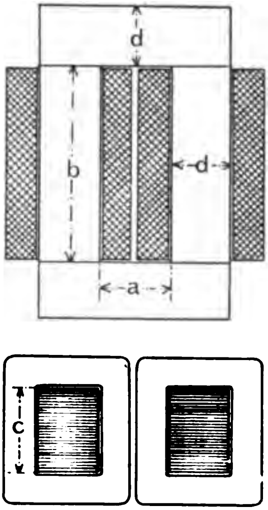


FIG. 117.—Rectangular core type transformer.

cross-section of iron in the core $=d\times c\times F_i$ has the values given in Table XI. The usual practice is to make the height of the yoke equal the core width, hence $h=d$.

As is the case with a circular core transformer,

$$\left(T_sa_s+T_pa_p\right)QB_m=\frac{2\ \text{K.V.A.}\times 10^{11}}{4.44\times\sim A}$$

Now, $T_s\ a_s+T_p\ a_p=a\ .\ b\ .\ F_c$ and $A=c\ .\ d\ .\ F_i$; hence, substituting these values in the above equation

$$QB_m.abcd=\frac{4.5\times 10^{10}\ \text{K.V.A}}{\sim F_i\ .\ F_c}=\alpha\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Now, the mass of iron expressed in kilogrammes is

$$K_i = F_i cd\,(2a + 2b + 4d) \times 7.8 \times 10^{-3}$$
$$= 15.6 \times 10^{-3} \,.\, F_i cd(a + b + 2d) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (32)$$

Since the volume of the space available for winding

$$= a \,.\, b\,(2a + 2c + 2d),$$

the weight of copper is expressed by

$$K_c = F_c ab \times 2\,(a + c + d) \times 8.9 \times 10^{-3}$$
$$= 17.8 \times 10^{-3} \,.\, F_c ab(a + c + d) \,. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (33)$$

Combining these expressions for K_i and K_c with equations 23 and 24 respectively, we have

$$\beta = \frac{C_0 W_i}{15.6 \times 10^{-3} \,.\, F_i} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c \,.\, p_i}{C_0 \,.\, C_1}} \quad . \quad . \quad . \quad . \qquad (34)$$

$$\gamma = \frac{W_i}{17.8 \times 10^{-3} \,.\, F_c} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c \,.\, p_i}{C_0 \,.\, C_1}} \quad . \quad . \quad . \quad . \qquad (35)$$

The cost of active material is

$$P_t = S_i K_i + S_c \,.\, K_c$$
$$= G_1 \,.\, cd(a + b + 2d) + G_2 ab(a + c + d),$$

where $G_1 = 15.6 \times 10^{-3} \,.\, F_i \,.\, S_i$, and $G_2 = 17.8 \times 10^{-3} \,.\, F_c \,.\, S_c$

An equation giving the relationship between a, c, and d for the best value of P_t is obtained by differentiating the latter equation with respect to b and placing the differential coefficient $= 0$. Thus

$$\frac{dP_t}{db} = G_1 cd + G_2 a(a + c + d) = 0$$

$$i.e., \; a(a + c + d) = -\frac{G_1}{G_2} \,.\, cd$$

The relation between a, b, and d for the best value of P_t is similarly obtained; thus—

$$\frac{dP_t}{dc} = G_1 d(a + b + 2d) + G_2 ab = 0$$

$$i.e., \; d(a + b + 2d) = -\frac{G_2}{G_1} \,.\, ab$$

For the most economical design cost of iron = cost of copper, therefore—

$$G_1 cd(a + b + 2d) = G_2 ab(a + c + d)$$

Substituting the above values for $d(a + b + 2d)$ and $a(a + c + d)$,

$$a = \frac{G_1 d}{G_2} = \frac{15.6 \times 10^{-3} \,.\, F_i \,.\, S_i}{17.8 \times 10^{-3} \,.\, F_c \cdot S_c} d = \frac{0.875\, F_i}{C_0 \,.\, F_c} \cdot d \,. \quad . \quad . \quad . \qquad (36)$$

From equations 31, 34, and 23—

$$\frac{a}{ab} = \frac{\beta}{(a+b+2d)}$$

$$\text{since } \frac{a}{ab} = QB_m cd$$

$$= \frac{C_0 W_t}{K_i} \cdot \frac{1}{\sqrt{C_2}} \cdot \sqrt{\frac{p_c p_i}{C_0 C_1}} \text{ from } 23$$

$$= \frac{\beta}{a+b+2d} \text{ from 32 and 34}$$

$$\text{Hence } b = \frac{a+2\,d}{\frac{\beta}{a}a - 1} \qquad (37)$$

From equations 31 and 35—

$$\frac{a}{cd} = \frac{\gamma}{a+c+d}$$

$$i.e.,\ c = \frac{d+a}{\frac{\gamma}{a}d - 1} \qquad (38)$$

Finally, from equation 31—

$$Q \,.\, B_m = \frac{a}{abcd} \qquad (39)$$

For several assumed values of d, the corresponding values of a, b, c, and QB_m are calculated successively. A curve is then plotted showing QB_m as a function of d, from which the value of the latter corresponding to the most economical design may be obtained. The individual values of Q and B_m are afterwards calculated from equation 30.

Three-Phase

For 3-phase transformers of unsymmetrical core type (see Figure 56), all the cores being arranged in one line, the equations are as follows:—

(1) **Circular Core.**—

$$a = QB_m b H D^2 = \frac{3.28 \times 10^{10} \times \text{K.V.A.}}{\sim .\, F_i \,.\, F_c} \qquad (40)$$

$$K_i = 6.1 \times 10^{-3} \,.\, F_i \,.\, D^2(6\,D + 4\,H + 3\,b) \qquad (41)$$

$$K_c = 42 \times 10^{-3} \,.\, F_c(DH + 0.5\,H^2)b \qquad (42)$$

$$K_i = \frac{C_0 W_t}{QB_m} \times \sqrt{\frac{p_c p_i}{C_0 C_1 C_2}}$$

$$K_c = \frac{W_t}{QB_m} \times \sqrt{\frac{p_c p_i}{C_0 C_1 C_2}}$$

$$\beta = \frac{C_0 W_t}{6.1 \times 10^{-8} . F_i} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c p_i}{C_0 C_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

$$\gamma = \frac{W_t}{42 \times 10^{-8} . F_c} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c . p_i}{C_0 . C_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

$$H = 2\,D\left(\frac{\gamma D}{a} - 1\right)$$

$$QB_m = \frac{\beta H - 3\,a}{D^2 H(6\,D + 4\,H)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$$b = \frac{a}{QB_m H D^2}$$

$$Q = \sqrt{\frac{W_c}{C_1 K_c}}$$

(2) Rectangular Core.—

$$a = \frac{2.6 \times 10^{10} . \text{K.V.A.}}{\sim F_i . F_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

$$K_i = 7.8 \times 10^{-8} . F_i . cd(4a + 3b + 6d) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

$$K_c = 27 \times 10^{-8} . F_c . ab(a + c + d) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

$$\beta = \frac{C_0 W_t}{7.8 \times 10^{-8} . F_i} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c . p_i}{C_0 . C_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

$$\gamma = \frac{W_t}{27 \times 10^{-8} . F_c} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c . p_i}{C_0 . C_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

$$a = \frac{0.29 F_i . d}{C_0 . F_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

$$b = \frac{2(2a + 3d)}{\frac{\beta}{a}a - 3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

$$c = \frac{d + a}{\frac{\gamma}{a}d - 1}$$

$$Q . B_m = \frac{a}{abcd}$$

Shell Type Single-Phase Transformers

Rectangular Core.—The outline of the magnetic circuit for a transformer of the shell type is shown in Figure 118. The cross-section of the core $= F_i . c . d$. As regards ventilating ducts, they are

given a width approximately $= 0.5$ cm., and the required number varies with the output as set forth in Table XIII., which also gives the corresponding values of F_i.

As the proportions of the coils will be approximately that given in

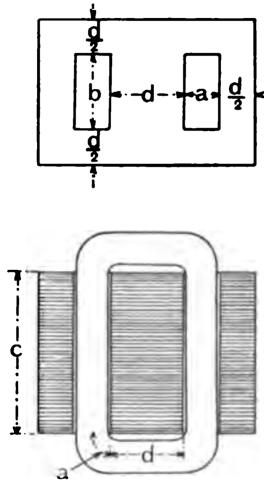


FIG. 118.—Rectangular shell type transformer.

the figure, the mean length per turn $= 4a + 2c + 2d$, and the winding space factors F_c will have about the same values as for the core type.

TABLE XIII.—IRON SPACE FACTORS F_i FOR RECTANGULAR SHELL TYPE TRANSFORMER, VENTILATING DUCTS 0.5 CM. WIDE.

Output in K.V.A.	Number of Ducts.	Space Factor F_i.	
		0.35 mm. Plates.	0.50 mm. Plates.
0 – 50	1	0.84	0.89
50 – 100	2	0.82	0.87
100 – 150	3	0.80	0.84
150 – 200	4	0.77	0.82
200 – 300	5	0.75	0.80
300 – 400	7	0.71	0.75
400 – 500	9	0.67	0.71
500 – 700	11	0.62	0.66
700 upwards	13	0.58	0.61

The design equations, derived in a similar manner to those for the transformer of the rectangular core type, are as follows :—

$$\alpha=\frac{4.5\times 10^{10}\text{ K.V.A.}}{\sim F_i\,.\,F_c}$$

$$\beta=\frac{C_0 W_t}{15.6\times 10^{-3} F_i}\times\frac{1}{\sqrt{C_2}}\times\sqrt{\frac{p_c\,.\,p_i}{C_0\,.\,C_1}}$$

$$\gamma=\frac{W_t}{17.8\times 10^{-3} F_c}\times\frac{1}{\sqrt{C_2}}\times\sqrt{\frac{p_c\,.\,p_i}{C_0\,.\,C_1}}$$

$$a=\frac{0.875\,F_i}{C_0\,.\,F_c}\,.\,d$$

$$b=\frac{a+d}{\frac{\beta a}{\alpha}-1}\quad\ldots\ldots\ldots\ldots\quad(53)$$

$$c=\frac{d+2a}{\frac{\gamma d}{\alpha}-1}\quad\ldots\ldots\ldots\ldots\quad(54)$$

$$Q.B_m=\frac{\alpha}{abcd}$$

Designs of Five Single-Phase, Oil-Immersed Transformers of the Core Type to give an Output of 100 K.V.A.

Two single-phase transformers of the core type have to be designed to the following specification. Transformer A is for 50 periods, while transformer B is for 25 periods. The design for the 50 period transformer is worked out in full.

Transformer A 50~.

Specification—

Rated output	= 100 K.V.A.
Primary terminal pressure	= 2000 volts.
Secondary terminal pressure	= 400 „
Full-load efficiency ($\cos\phi = 1$) . . .	= 98 per cent.

Voltage drop for $\cos\phi = 0.80$ not to exceed 2.9 per cent.
Temperature rise of any part not to exceed 50° C.
Transformer to be operated continuously at full-load.

Design.—The transformer will be designed on the assumption that ordinary transformer iron 0.35 mm. thick is employed for the construction of the core. Since the load factor is approximately 100 per cent., the losses,* $W_t \cong 2000$ watts, will be distributed equally between copper and iron so that $p_c = p_i = 0.5$, and $W_c = W_i = 1000$ watts. For a primary pressure of 2000 volts the copper space factor will be

* The exact value of $W_t = 2040$ watts.

about 0.425, *i.e.* $F_c = 0.425$. Adopting the circular cross-section of Figure 112, $F_i = 0.67$. From pages 144 and 145 respectively, $C_1 = 2.7 \times 10^{-4}$, and $\sqrt{C_2} = 1.75 \times 10^{-4}$. Suppose the price of insulated copper wire and stamped iron sheets is 1s. 10d. and 7d. per kilogramme respectively, then $C_0 = 3.15$. From this data the design is worked out as follows:—

Design Coefficients—

$$\alpha = \frac{5.7 \times 10^{10} \,.\, \text{K.V.A.}}{\sim F_i \,.\, F_c} = \frac{5.7 \times 10^{10} \times 100}{50 \times 0.67 \times 0.425} = 40 \times 10^{10}$$

$$\beta = \frac{C_0 W_t}{12 \times 10^{-3} \,.\, F_i} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c \,.\, p_i}{C_0 \,.\, C_1}}$$

$$= \frac{3.15 \times 2000}{12 \times 10^{-3} \times 0.67} \times \frac{1}{1.75 \times 10^{-4}} \times \sqrt{\frac{0.5 \times 0.5}{3.15 \times 2.7 \times 10^{-4}}} = 7.7 \times 10^{10}$$

$$\gamma = \frac{W_t}{28 \times 10^{-3} \,.\, F_c} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c \,.\, p_i}{C_0 \,.\, C_1}}$$

$$= \frac{2000}{28 \times 10^{-3} \times 0.425} \times \frac{1}{1.75 \times 10^{-4}} \times \sqrt{\frac{0.5 \times 0.5}{3.15 \times 2.7 \times 10^{-4}}} = 1.65 \times 10^{10}$$

Main Dimensions.—Next assume various values of D, and determine the corresponding ones of H and QB_m from equations 27 and 28, thus—

Let D = 28 cms.	then H =	8.4 cms.	and	QB_m =	6.0×10^5
29 ,,	,,	11.0	,,	,,	7.1×10^5
29.5 ,,	,,	12.4	,,	,,	7.2×10^5
30 ,,	,,	13.8	,,	,,	7.2×10^5
31 ,,	,,	16.7	,,	,,	7.0×10^5
32 ,,	,,	19.8	,,	,,	6.6×10^5

In Figure 119 the values QB_m have been plotted as a function of D. The most economical values for D and QB_m are 300 mms. and

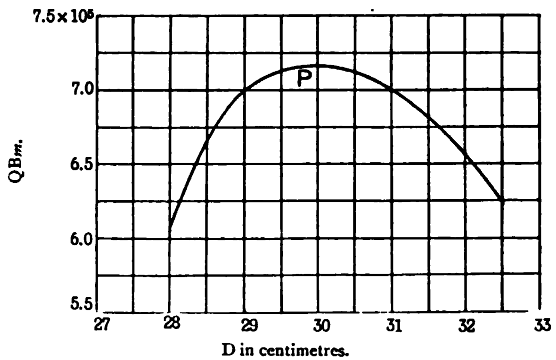


FIG. 119.

7.2×10^5 respectively denoted by the peak P of the curve. These values now form the basis of the subsequent calculations.

$$b = \frac{a}{QB_m \; H.D^2} = \frac{40 \times 10^{10}}{7.2 \times 10^5 \times 13.8 \times 30^2} \text{ cms.} = 450 \text{ mms.}$$

$$K_c = \frac{W_t}{QB_m} \times \frac{1}{\sqrt{C_2}} \times \sqrt{\frac{p_c p_i}{C_0 \cdot C_1}}$$

$$= \frac{2000}{7.2 \times 10^5} \times \frac{1}{1.75 \times 10^{-4}} \times \sqrt{\frac{0.5 \times 0.5}{3.15 \times 2.7 \times 10^{-4}}} = 270 \text{ kgs.}$$

$$K_i = 3.15 \times 270 = 850 \text{ kgs.}$$

$$Q = \sqrt{\frac{W_c}{C_1 K_c}} = \sqrt{\frac{1000}{2.7 \times 10^{-4} \times 270}} = 115 \text{ ampères per sq. cm.}$$

$$B_m = \frac{QB_m}{Q} = \frac{7.2 \times 10^5}{115} = 6300$$

A dimensioned sketch of the magnetic circuit is given in Figure 120,

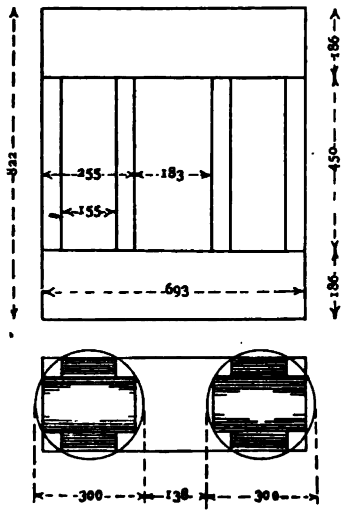


FIG. 120.—Magnetic circuit of 100-K.V.A. 50~ transformer.

the cross-sectional area of the yoke being made equal to that of the core.

Windings.—The main dimensions having thus been settled, the details of the windings are then calculated as follows:—

Area of cross-section of core

$$= A = F_i \times \frac{\pi D^2}{4} = \frac{0.67 \times \pi \times 30^2}{4} = 475 \text{ sq. cms.}$$

Magnetic flux $= \Phi = B_m \times A = 6300 \times 475 = 3.0 \times 10^6$ lines.

$$\text{Number of secondary turns} = T_s = \frac{E_s \times 10^8}{4.44 \sim \Phi}$$

$$= \frac{400 \times 10^8}{4.44 \times 50 \times 3.0 \times 10^6} = 60$$

$$\text{Ratio of transformation} = \frac{T_p}{T_s} = \frac{2000}{400} = 5$$

Hence number of primary turns $= T_p = 60 \times 5 = 300$

$$\text{Secondary current} = I_s = \frac{10^5}{400} = 250 \text{ ampères}$$

$$\text{Primary current} = I_p = I_s \times \frac{T_s}{T_p} \times \frac{100}{\text{efficiency per cent.}}$$

$$= 250 \times \frac{1}{5} \times \frac{100}{98} = 51 \text{ ampères}$$

Cross-section of secondary conductor $= I_s/Q = 217$ sq. mms.
Cross-section of primary conductor $= I_p/Q = 44$ sq. mms.

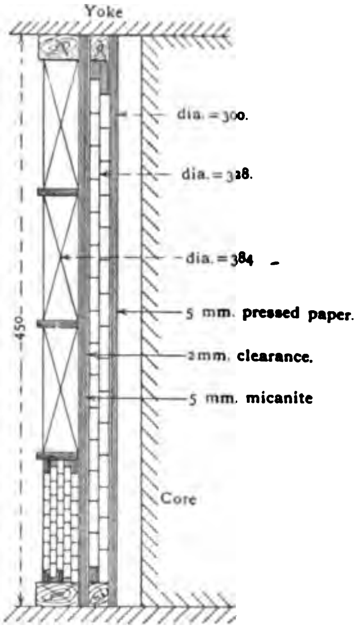


FIG. 121.—Design of windings for 100-K.V.A. transformer, 2000/400 volts.

The primary and secondary windings will be arranged concentrically, with the low-tension coils nearest the core. The bobbins, on which the secondary is wound, would probably consist of a fibre or compressed paper cylinder (see Figure 121) about 5 mms. in thickness and provided with teak or fibre flanges. If the latter have a thickness of 20 mms., then the length of the bobbin available for the winding $= 410$ mms.

In order to keep down the eddy-current loss in the copper and obtain flexibility in winding, it is not advisable to use a conductor of much greater section than 100 sq. mms. The conductor for the secondary winding will therefore consist of three copper strips connected in parallel; each strip will have a cross-sectional area of 73 sq. mms. While being wound on the bobbin, the strips would be laid one on the top of the other and bound together with insulation to the thickness of 0.3 mm. Each secondary coil must contain 30 turns, and these will be disposed in two layers, *i.e.* there are 15 turns per layer. In crossing over from one layer to the next, the space of one turn will be lost, hence the width of insulated conductor must be $\frac{1}{16}$ of the net winding length, *i.e.* $\frac{410}{16} = 25.6$ mms. Since the tape is wound on to the thickness of 0.3 mms. the width of the copper strip $= (25.6 - 2 \times 0.3) = 25$ mms., and thickness $= \frac{73}{25}$ $= 2.9$ mms. The size of each insulated conductor therefore $= 25.6 \times 9.3$ mms., and the radial depth of secondary winding 18.6 mms.

The primary coil will also be wound on a bobbin just large enough to slip easily over the secondary. For a pressure of 2000 volts, the bobbin would consist of a micanite cylinder 5 mms. thick. Each coil will have 150 turns, wound in four sections of 38, 38, 37, and 37 turns each, adjacent sections being separated from each other by a split mica ring 3.3 mms. thick. The winding length available per section will be 100 mms., and if the winding be arranged in 5 layers with 8, 8, 8, 7, 7 turns per layer, then the size of conductor will be 10.4×4.0 mms. (bare) or 11.0×4.6 mms. (insulated). If between adjacent layers there be inserted press-spahn or micanite insulation of 1 mm. thickness, then the radial depth of primary winding $= 27$ mms. The overall diameter of the windings is therefore 413 mms., leaving a space of 25.5 mms. between the windings of each core.

No-load Current and Power Factor.—Mean length of magnetic circuit $= 2(H + D + h + b) = 215$ cms. For $B_m = 6300$, the virtual ampère-turns per cm. $= \frac{2}{\sqrt{2}} = 1.4$ (from Fig. 57), hence magnetising ampère-turns $= 1.4 \times 215 = 300$, and magnetising current $= I_m = \frac{300}{T_p}$ $= \frac{300}{300} = 1$ ampère.

The energy component of the no-load current $I_e =$ core loss $\div$ primary volts $= \frac{1000}{2000} = 0.5$ ampères. Therefore no-load currrent $=$ $I_o = \sqrt{I_e^2 + I_m^2} = \sqrt{0.5^2 + 1.0^2} = 1.1$ ampère $= 2.2$ per cent. of full-load primary current.

Regulation.—Secondary ohmic drop $= I_s R_s$

$$= \frac{I_s \times 2 \times 10^{-6} \times k \times l_s \times T_s}{A_s} = 250 \times \frac{2 \times 10^{-6} \times 1.2 \times 103 \times 60}{2.19} = 1.7 \text{ volts},$$

where mean length of turn $l_s = \pi \times 32.8 = 103$ cms.

Primary ohmic drop transferred to secondary $= I_p R_p \frac{T_s}{T_p}$

$= 51 \times \frac{2 \times 10^{-6} \times 1.15 \times 120 \times 300}{0.416} \times \frac{1}{5} = 2$ volts, where mean length of primary turn $= 120$ cms.

Secondary drop due to total ohmic resistance $= 3.7$ volts $= 0.92$ per cent.

Maximum inductive drop per cent. by Kapp's formula (see p. 94).

$$= \frac{100 \text{ KAT}_s}{\Phi}\left(\Delta + \frac{\Delta_1 + \Delta_2}{3}\right)\frac{p}{L}$$

$$= \frac{100 \times 1 \times 250 \times 60}{3.0 \times 10^6}\left(0.56 + \frac{1.86 + 2.7}{3}\right)\frac{\pi \times 34.3}{41}$$

$= 2.6$ per cent.

Percentage regulation (*vide* page 100)

$$= V_2 \sqrt{1 - \cos^2 \phi} + V_3 \cos \phi.$$

For $\cos \phi = 0.8$, regulation

$$= 2.6 \sqrt{1 - 0.8^2} + .092 \times 0.8 = 2.1 + 0.72 = 2.29 \text{ per cent.}$$

Had the percentage regulation exceeded the specified value it would have been necessary to modify the design. The above equation for inductive drop shows that one method of improving the regulation, should it exceed the specified limit, is to diminish the secondary turns T_s and increase the flux Φ proportionally. The price of active material will then for the same efficiency be somewhat greater than the minimum cost. The regulation could also be improved by employing a double concentric winding, with one-half of the secondary inside, and the other half outside the primary. This latter procedure would, however, add considerably to the manufacturing cost.

Heating.—(1) *Winding.*—The outside cooling surface of winding $= 2\pi \times 42.0 \times 41 = 11{,}000$ sq. cms. $= 110$ sq. decimetres. The inside cooling surface of winding $= 2\pi \times 30 \times 41 = 7700$ sq. cms. $= 77$ sq. decimetres. Owing to the comparatively thick flanges on the bobbins the cooling effect of the ends of the coils will be exceedingly small, and may be neglected. Total cooling surface for windings $= 187$ sq. decimetres. Since the copper loss $= 1000$ watts, the watts per square decimetre of cooling surface $= 1000 \div 187 = 5.4$. If this transformer be air cooled with natural draught, the temperature rise of winding would $= K \cdot \frac{W}{A} = 14 \times 5.4 = 75°$ C., the value of K

being taken from Table IX. page 129. This rise of temperature is excessive, so that oil cooling must be adopted. Temperature rise of winding above oil $= 2 \times 5.4 = 10.8°$ C., say $11°$ C.

(2) *Iron.*—From Figs. 112, 116 and 120, edge-cooling surface of core + yoke, since the breadth of the core $= 0.85$ D is

$$\begin{aligned} &= 2(2 \times 0.85\ D \times b) + 2\{0.85\ D\ (2\ h + A)\} \\ &= 1.7\ D\ (2\ b + 2\ h + A) = 1.7 \times 30\ (90 + 37.2 + 69.3) \\ &= 10{,}000 \text{ sq. cms.} = 100 \text{ sq. decimetres.} \end{aligned}$$

The inside edge surface of yoke is here neglected as its cooling effect will be very small owing to the bobbin flanges lying hard against it.

Watts per square decimetre of cooling surface $= 1000 \div 100 = 10$.
Estimated temperature rise of iron above oil $= 1.7 \times 10 = 17°$ C.

Since the temperature rise of any part has not to exceed $50°$ C., the difference in temperature between the cooling oil and the external air must not exceed about $33°$ C. Hence the containing tank must be designed to have an external radiating surface not less than $A = \dfrac{K.W_t}{T^0} = \dfrac{15 \times 2000}{33} = 910$ sq. decimetres.

The case for this transformer would have the dimensions 100×60 cms. by 100 cms. high, and the sides would be corrugated so as to give the required radiating surface.

Weights and Costs—

Weight of copper $= 270$ kgs.
Weight of iron $= C_0 K_c = 3.15 \times 270 = 850$ kgs.
Total weight, iron + copper $= 1120$ kgs.
Weight of active material per kilowatt at unity power factor

$$= \frac{1120}{100} = 11.2 \text{ kgs.}$$

Cost of active material $= 1.83\left(270 + \dfrac{850}{3.15}\right) = 1000$ shillings $= £50$.

Cost of active material per kilowatt at unity power factor $= 10$ shillings.

Transformer B 25~.

The second design for 25 periods is calculated in a similar manner to the above, and in order to effect a comparison of the two designs the principal dimensions and constants have been tabulated in Table XIV. The laminations for a 25~ transformer will have a thickness of 0.5 mm., so that adopting the same cross-section of core as before, the iron space factor $F_t = 0.7$. The new values of C_1 and $\sqrt{C_2}$ will be 2.45×10^{-4}, and 1.15×10^{-4} respectively.

TABLE XIV.—COMPARATIVE DESIGNS OF 100-K.V.A. TRANSFORMERS FOR 50~ AND 25~.

Ratio of transformation 2000/400 volts. Full-load efficiency, 98 per cent.
Full-load copper loss = iron loss $\simeq$ 1000 watts.

	Transformer A 50~.	Transformer B 25~.
Constant a	40×10^{10}	76×10^{10}
,, β	7.7×10^{10}	11.8×10^{10}
,, γ	1.65×10^{10}	2.65×10^{10}
Best value of QB_m	7.2×10^5	6.7×10^5
Dimension D	300 mms.	350 mms.
,, H	138 ,,	160 ,,
,, b	450 ,,	580 ,,
,, h	186 ,,	216 ,,
Weight of copper K_c	270 kgs.	465 kgs.
,, iron K_i	850 ,,	1460 ,,
Current density Q, ampères per sq. cm.	115	93
Flux density in lines per sq. cm. ($= B_m$)	6300	7300
Cross-sectional area of core	475 sq. cms.	643 sq. cms.
Magnetic flux Φ	3.0×10^6	4.7×10^6
Number of secondary turns	60	76
,, primary turns	300	384
Cross-sectional area of primary conductor	218 sq. mms.	270 sq. mms.
,, ,, secondary ,,	28 ,,	35 ,,
No-load current	1.10 ampères	1.20 ampères
No-load in per cent. of full-load current	2.2 per cent.	2.4 per cent.
Total weight of active material	1120 kgs.	1925 kgs.
Weight of active material per K.W. at unity power factor	11.2 ,,	19.25 ,,
Cost of active material	£50	£85
,, ,, per K.W. for $\cos \phi = 1$	10 shillings	17 shillings.

When these two designs are compared, two important features should be noted. (1) A high-frequency transformer can be constructed with less active material than that required for one of a lower frequency, consequently a 25~ transformer will be larger and more expensive than one for 50~, the difference in cost of active material for the above designs being as great as 70 per cent. (2) The flux density in a low-frequency transformer will, for the same output and ratio of transformation, be greater than that in one designed for a higher frequency. This is quite logical, for, to obtain the same specific iron loss at 25~ as at 50~, the induction must be greater.

COMPARATIVE DESIGNS OF A 100-K.V.A., 50~, 2000/400 VOLTS, OIL-COOLED, SINGLE-PHASE TRANSFORMERS FOR 98, 98.2, AND 98.4 PER CENT. EFFICIENCY.

In order to show the influence of alloyed iron on the design of a transformer, and also the relationship between the efficiency and

the cost of active material, the data of three 100-K.V.A. 50~ transformers constructed of alloyed iron is set forth below, the calculations having been made for efficiencies of 98, 98.2, and 98.4 per cent.

Data upon which the designs are based—

Copper loss = iron loss, i.e. $p_c = p_i$.

Core constructed of alloyed iron sheets, 0.5 mm. in thickness.

Copper space factor $= F_c = 0.425$.

Iron space factor $= F_i = 0.70$.

Price of insulated copper = 1s. 10d. per kilogramme.

Price of alloyed iron = 11d. ,, ,,

$C_0 = 2$, $C_1 = 2.7 \times 10^{-4}$, and $\sqrt{C_2} = 1.38 \times 10^{-4}$.

	C.	D.	E.
Efficiency at full-load, per cent.	98.0	98.2	98.4
Total loss in watts ($= W_t$)	2000	1800	1600
Approximate copper loss in watts ($= W_c$)	1000	900	800
Approximate iron loss in watts ($= W_i$)	1000	900	800
Design Coefficients—			
α	38.3×10^{10}	38.3×10^{10}	38.3×10^{10}
β	7.45×10^{10}	6.65×10^{10}	6.0×10^{10}
γ	2.62×10^{10}	2.35×10^{10}	2.1×10^{10}
Main Dimensions and Magnetic Flux—			
Best value of QB_m	22.8×10^5	14.5×10^5	9.6×10^5
D in mms.	190	210	240
H ,,	114	118	154
b ,,	406	510	450
h ,,	123	135	154
Weight of copper in kilogrammes ($= K_c$)	137	190	260
,, iron ,, ($= K_i$)	274	380	520
Current density, ampères per sq. cm. ($= Q$)	164.5	130	107
Flux density lines per sq. cm. ($= B_m$)	13,900	11,000	9000
Shape of cross-section of core	See	Figure 112	
Area of cross-section of core in sq. cms.	198	240	315
Magnetic flux Φ	2.75×10^6	2.6×10^6	2.84×10^6
Secondary Winding—			
Number of turns in series	66	70	64
Current in ampères	250	250	250
Cross-sectional area of conductor in sq. mms.	152	190	235
Size of conductor (bare)	$3(19.7 \times 2.6)$	$3(24.2 \times 2.6)$	$3(23.5 \times 3.3)$
,, ,, (insulated)	20.3×8.3	24.8×8.4	24.1×10.6
Number of turns per core	33	35	32
Number of layers per core	2	2	2
Turns per layer	16 and 17	18 and 17	16 and 16
Radial depth of winding in mms.	16·6	16.8	21.2

	C.	D.	E.
Primary Winding—			
Number of primary turns	330	352	320
Primary current in ampères	51	51	51
Cross-sectional area of conductor in sq. mms.	31.0	39.0	47.5
Size of conductor (bare)	10.5 × 3	10.9 × 3.6	10.4 × 4.55
„ „ (insulated)	11.1 × 3.6	11.5 × 4.2	11.0 × 5.15
Number of sections	8	8	8
Turns per section	41,41,41,42	44	40
Number of layers per section	6	5	5
Thickness of insulation between layers (mm.)	1.0	1.0	1.0
Radial depth of winding in mms.	31.6	25	30
Copper Drop—			
Mean length of secondary turn in cms.	67	75	85
Secondary IR drop in volts	1.75	1.66	1.38
Mean length of primary turn	85	95	105
Primary IR drop transferred to secondary	2.12	2.0	1.65
Total secondary drop due to ohmic resistance	3.87	3.66	3.03
Secondary drop expressed as a percentage	0.97 %	0.915 %	0.76 %
Inductive Drop—			
Secondary ampère turns	250 × 66	250 × 70	250 × 64
Distance between windings ($=\Delta$) in cms.	0.56	0.56	0.56
Radial depth of secondary winding ($=\Delta_2$), cms.	1.66	1.68	2.16
Radial depth of primary winding ($=\Delta_1$), cms.	3.16	2.5	3.0
Mean length per turn for both windings ($=p$)	80	88	100
Length of winding space (L), cms.	36	47	41
Inductive drop expressed as a percentage	2.9 %	2.47 %	3.1 %
Regulation—			
Regulation, $\cos\phi=0.8$	2.52 %	2.21 %	2.46 %
No-load Current—			
Mean length of magnetic circuit in cms.	166	195	199
Ampère-turns per cm. (RMS. value)	5.8	3.1	2.26
Magnetising current (I_m)	2.9 amps.	1.72 amps.	1.4 amps.
Energy component of no-load current (I_e)	0.5 „	0.45 „	0.4 „
No-load current $=\sqrt{I_m^2+I_e^2}$	2.95	1.78	1.45
No-load current as a percentage of full-load current	5.9 %	3.56 %	2.90 %
Heating.—(1) *Windings—*			
Outside cooling surface of winding in sq. decimetres	72	94	92
Inside	44	62	62
Total	116	156	154
Watts per sq. decimetre of cooling surface	8.6	5.8	5.2
Probable temperature rise above oil	17.2° C.	11.6° C.	10.4° C.
(2) *Iron—*			
Edge cooling surface of core and yoke	50 sq. dcms.	64 sq. dcms.	68 sq. dcms.
Watts per square decimetre	20	14	11.5
Probable temperature rise above oil	34° C.	24° C.	20° C.

	C.	D.	E.
Weights and costs—			
Weight of copper in kilogrammes . .	137	190	260
,, iron laminations in kgs. .	274	380	520
,, active material in kgs. . .	411	570	780
,, active material in kgs., per K.W. at unity power factor . .	4.11	7	7.8
Cost of active material	£25	£35	£48
,, active material per K.W. at unity power factor	5s.	7s.	9s. 7d.

Efficiency and Cost.—From the above data it will be seen that the cost of active material increases rapidly with the efficiency. At 98.4 per cent. the cost of active material is about 1.9 time greater than for an efficiency of 98.0 per cent. The degree of efficiency obtained from a well-designed transformer is thus largely a matter of cost, and in determining the most economical efficiency for any particular working conditions the relation between capital cost of the transformer and cost of energy must be taken into account. In all cases the efficiency of a transformer must be such that the working cost is a minimum. This efficiency for known working conditions may be derived as follows:—

The cost of active material for a transformer is a function of the efficiency ϵ, and is expressed by

$$P_t = f(\epsilon)$$

The cost of inactive material, such as tanks, clamping plates, cooling oil—if any—and the expenditure caused by wages, etc., are partly proportional to P_t, partly constant, so that the total cost of the transformer is expressed by

$$P = C + c\,P_t \text{ shillings}$$

where C and c are coefficients. The former varies in value from 1.5 P to 0.5 P, and the latter from 2.0 to 1.2 according to the size of the transformer, the mode of construction, and locality of the manufacturing works, being constant for a given size and known manufacturing conditions.

The expenditure per annum caused by interest and depreciation, at x per cent., is

$$xC + xcP_t = xC + xcf(\epsilon)$$

The cost for electrical energy per annum required to cover the losses is also a function of the efficiency and $= F(\epsilon)$. The total working costs per annum are therefore equal to

$$xC + xcf(\epsilon) + F(\epsilon)$$

When this is a minimum

$$f'(\epsilon) = -\frac{1}{xc} \times F'(\epsilon)$$

i.e. cost of active material $= -\dfrac{\text{cost of energy per annum to cover losses}}{xc}$

Referring to Figure 122—

$F'(\epsilon) \quad = \tan \alpha$

and $f'(\epsilon) \quad = \tan \beta$

hence for minimum working costs $\dfrac{\tan \beta}{\tan \alpha} = -\dfrac{1}{xc}$

That is, the most favourable point on the curve $P_t = f(\epsilon)$ is that point the tangent at which forms, with the axis of abscissæ, an angle β, the negative sign denoting that the angles α and β are to be plotted in opposite directions with regard to the axis of abscissæ. All that it is necessary to know is therefore the values of x and c, the cost per kilowatt hour, and the load factor at which the transformer is operated.

Example.—Suppose the above transformers C, D, E are to be connected to a circuit the nature of whose load is such that they

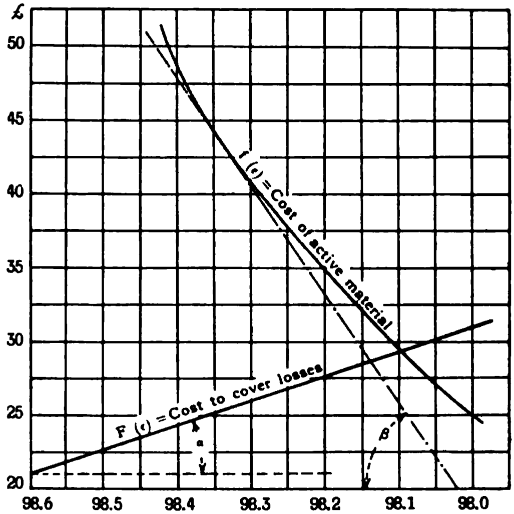


FIG. 122.

will be operated continuously at full-load for 10 hours per day and 365 days per year. It is required to determine the most suitable transformer for two conditions, namely, when the price per K.W.-hour to be charged to the energy lost in the transformer is 1d. and 3d. respectively.

In Figure 122 curve $f(\epsilon)$ gives, for the three designs, the cost of active material as a function of the efficiency. For energy at 1d. per unit curve F (ϵ) shows how the cost of energy lost in the transformer per annum varies with the efficiency. If 12 per cent. be allowed for interest and depreciation, and the value of the

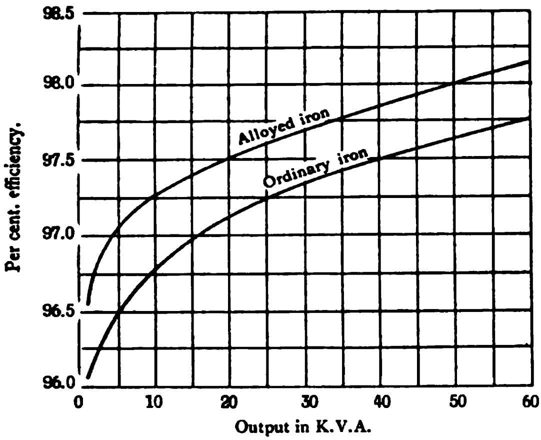


FIG. 123.—Efficiency curves for transformers cooled by natural draught.

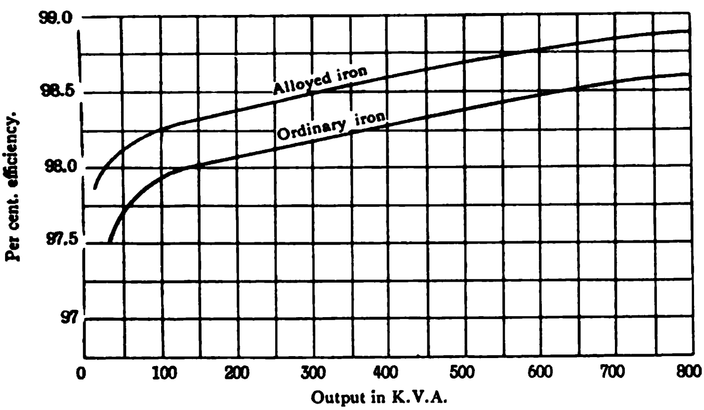


FIG. 124.—Efficiency curves for oil-immersed transformer.

coefficient c be taken as 1.8, then since $\tan \alpha = 0.33$, the angle β is given by

$$\beta = \tan^{-1}\left(-\frac{\tan \alpha}{xc}\right) = \tan^{-1}\left(-\frac{0.33}{0.12 \times 1.8}\right) = -57^\circ$$

When a tangent is drawn to curve $f(\epsilon)$ such that it makes with the axis of abscissæ an angle $\beta = 57^\circ$, it will be found that the most favourable efficiency is 98.3 per cent. Hence either design D or

design E may be selected. When energy is 3d. per unit the transformer design having an efficiency of 98.4 per cent. would be the most suitable.

For unknown working conditions the efficiency has to be assumed, for which purpose the curves in Figures 123 and 124 may be employed. As the temperature rise is approximately inversely proportional to the efficiency, the latter must be high enough to prevent undue heating. Figure 123 gives the efficiencies of transformers with natural draught cooling, the temperature rise of which is limited to 50° C., whilst the curves of Figure 124 are for oil cooling.

Design of Transformers as Influenced by the Employment of Alloyed Iron

A study of the four designs for a 50~ transformer indicates the great advance that has been made due to the introduction of alloyed iron. From designs A and C, it will be seen that in the case of a 100-K.V.A. transformer built with alloyed iron, the same full-load efficiency can be obtained with 37 per cent. of the active material required for a transformer using ordinary iron, the product QB_m being now 3.2 times greater than in the original transformer. As regards the no-load current the alloyed iron transformer is somewhat inferior, whilst as to regulation there exists no material difference between the two designs.

If the alloyed iron transformer could be worked at as low an efficiency as the transformer with ordinary iron, the reduced cost of active material would be considerable. This, however, is not permissible unless more efficient cooling methods are adopted. Consequently what may be expected from the use of alloyed iron in transformers of 40~ and upwards is a somewhat increased efficiency and a reduction in cost of active material by about 20 or 30 per cent. as compared with transformers built with ordinary iron. For instance, of the three transformers with alloyed iron, the second would be the one selected as being the cheapest design which complied with the heating limits given in the specification on page 159, the first design being excluded owing to the excessive temperature rise of the iron above the oil. When transformer D is compared with transformer A it will be observed that the efficiency is 98.2 per cent. as against 98.0, while the cost of active material is 30 per cent. less. Another point to be remembered is that the mechanical construction and the means of cooling will be cheaper for the transformer with alloyed iron, as the dimensions are smaller, and smaller losses have to be dissipated.

Although a reduction in cost of active material of the above order can be obtained for the present standard efficiencies at 50~, this

reduction in cost is not possible with transformers designed for a frequency of 30~ or less. Below this frequency, the magnetic induction cannot be increased much above the value at present used in transformers built with ordinary iron, owing to the excessive no-load current which results from an increase in the induction. Hence the employment of alloyed iron in low-frequency transformers results in a considerable increase in efficiency, there being very little, if any, reduction in price.

Design of a 500-K.V.A. single-phase Transformer of the shell type—Oil insulated and Water cooled

As an example of a single-phase transformer of the shell type, a design to fulfil the following specification will now be calculated.

Specification—

Rated output at 50~	= 500 K.V.A.
Primary terminal pressure	= 6600 volts.
Secondary " "	= 390 "

Full-load efficiency, for $\cos\phi = 1$, not to be less than 98.7 per cent.
Voltage drop for $\cos\phi = 0.80$ not to exceed 2.5 per cent.
Temperature rise of any part not to exceed 50° C.
Transformer to be operated continuously at full load.

Design.—For this frequency alloyed iron 0.5 mm. in thickness will be employed. The loss at full load will be divided equally between iron and copper so that $p_c = p_i = 0.5$. As the total loss $W_t =$ 6600 watts, $W_i = W_c = 3300$ watts. For a primary pressure of 6600 volts, $F_c = 0.35$, and from Table XIII. page 158, $F_i = 0.71$, this allows for 9 ventilating ducts. The constants C_1, $\sqrt{C_2}$ and C_0 will have the following values: $C_1 = 2.7 \times 10^{-4}$, $\sqrt{C_2} = 1.38 \times 10^{-4}$, and $C_0 = 2$. In deriving the value of C_0 it is assumed that insulated copper wire and stamped alloyed sheets cost 1s. 10d. and 11d. per kilogramme respectively.

Design Coefficients (from p. 159)—

$$\alpha = \frac{45 \times 10^{10} \times 500}{50 \times 0.71 \times 0.35} = 180 \times 10^{10}$$

$$\beta = \frac{2 \times 6500}{15.6 \times 10^{-3} \times 0.71} \times \frac{1}{1.38 \times 10^{-4}} \times \sqrt{\frac{0.5 \times 0.5}{2 \times 2.7 \times 10^{-4}}} = 18.2 \times 10^{10}$$

$$\gamma = \frac{6500}{17.8 \times 10^{-3} \times 0.35} \times \frac{1}{1.38 \times 10^{-4}} \times \sqrt{\frac{0.5 \times 0.5}{2 \times 2.7 \times 10^{-4}}} = 16.2 \times 10^{10}$$

From the equations on page 159 the values of a, b, and c expressed in terms of d are as follows:—

$$a = 0.875 \times \frac{F_i d}{C_0 . F_c} = \frac{0.875 \times 0.71}{0.35 \times 2} d = 0.9\ d$$

$$b = \frac{a+d}{\frac{\beta}{\alpha} a - 1} = \frac{0.9\ d + d}{\left(\frac{18.2 \times 10^{10}}{180 \times 10^{10}} \times 0.9\ d\right) - 1} = \frac{1.9\ d}{0.09\ d - 1}$$

$$c = \frac{d + 2a}{\frac{\gamma}{\alpha} d - 1} = \frac{d + (2 \times 0.9\ d)}{\frac{16.2 \times 10^{10}}{180 \times 10^{10}} . d - 1} = \frac{2.8\ d}{0.09\ d - 1}$$

For various assumed values of d, the corresponding values of a, b, c, and QB_m are calculated, and from the following table it will be seen that the linear dimensions corresponding to a maximum value of QB_m ($= 1.53 \times 10^6$) are

$a = 200$ mms.; $b = 425$ mms.; $c = 630$ mms.; and $d = 220$ mms.

d (assumed).	a.	b.	c.	QB_m.
cms.	cms.	cms.	cms.	
20	18	47.5	70	1.50×10^6
21	19	44.5	66	1.52×10^6
22	20	42.5	63	1.53×10^6
23	20.7	41.0	60.5	1.51×10^6

A dimensioned sketch of the core plates is given in Figure 125A, corresponding to the lettering given in Figure 118.

$$\text{Weight of copper} = K_c = \frac{6500}{1.53 \times 10^6} \times \frac{1}{1.38 \times 10^{-4}} \times \sqrt{\frac{0.5 \times 0.5}{2 \times 2.7 \times 10^{-4}}} = 660 \text{ kgs.}$$

Weight of iron $= K_i = 1320$ kgs.

$$\text{Current density} = Q = \sqrt{\frac{3250}{2.7 \times 10^{-4} \times 660}} = 135$$

$$\text{Flux density} = B_m = \frac{1.53 \times 10^6}{135} = 11{,}300 \text{ lines per sq. cm.}$$

Windings—

Area of cross-section of core $= A = F_i . d . c. = 0.71 \times 22 \times 63 = 980$ sq. cms.

Magnetic flux $= \Phi = B_m \times A = 11{,}300 \times 980 = 11.1 \times 10^6$ lines.

$$\text{Number of secondary turns} = T_s = \frac{390 \times 10^8}{4.44 \times 50 \times 11.1 \times 10^6} = 16$$

$$\text{Ratio of transformation} = \frac{6600}{390} = 17$$

Number of primary turns $= T_p = 17 \times 16 = 272$.
Secondary current $= I_s = 1300$ ampères.
Cross-sectional area of secondary conductor $= 960$ sq. mms.
Primary current $= I_p = 77$ ampères.
Cross-sectional area of primary conductor $= 57$ sq. mms.

For the present, it will be assumed that the primary and secondary are each wound in 8 sections and arranged thus—

$$S_1 \ PP \ S_2S_2 \ PP \ S_3S_3 \ PP \ S_4S_4 \ PP \ S_1$$

The primary sections are all connected in series, while the secondary sections with the same suffix numbers are in parallel. Each secondary section, consisting of 4 turns and carrying one-half the total current, is wound with 9 copper strips, wound on the flat, 10×4.8 mms. connected in parallel. The strips are bound together with tape, and when insulated the size of the conductor $= 11 \times 44$ mms. After being wound each section is separately insulated with press-spahn and tape to a width of about 13 mms.

The primary sections are wound with 34 turns of 14×4 mm. copper strip, there being 1 turn per layer. The strip is double cotton covered, and the insulation between the layers is reinforced with a continuous strip of special insulating material about 0.5 mm. thick. Each section is finally insulated to a thickness of about 3 mms. with oiled-linen and press-spahn.

The spacing of the windings inside the iron openings is shown in Figure 126 (p. 185). Adjacent primary sections and adjacent secondary sections are separated by barriers of insulating material 5 mms. and 3 mms. thick respectively. Spacing blocks of paraffined teak are inserted between neighbouring primary and secondary sections, thus giving the cooling medium free access to the windings. When the latter have been assembled they are bound together with press-spahn and micanite insulation, so that ultimately the primary and secondary coils are separated from the core by 10 mms. thickness of insulation. To facilitate the circulation of oil at that portion of the winding which extends beyond the core, the ends of the coils are spread out as shown in Figure 53.

Heating.— *Windings*—

Surface of primary winding in contact with oil $= 350$ sq. decimetres.
„ secondary winding in contact with oil $= 370$ „
Total cooling surface of windings $= 720$ „

Hence temperature drop from copper to oil $= K \cdot \dfrac{W}{A}$

$$= \frac{2 \times 3250}{720} = 9^\circ \text{ C.}$$

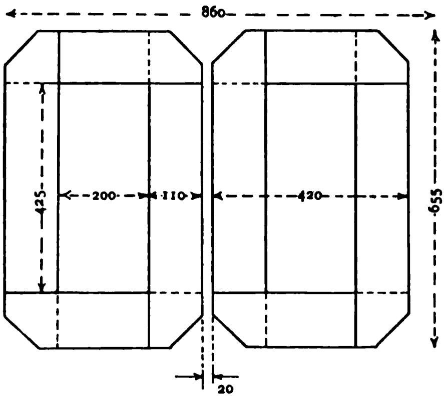
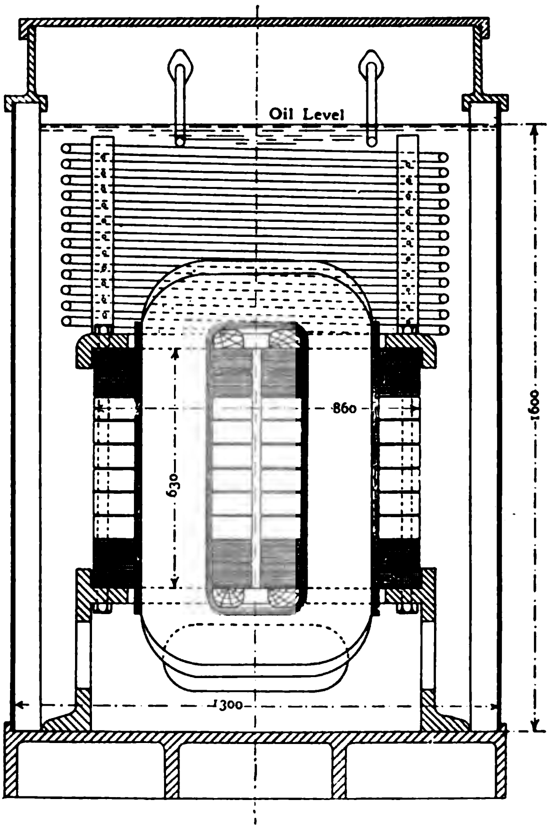


FIG. 125A.—Core plates for 500-K.V.A. shell transformer.



6600/390 volts 50~.

FIG. 125B.—500-K.V.A. shell-type transformer.

Iron.—Since the iron edge surface inside the core is closely packed with insulation, it cannot contribute to the cooling surface, so that it is only necessary to calculate the outer edge surface. As the latter $=250$ sq. decimetres, the temperature drop between iron and oil will be about $\frac{1.7 \times 3250}{250} = 22^\circ$ C. The drop in temperature between oil and outside air must not exceed 28° C.

Case and Cooling Worm.—A suitable size of case for this transformer would be 130 cms. × 110 cms. by about 160 cms. high (see Figure 125B). The surface in contact with oil $=140$ sq. decimetres for bottom of tank $+770$ sq. decimetres for sides. As the tank is corrugated so as to increase the cooling surface of the sides by about 100 per cent., the total cooling surface of tank $=1650$ sq. decimetres. Since the temperature rise of oil above external air has not to exceed 28° C., the case can dissipate $\frac{T \times A}{K}$ watts $= \frac{28 \times 1650}{15} = 3000$ watts. The remaining 3500 must be dissipated by artificial cooling, that is to say, by means of water or forced draught. If the entire loss of 6500 watts were dissipated by natural cooling an excessive size of case would be necessary. The additional cooling facilities will be provided by means of a cooling worm.

From formula on page 125, the volume of water required $=0.48 \times 10^{-4} \times 3.5 = 1.7 \times 10^{-4}$ cubic metres per second $=61$ cubic metres per hour. Assuming the rate of flow to be 0.6 metre per second, the cross-sectional area of tube $= \frac{1.7 \times 10^{-4}}{0.6} = 2.83 \times 10^{-4}$ sq. metres $=2.83$ sq. cms. The tube must therefore have a bore of 1.91 cms., say 2 cms. If the average temperature of the cooling water be the same as that of the external air, then, for a drop in temperature from oil to cooling water of 28° C., the surface area of cooling worm required is

$$A = K \,.\, \frac{W}{T^\circ} = \frac{2.3 \times 3500}{28} = 290 \text{ sq. decimetres} = 29{,}000 \text{ sq. cms.}$$

For a pipe of 2 cm. bore the length required $= \frac{29{,}000}{\pi \times 2} = 4600$ cms. The cooling worm will therefore consist of twelve turns. A sectional elevation of the transformer is given in Figure 125.

Inductive Drop.—From equation (9), page 96, the inductive drop expressed as a percentage is

$$V \text{ per cent.} = 50 \,.\, \frac{n+2}{n} \,.\, \frac{AT}{\Phi} \,.\, \left(b + \frac{a_2 + a_1}{6}\right)\frac{p}{l}$$

where $n =$ number of double sections $= 8$.

$AT =$ secondary current × turns per double section $= 5200$.

$\Phi = 11.1 \times 10^6$.

b = distance of copper to copper between a primary and its neighbouring secondary = 1.8 cms.
a_1 = thickness of a double section of primary = 4.4 cms.
a_2 = thickness of a double section of secondary = 2.8 cms.
p = mean length of two coils = 230 cms.
l = depth of coil = 18 cms.

Substituting these values in the above equation

$$\text{V per cent.} = 50 \times \frac{10}{8} \times \frac{5200}{11.1 \times 10^6} \times \left(1.8 + \frac{2.8 + 4.4}{6}\right) 230/18$$
$$= 1.12 \text{ per cent.}$$

If the 16 sections into which the windings are divided be interleaved as follows:—

SS PPPP SSSS PPPP SS
SSSS PPPPPPPP SSSS

Then the values of the inductive drop would be 5.3 per cent. and 15 per cent. respectively, thus showing that with suitably designed windings any degree of regulation can be obtained.

Comparison of Types and Effect of Space Factor of Windings on Cost of Material

In the following tables are given the principal design data for a 50-K.V.A., 50-period, single-phase transformer of the following types: (1) circular core, (2) rectangular core, and (3) rectangular shell. The calculations for each type have been made for (1) a low-tension transformer having a copper space factor $F_c = 0.5$, and (2) a high-tension transformer having $F_c = 0.25$. In all cases it is assumed that the core is constructed of alloyed iron 0.5 mm. thick, and that the ratio between price per kilogramme of copper and iron $= C_0 = 2$. The other constants $\sqrt{C_2} = 1.38 \times 10^{-4}$, $C_1 = 2.7 \times 10^{-4}$, and cost of insulated copper wire = 1s. 10d.

From the following data it will be seen that the difference in the amount of material for the three types is very small. For high copper space factors, the rectangular shell type is slightly superior to the other types, while for low space factors the reverse is the case. The latter designs—the conclusions here drawn are applicable to all sizes—therefore confirm a statement made in the previous chapter, namely, that there is very little to choose between the various types, as when the same amount and quality of active material is employed, and the designs are correctly made, the electrical data of the various types will not vary appreciably.

TABLE XV.—COMPARATIVE DESIGNS OF 50-K.V.A. SINGLE-PHASE TRANSFORMERS.

Efficiency 98 per cent., iron loss copper loss = 500 watts.

Winding Space Factor.	$F_c = 0.5$.			$F_c = 0.25$.		
Type.	Circular Core.	Rectangular Core.	Rectangular Shell.	Circular Core.	Rectangular Core.	Rectangular Shell.
Cross section of core	As fig. 112.	As fig. 117.	As fig. 118.	As fig. 112.	As fig. 117	As fig. 118.
Number of ventilating ducts	..	..	..	..	..	..
Iron space factor F_i	0.7	0.9	0.9	0.7	0.9	0.9
Most economical value of QB_m	12×10^5	13.0×10^5	14.2×10^5	7.2×10^5	7.0×10^5	6.5×10^5
Dimensions of Core in mms.						
a	..	103	100	..	215	190
b	380	310	200	500	390	300
c	..	185	270	..	255	450
d	..	130	130	..	135	120
D	190	..	..	215	..	..
H	100	..	..	190	..	..
Copper weight in kgs.	130	120	102	220	220	240
Iron weight in kgs.	260	240	204	440	440	480
Weight in kgs. per K.W.	7.8	7.2	6.1	13.0	13.0	14.4
Cost of active material in shillings	480s.	440s.	380s.	800s.	800s.	880s.
Cost of active material per K.W.	9.6s.	8.8s.	7.5s.	16s.	16s.	17.5s.
Current density Q	120	125	135	90	90	88
Flux density B_m	10,000	10,400	10,500	8000	8000	7400

It is of further interest to note that a reduction in value of the copper space factor from 0.5 to 0.25, *i.e.* to 50 per cent. of its original value, corresponds to an increase in the cost of active material of approximately 45 to 57 per cent. for the different types. Hence high-tension transformers will, for the same efficiency, be more expensive than low-tension ones, or if the costs have to be the same, then the former must be designed for a much lower efficiency than the latter.

EXAMPLES OF DESIGNS

100-*K. V.A.*, 50~, 6000/2200 *volts, single-phase, oil-immersed transformer of the core type: constructed by Johnson & Phillips Ltd.*

The drawings of this transformer are given in Plate I. The core, of rectangular cross-section, is built up of alloyed sheets 0.5 mm. thick, and is divided into four equal parts by three 9.5 mm. ventilating ducts. The cores and lower yoke are in one piece, while the top yoke, built up of the laminations stamped out from between the cores of the ⊔-stamping, is laid across with butt joints. The clamping plates are cast in the form of a grid, and from the drawings it will be observed that none of the clamping bolts pass through the laminations.

The coils are wound concentrically with the secondary inside. The latter is divided into two sections per core, and spaced out from the core by 9.5 mms. wood distance pieces. Each section is wound with 75 turns,

arranged in 5 layers with 15 turns per layer. The conductor is bound over with tape to a thickness of 1 mm., and adjacent layers separated by a 3 mm. layer of press-spahn. Each section is finally insulated with a 1.6 mm. thickness of empire cloth.

The primary winding on each core is divided into 5 sections, each wound with 81 turns in 9 layers. The details of the insulation for the primary are as follows :—

Thickness of tape round conductor	= 0.5 mms.
Thickness of press-spahn between layers . .	= 3.0 ,,
Thickness of empire cloth forming external insulation	= 1.6 ,,
Distance between adjacent sections	= 9.5 ,,

The primary sections are placed over the secondary, but separated therefrom by 9.5 mm. fibre distance pieces. The spacing between the core and the secondary and between the primary and earth is clearly shown in the drawings.

The containing tank is of plain wrought iron, 130 cms. × 90 cms. by 135 cms. high. The distance from bottom of tank to oil level is approximately 110 cms.

SPECIFICATION—

Output in K.V.A.	100
Frequency in cycles per second	50
$\frac{\text{Primary volts}}{\text{Secondary volts}}$	$\frac{6000}{2200}$

MAGNETIC CIRCUIT—

Net cross-section of iron in core and yoke	353 cms.^{2}
Flux density in core and yoke (B_m)	9500 lines per cm.^{2}
Magnetic flux Φ	3.35×10^6 lines
Weight of core and yokes	475 kgs.

WINDINGS—

	Primary.	*Secondary.*
Turns in series per phase	810	300
Number of sections	10	4
Turns per section	81	75
Mean length per turn	130 cms.	100 cms.
Size of conductor (bare)	6.1 mm. × 2.03 mm.	10.16 mm. × 3.43 mm.
Current in ampères	17	45.5
Current density in ampères per cm.^{2}	137	130
Resistance per phase at 60° C.	1.62 ohms	0.182 ohms
Copper drop at 60° C.	27.5 volts	8.3 volts
Copper loss at 60° C.	470 watts	375 watts
Space factor		0.305

LOSSES AND EFFICIENCY AT FULL-LOAD—

Iron loss (observed)	965 watts
Loss per kg. at 50~	2.03 ,,
Total copper loss at 60° C.	845 ,,
Efficiency at unity P.F.	98.3 per cent.
Fractional copper loss (p_c)	0.47
Fractional iron loss (p_i)	0.53

HEATING—

Windings—

Copper loss per limb	422 watts
Effective cooling surface per limb	100 decimetres²
Watts per square decimetre of cooling surface	4.22

Core—

Total iron loss	965 watts
Edge cooling surface	95 decimetres²
Watts per square decimetre of cooling surface	10.2

Oil—

Total loss for iron and copper	1810 watts
Effective cooling surface of tank	600 decimetres²
Watts per square decimetre	3
Temperature rise above air	46° C.
Heating coefficient (K)	15

WEIGHTS—

Weight of copper	208 kgs.
Weight of iron	475 ,,
Weight of iron ÷ weight of copper = C_0	2.3
Weight per K.W. for cos $\phi = 1$	6.8 kgs.

235-*K.V.A.*, 25~, 6450/390 *volts, single-phase, oil-immersed transformer of the core type: constructed by Dick, Kerr & Co. Ltd.*

The following data relates to three single-phase transformers for supplying current to a 6-phase 650-K.W. rotary converter. The primaries are connected in star to an 11,000 volts, 3-phase supply. The secondary of each unit is wound in two sections, and the three sets of secondaries connected in double mesh. For the constructional details, see Plate II.

The core, of rectangular cross-section, is constructed of ordinary transformer iron 0.35 mm. thick. The laminations are made up into four blocks (two cores and two yokes), which are put together with imbricated joints and clamped between suitable end-plates, all bolts passing through the core being insulated therefrom with press-spahn tubes.

The secondary, consisting of two sections per limb (one for each phase), and 28 turns per section in one layer, is wound over a press-spahn former. For the primary winding there are four sections per limb, each section being wound in 5 layers with 18, 18, 18, 18, and 16 turns per layer. The bottom section of the primary rests on a paraffined teak flange, which in turn is supported on porcelain insulators fixed to the clamping plates. By means of suitable distance pieces, a space of about 6.5 mms. is left between adjacent sections, and to facilitate the circulation of the cooling oil a clearance of 12.5 mms. and 25.4 mms. respectively is left between the secondary and the core, and between primary and secondary.

SPECIFICATION—

Output in K.V.A.	235
Frequency in cycles per second	25
$\frac{\text{Primary volts}}{\text{Secondary volts}}$	$\frac{6450}{390}$

MAGNETIC CIRCUIT—

Net cross-section of iron in core and yoke	730 cms.^{2}
Flux density in core and yoke (B_m)	8680 lines per cm.^{2}
Magnetic flux Φ	6.3×10^6 lines
Weight of cores and yokes	1700 kgs.

WINDINGS—	*Primary.*	*Secondary.*
Turns in series per phase	880	56
Number of sections per phase	10	2
Turns per section	88	28
Mean length per turn	174 cms.	140 cms.
Size of conductor (bare)	4.7 mm. × 6.2 mm.	7(10 mm. × 3.4 mm.)
Current per phase	36.5 ampères	300 ampères
Current density in ampères per cm.^{2}	120	125
Resistance per phase at 60° C.	1.05 ohm	0.00633 ohm
Copper loss per phase	680 watts	570 watts
Space factor (F_c)		0.33

LOSSES AND EFFICIENCY AT FULL-LOAD—

Iron loss (observed)	1750 watts
Loss per kg. at 25~	1.0 ,,
Total copper loss at 60° C.	2500 ,,
Efficiency at unity power factor	98.2 per cent.
Fractional copper loss (p_c)	0.60
Fractional iron loss (p_i)	0.40

HEATING—

Windings—

Copper loss per limb	1275 watts
Effective cooling surface per limb	430 decimetres^2
Watts per square decimetre of cooling surface	3.0

Core—

Total iron loss	1750 watts
Edge cooling surface	194 decimetres^2
Watts per square decimetre of cooling surface	9.0

Oil—

Total loss for iron and copper	4300 watts
Effective cooling surface of tank	1400 decimetres^2
Watts per square decimetre	4.3
Final temperature rise above air (attained after twenty-four hours' run at full-load)	50° C.
Heating coefficient K	16

WEIGHTS—

Weight of copper	730 kgs.
Weight of iron	1700 kgs.
Weight of iron ÷ weight of copper = C_0	2.35
Weight per K.W. for $\cos \phi = 1$	8.5 kgs.
Volume of oil	250 gallons
Weight of oil	1000 kgs.

550-K.V.A., $33\frac{1}{3}\sim$, 11000/372 *volts, single-phase, air-blast transformer of the shell type: constructed by the British Westinghouse Co. Ltd.*

This transformer, the drawings of which are given in Plate III., is one of several which have been installed in the sub-stations of the London Metropolitan Railway, for supplying current to rotary converters.

The core is of rectangular cross-section, and is built up of ordinary transformer iron 0.35 mm. thick, the core and yokes being interleaved at the corners. For ventilation the core is divided by nine 6.5 mm. gaps into 10 equal parts.

The primary and secondary windings are wound in four and eight sections respectively, and arranged thus—

SSSS PPPP SSSS

Each primary section is made up of two coils, the conductor, of flat copper strip, being wound concentrically with one turn per layer. In addition to the double cotton-covered insulation, adjacent layers are separated by specially treated fuller-board 0.6 mm. thick. The fuller-board is also used in the manufacture of the insulating shields between adjacent coils. When completed the two coils forming a section are bound together with tape and insulated to a depth of 4 mms. with several layers of empire cloth. The high-tension coils are all connected in series across the primary terminals.

The secondary sections, each consisting of 4 turns and wound with eight copper strips in parallel, are insulated in a similar manner to the high-tension coils, with the one exception that the final outside insulation is only 1 mm. thick. The low-tension windings consist of two circuits in parallel, each circuit having four sections in series.

After being specially treated to render them perfectly moisture proof, the sections are assembled, as shown, in adjacent sections, being separated by 9.5 mm. teak distance pieces. The insulation between the windings and the case is entirely of fuller-board.

The transformer is mounted in a cast-iron housing, the different parts of which are held together by bolts which pass from the upper to the lower casting. These bolts also serve to clamp the iron of the transformer, which rests on the lower casting. At each end of the base are parallel openings which give access to the high-tension terminals, these being supported on porcelain insulators fixed to the base of the lower casting. The low-tension terminals are mounted on, but insulated from, an iron bar fastened in the top casting.

The air blast is sent into the transformer through an air duct in the floor, the iron and copper being separately cooled. The air for cooling the coils passes up through the transformer between the windings and discharges through an opening in the top of the case, while that for cooling the iron passes from the lower housing, through a regulator at

one side of the transformer, and then horizontally through the ducts in the core.

SPECIFICATION—

Output in K.V.A.	550
Frequency in cycles per second	$33\frac{1}{3}$
$\frac{\text{Primary volts}}{\text{Secondary volts}}$	$\frac{11,000}{372}$

MAGNETIC CIRCUIT—

Net cross-section of iron	1460 cms.^{2}
Flux density (B_m)	10,700 lines per cm.^{2}
Magnetic flux Φ	15.75×10^6 lines
Weight of core and yokes	2460 kgs.

WINDINGS—	*Primary.*	*Secondary.*
Turns in series per phase	473	16
Number of sections	4	2×4
Mean length per turn	280 cms.	270 cms.
Section of conductor (bare)	32 mm.^{2}	78 mm.^{2} – 8 in parallel
Current per phase	50 ampères	1480 ampères
Current density in ampères per cm.^{2}	160	120
Resistance per phase at 60° C.	0.74 ohm	0.00077 ohm
Copper loss	1850 watts	1700 watts
Space factor		0.29

LOSSES AND EFFICIENCY AT FULL-LOAD—

Iron loss (observed)	4400 watts
Loss per kg. at $33\frac{1}{3}\sim$	1.8 ,,
Total copper loss at 60° C.	3550 ,,
Efficiency at unity power factor	98.6 per cent.
Fractional copper loss (p_c)	0.45
Fractional iron loss (p_i)	0.55

WEIGHTS—

Weight of copper	865 kgs.
Weight of core	2460 ,,
Weight of iron ÷ weight of copper $= C_0$	3.1
Weight per K.W. for $\cos\phi = 1$	5.9 kgs.

TEST RESULTS—

Heating.—With an air blast of 46 cubic metres per minute the temperature rise above external air was as follows:—

Core	15° C.
Low-tension winding	17° C.
High-tension winding	29° C.

Regulation—

Regulation for unity power factor	5.75 per cent.
Regulation for $\cos\phi = 0.8$	11.5 ,,

This transformer was designed for a high reactance, and hence poor regulation, so that the voltage on the direct current side of

the converter could be varied over a wide range by simply adjusting the exciting current (see p. 445). The requisite amount of leakage is obtained by grouping all the primary sections together and surrounding them with a sheet of high permeability iron, marked *reactive iron* in the drawing.

4450-K. V.A., 25~, single-phase, air-blast transformer of the core type: constructed by the Oerlikon Company.

This transformer is one of three which supply 3-phase currents to furnaces used for the preparation of calcium carbide. Besides being a good example of the application of transformers to chemical furnaces where the electrical energy is transmitted from a waterfall situated at some considerable distance from the chemical works, this design is of

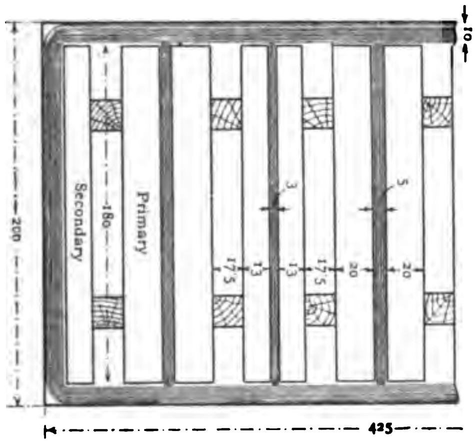


FIG. 126.—Design of windings and insulation of 500 K.V.A. shell transformer.

special interest, as it represents one of the largest transformers made up to the present. The drawings are given in Plate IV. The transformer is forced ventilated, the air blast being admitted from below through a culvert in the floor of the transformer house. The blast ascends through the windings and core, the latter being constructed with six vertical air ducts. The base plate of the containing case is mounted on wheels, which run on rails laid over the ventilating culvert.

The primary voltage is 15,000, and by selecting a suitable number of primary turns, any secondary voltage between 115 and 155 can be obtained. On account of the exceptionally large secondary current—28,700 ampères—it is necessary to winding the low-tension coils of massive copper strip which, when the transformers are in use, will have an eddy current loss almost equal to that due to the ohmic resistance itself. The joints in the secondary winding must be very carefully

riveted and soldered, as these connections always form a delicate part of the transformer. If the contact be bad local heating will result, causing the contact to become worse and worse and the heating to increase, until finally the solder melts and the insulation between primary and secondary breaks down.

FIG. 127.

When used for furnace work, transformers are frequently subjected to severe short circuits producing extremely heavy momentary currents, which may cause serious damage to the windings. In order to limit the magnitude of the current on short circuit, transformers for chemical work should be designed for an inductive drop of from 6 to 8 per cent. For this reason the distance between primary and coils is made somewhat greater than that actually required for insulating

purposes. This expedient considerably increases the facilities for cooling, and, moreover, as the furnaces are always operated at full-load, the large inductive drop will not occasion variations in the secondary terminal P.D.

The high-tension winding, placed inside, has the coils of each core in series, each coil being sub-divided into twenty-one sections insulated from one another with fibre arranged so as to permit an easy circulation of the air blast. The total number of turns in series is 492. By special arrangement of connections to the primary winding, three of which are shown in Figure 126, it is possible to obtain the following secondary volts—

115, 121, 127, 135, 145, and 155.

The low-tension winding consists of 8 turns in series arranged 4 on each core, the conductor consisting of 4 copper strips in parallel. A micanite cylinder 8 mm. in thickness separates the primary coils from the core and from the exterior secondary winding. The insulation details are given in Plate IV., and a view of the transformer with case removed is shown in Figure 127.

DESIGN DATA (155 Volts at Secondary Terminals).

Magnetic Circuit—		
Quality of iron	Ordinary	
Net cross-section of iron	2500 cm.^{2}	
Flux density (B_m)	11,000 lines per cm.^{2}	
Magnetic flux (Φ)	27×10^6 lines	
Weight of core and yokes	9200 kgs.	
Windings—	*Primary.*	*Secondary.*
Turns in series per phase	492	8
Number of sections	42	2
Mean length per turn	310 cms.	350 cms.
Size of conductor (bare)	50 × 2.2 mms.	300 × 3 mms. (12 Bleche).
Current in ampères	392	28.700
Ampères per cm.^{2}	2,76	2,66
Copper loss	50,000 watts	
Losses and Efficiency at Full-Load—		
Iron loss	28,600 watts	
Loss per kg. at 25~	3.1 ,,	
Total copper loss at 60° C.	50,000 ,,	
Efficiency at unity power factor	98.2 per cent.	
Fractional copper loss	0.64	
Fractional iron loss	0.36	
Weights—		
Weight of copper	2900 kgs.	
Weight of iron	9200 ,,	
Weight of iron ÷ weight of copper = C_0	12,100 ,,	
Weight per K.W. cos $\phi = 1$	2.7 ,,	

Heating.—With an air blast of 420 cubic metres per minute the temperature rise (max.) above external air was as follows:—

Core	60°
Low-tension winding	50°
High-tension winding	50°

Regulation—

Regulation for cos $\phi = 1$	1.2 per cent.
Regulation for cos $\phi =$	4.4 ,,

The total weight of the transformer complete was 14,600 kgs. For 50~ and without a very close degree of regulation being required the same type of transformer would have a capacity of 9000 K.V.A., which corresponds to a weight of 1.62 kg. per K.V.A.

90-*K.V.A.*, 18~, 130/2200 *volts*, 3-*phase, oil-immersed transformer of the core tyre: constructed by the Electric Construction Co. Ltd.*

This transformer, illustrated in Plate V., is given as an example of one designed for an exceptionally low frequency.

SPECIFICATION—

Output in K.V.A.	90
Frequency in cycles per second	18
$\frac{\text{Primary volts}}{\text{Secondary volts}}$	$\frac{130}{2200}$

MAGNETIC CIRCUIT—

Quality of iron	Ordinary
Thickness of sheet	0.5 mm.
Net cross-sectional area of iron in core	350 cms.^{2}
Net cross-sectional area of iron in yoke	360 ,,
Flux density in core (B_m)	9100 lines per cm.^{2}
Flux density in yoke	8900 ,,
Magnetic flux Φ	3.2×10^6 lines

WINDINGS—

	Primary.	Secondary.
Windings connected in	delta	delta
Volts per phase	130	2200
Turns in series per phase	50	864
Mean length per turn	90 cms.	114 cms.
Size of conductor (bare)	4(8.0 mm. × 8.0 mm.)	4.2 mm. × 4.2 mm.
Current per phase	230 ampères	14 ampères
Current density in ampères per cm.^{2}	90	80
Resistance per phase at 60° C.	0.0035 ohm	1.1 ohm
Copper loss at 60° C.	555 watts	645 watts
Space factor		0.43

LOSSES AND EFFICIENCY AT FULL-LOAD—

Iron loss (observed)	1100 watts
Total copper loss at 60° C.	1200 ,,
Efficiency at unity power factor	97.6 per cent.
Fractional copper loss (p_c)	0.45
Fractional iron loss (p_i)	0.55

HEATING—

Windings

Copper loss per phase	400 watts
Effective cooling surface per phase	100 decimetres^2
Watts per square decimetre	4

Iron—

Total iron loss	1100 watts
Edge cooling surface	100 decimetres^2
Watts per square decimetre of cooling surface	11

Oil—

Total loss for iron and copper	2300 watts
Cooling surface of tank	650 decimetres^2
Watts per square decimetre of cooling surface	3.5
Temperature rise above air	34° C.
Heating coefficient (K.)	10

WEIGHTS—

Weight of copper	765 kgs.
Weight of iron	1100 kgs.
Weight of iron ÷ weight of copper (C_0)	1.46
Weight per K.W. for cos $\phi = 1$	32 kgs.

600-*K. V.A.*, 50~, 6300/356 *volts*, 3-*phase, oil-immersed transformer of the core type: constructed by Johnson and Phillips Ltd.*

The drawings are given in Plate V^A. The laminations are stamped out of alloyed sheet 0.5 mm. thick, and are assembled to form three vertical cores connected by horizontal yokes with interleaved joints at the bottom and a butt joint at the top. The core is of rectangular cross-section, and is divided in twelve parts by eleven ventilating ducts.

The coils are wound in a similar manner to those of the first example, except that the secondary is in one section and the primary in four sections per core. The following table gives the insulation details for the winding:—

	Primary.	*Secondary.*
Number of turns per section	51	20
Number of layers	4	1
Insulation round conductor	D.C.C. and braided	bare
Thickness of insulation round conductor	0.8 mm.	0.75 m.m., varnished press-spahn between turns
Insulation between adjacent layers	3 mm. press-spahn	...
External insulation for sections	1.6 mm. empire cloth	1.6 mm. empire cloth
Distance between adjacent sections	12 mm.	...
Thickness of hard-wood distance piece between secondary and core	9.5 mms.	
Thickness of fibre distance piece between primary and secondary	9.5 mms.	

The lower section of each high-tension winding rests on four porcelain insulators fixed to a hard-wood button supported by the clamping plates.

The containing tank, constructed with corrugated sheet iron sides secured in top and bottom castings, had inside dimensions as follows: 120 cms. × 100 cms. by 190 cms. high, the depth of the oil being

160 cms. The shape of the corrugations was such as to give an external cooling surface = 3000 square decimetres.

SPECIFICATION—

Output in K.V.A.	600
Frequency in cycles per second	50
$\frac{\text{Primary volts}}{\text{Secondary volts}}$	$\frac{6300}{356}$

MAGNETIC CIRCUIT—

Net cross-section of iron in core and yoke	693 cms.^{2}
Flux density in core and yoke (B_m)	11,700 lines per cm.^{2}
Magnetic flux Φ	8.1×10^6 lines
Weight of core and yokes	1700 kgs.

WINDINGS—

	Primary.	*Secondary.*
Windings connected in	Star	Delta
Volts per phase	3640	356
Turns in series per phase	204	20
Mean length per turn	188 cms.	162 cms.
Size of conductor (bare)	6.0 mm. × 4.57 mm.	4(20.8 mm. × 3.3 mm.)
Current per phase	56 ampères	560 ampères
Current density in ampères per cm.^{2}	204	204
Resistance per phase at 60° C.	0.254 ohm	0.0028 ohm
Copper loss at 60° C.	2400 watts	2640 watts
Space factor		0.28

LOSSES AND EFFICIENCY AT FULL-LOAD—

Iron loss (observed)	5020 watts
Loss per kg. at 50~	2.95 ,,
Total copper loss at 60° C.	5040 ,,
Efficiency at unity power factor	98.36 per cent.
Fractional copper loss (p_c)	0.5
Fractional iron loss (p_i)	0.5

HEATING—

Windings—

Copper loss per phase	1680 watts
Effective cooling surface per phase	300 decimetres^2
Watts per square decimetre of cooling surface	5.6

Core—

Total iron loss	5020 watts
Edge cooling surface	260 decimetres^2
Watts per square decimetre of cooling surface	19

Oil—

Total loss for iron and copper	10,600 watts
Effective cooling surface of tank	3000 decimetres^2
Watts per square decimetre	3.5
Temperature rise above air	50° C.
Heating coefficient (K.)	14

WEIGHTS—

Weight of copper	540 kgs.
Weight of iron	1700 ,,
Weight of iron ÷ weight of copper = C_0	3.1
Weight per K.W. for $\cos \phi = 1$	3.73

CHAPTER VI

ALTERNATORS—MECHANICAL CONSTRUCTION AND ARMATURE WINDINGS

Mechanical Construction.—The standard type of alternator is one in which the armature is stationary and the field magnets revolve, the latter being excited with a direct current. This method of construction has several distinct advantages:—(1) The field magnet can be given greater mechanical strength than is possible with an armature the coils of which are disposed over its periphery. (2) Since the majority of alternators are designed for pressures exceeding 2000 volts, and the E.M.F. of the exciting circuit need never exceed 250 volts, the most satisfactory arrangement will be to have the sliding contacts in the low voltage circuit. (3) A stationary armature will relieve the high-tension insulation from the strains caused by centrifugal force.

According to the speed of the revolving part, electric generators may be divided into three classes—

(1) Slow Speed	. .	80–200 revolutions per minute.
(2) Medium Speed	. .	200–800 " "
(3) High Speed	. .	1000–3000 " "

Since the frequency of the induced E.M.F. $= \frac{Rp}{60}$, where R = revolutions per minute, and p = pairs of poles, the number of pairs of poles to give a frequency $\sim$ is

$$p = \frac{60\sim}{R}$$

The frequencies for which alternators are commercially designed range from 100 to 25, and even lower values have been considered for railway work. Slow and medium speed machines will therefore have a large number of poles, and consequently a rotor of large diameter and comparatively small axial length. High-speed generators are those direct-coupled to steam turbines and, owing to the peripheral speed limit, their proportions are much different from alternators driven by reciprocating engines.

In turbo-alternators, the air-gap diameter rarely exceeds 2 metres,

even in 10,000 K.W. sizes. The axial length of the armature has to be increased accordingly, and a length of 2 metres is not uncommon in alternators for large outputs.

The general construction of slow and medium speed alternators is illustrated in Figure 128. The armature laminations A are supported by a hollow cast-iron case or frame B, the lower part of which is cast with feet for bolting to either a bed-plate or concrete foundation. The rotor consists of a cast-iron or cast-steel flywheel, to the rim of which are bolted the poles N, S. The terminal ends of the field winding are connected to insulated metal slip-rings, which rotate with the shaft,

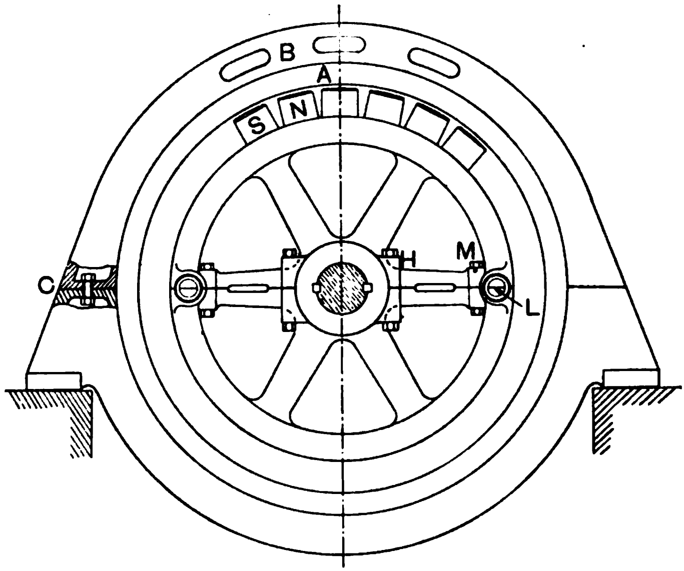


FIG. 128.—Slow speed alternator.

connection being made with the direct current supply through brushes bearing on the rings.

SLOW AND MEDIUM SPEED ALTERNATORS

Stators.—Armature cores are built up of soft annealed wrought iron or mild steel laminations, about 0.5 mm. in thickness. Except with very small diameters the laminations are in segments, so fitted that the joints of adjacent layers lie on different radii. Frames for supporting the laminations are always of cast-iron, and to provide sufficient ventilation, should be of as skeleton a construction as is consistent with mechanical strength. For convenience in handling they are generally divided into two sections across a horizontal diameter, the sections being bolted together, as indicated at C in

Figure 128. When the stator case exceeds 5 metres in diameter, the number of sections may be increased to four. The design of stator frame shown in Figure 129 is suitable for medium-sized alternators up to 200 K.W.

The armature laminations L are held between two end-flanges F, F, one of which is secured by a number of short keys K, driven in circumferentially between the flange and the circular rib R. To provide adequate ventilation the core is formed with a number of radial air-ducts, through which is driven a current of air, set up by the revolving

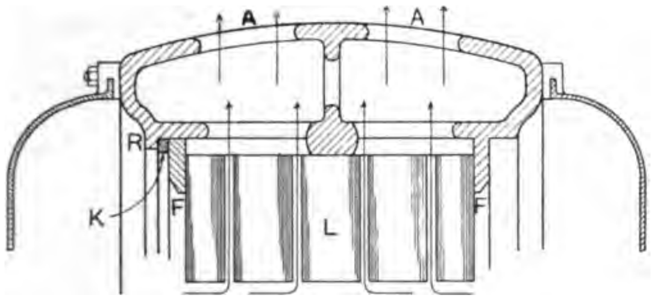


FIG. 129.—Stator frame for alternators up to 200 K.V.A.

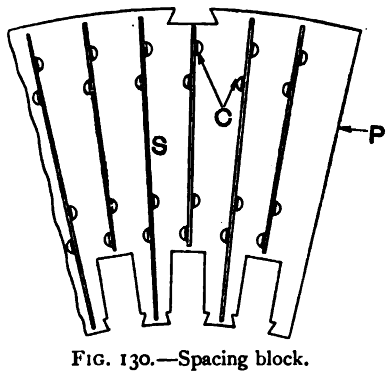


FIG. 130.—Spacing block.

field magnets. After passing through the ducts the air escapes by the apertures A formed in the shell of the frame; the paths taken by the air currents being indicated by the arrows. The ducts range in width from 10 to 15 mms., and are formed by inserting spacing blocks at intervals of from 4 to 7 centimetres. A type of spacing block in very general use is shown in Figure 130. The steel plate P, about 1 mm. in thickness, has cut in it a number of semi-circular pieces C which are bent at right angles to the plane of the plate. Radial brass or steel strips S form the distance pieces, and are held between the supports C.

In large alternators, where the diameter of core is great compared

with its length, the design of frame would be somewhat as in Figure 131. The frame consists of an outer shell cast with a number of radial ribs R, which are webbed together by an inner shell of about the same thickness. The laminations are bolted between the two end clamps, one of which is cast to the frame. Interposed between the clamps and the core are brass end grids G, cast with projections to fit over the sides of the teeth and thus prevent the latter from bulging outwards. The cast-iron end shields E protect the armature coils from mechanical injury, the former being of a perforated design so as not to interfere with the ventilation of the winding.

The stator shown in Figure 132 is of an entirely different construc-

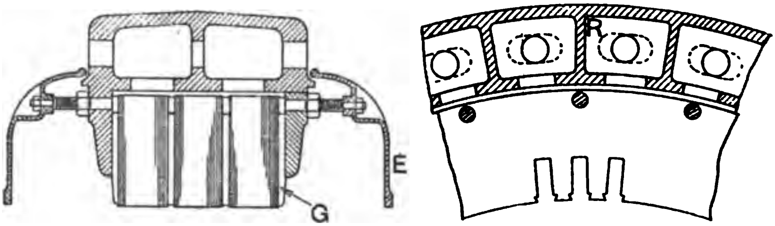


FIG. 131.—Stator frame for large alternator.

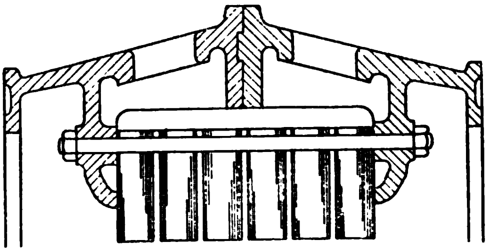


FIG. 132.—Stator frame.

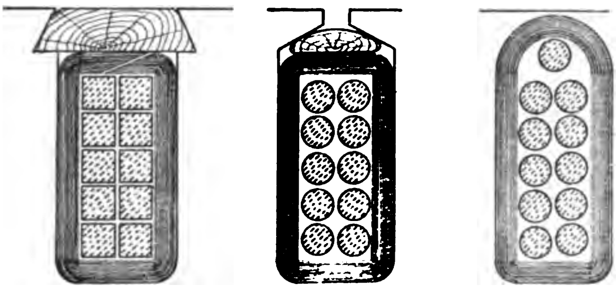
tion, and is typical of continental machines. The frame is cast in two exactly similar halves, so as to divide it in a plane perpendicular to the shaft. In building, all the laminations are assembled on one half; the other half is then laid on and the two sections clamped together. End flanges are cast to each section, between which the laminations are secured.

Slots.—Slots for retaining the armature winding are of three types:—(1) open; (2) semi-closed; (3) totally closed. Each of these is shown in Figure 133. The open slot is the one in most general use because heavily insulated former-wound coils may then be employed. The latter have the advantage that they are easy to wind, can be thoroughly impregnated with insulating varnish prior to assembling, and, should any coil become faulty, it can be readily removed for repairs.

The chief objection to open slots is that they cause an unequal distribution of flux under the pole face, and so introduce higher harmonics in the E.M.F. wave. Open slot armatures also necessitate the use of laminated pole shoes (see p. 316).

The materials generally employed for slot linings are:—leatheroid, press-spahn, manila paper, and micanite, the latter being almost exclusively used for pressures exceeding 2000 volts. When employed, the micanite is formed into long rectangular or oval-shaped tubes with a scarfed opening at the mouth of the slot, thus enabling the straight side of the coil to be inserted in its insulating envelope before the coils are assembled. The conductors are held in place by wood keys driven into grooves near the top of the teeth.

When totally closed slots are used the conductors must be threaded through by hand, the slots being previously lined with seamless micanite or press-spahn tubes, which project well out beyond the



Open. Semi-closed. Closed.

FIG. 133.—Design of slots.

flanges of the core. Closed slots are of advantage in that they eliminate ripples in the E.M.F. wave and avoid the necessity for laminated pole shoes. Coils placed in closed slots are, however, more expensive to wind, and the winding space is not so well utilised.

The self-induction of a closed slot winding is, owing to the low reluctance of the path of local flux, much greater than with the corresponding open slot winding. This latter objection may to some extent be overcome by adopting semi-closed slots. Armatures with this type of slot are easier to wind than those with totally closed slots, as the conductors can be passed singly through the mouth of the slot. They still, however, involve more labour than open slots with former-wound coils.

Rotors.—The rim and spider forming the fly-wheel of an alternator are, for diameters up to about 2.5 metres, generally cast in one. With flywheels of larger diameter, the best practice is to cast them in two or more sections. This diminishes the possibility of the structure

becoming permanently weakened, due to enormous internal stresses caused by the non-uniform cooling after casting.

The construction of a large flywheel, cast in halves, is shown in Fig. 128. The rim and hub parts are bolted together at M and H respectively, and in order to increase the strength of the rim joints, wrought-iron rings are shrunk over pops L. As a further security steps are turned at each face of the hub, and the steel rings shrunk on after the shaft has been fitted. Excessive cooling strains can also be obviated by casting the rim and spider separately, and bolting together as in Fig. 134. With this construction the invariable practice is to make the spider of cast-iron and the rim of cast-steel. Another con-

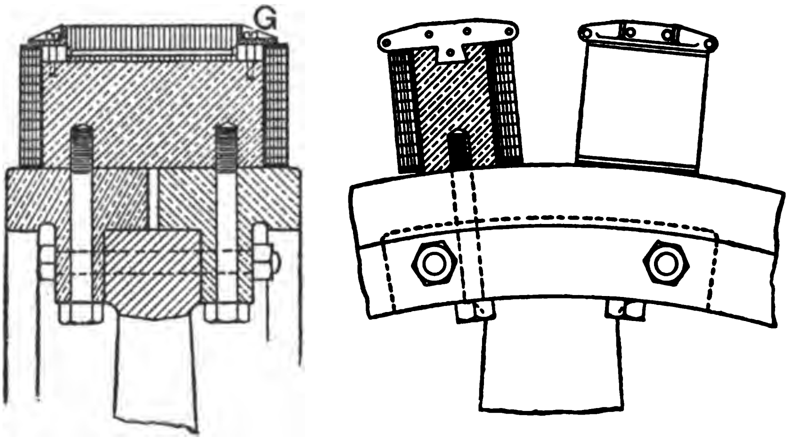


FIG. 134.

struction, applicable to large alternators of 1500 K.W. or upwards, is shown in Fig. 135. The yoke part of the rim is of laminated iron secured to the cast-iron supporting rim R by means of keys K and cast iron flanges F. As is usual in this construction, dovetails are machined on the periphery of the rim for the reception of the pole pieces.

The number of spider arms varies from 6 in small alternators to 10 or 12 in those of very large diameter. Where the pole cores are fixed by bolts passing through the rim, the spokes are sometimes cast with double arms, as in Plate X. This facilitates the fixing or removing of the poles. Rims are generally of rectangular cross-section, and should be of sufficient sectional area to carry half the flux of each pole. When alternators are direct-coupled to reciprocating engines a frequent practice is to make the moment of inertia of the rotor such that a flywheel between the engine and the alternator may be dispensed with. The mass of the flywheel rim must then be great enough to store the requisite kinetic energy. If the flywheel be separate, the

whole of the energy stored in it, and available for keeping a uniform peripheral speed with sudden changes in load, has to be transmitted through the shaft and spider, producing large torsional strains, which are avoided when the rotor and flywheel are combined. This practice, however, is only to be recommended when the most favourable diameter of rotor and flywheel naturally approximate to each other, and this is usually the case only with machines of outputs over 1000 K.W. In other cases, if the flywheel be given the dimensions which would best suit the alternator, an unduly large weight of rim may be necessitated and a larger shaft to carry it; on the other hand, if the diameter of rotor be increased to suit the flywheel requirements

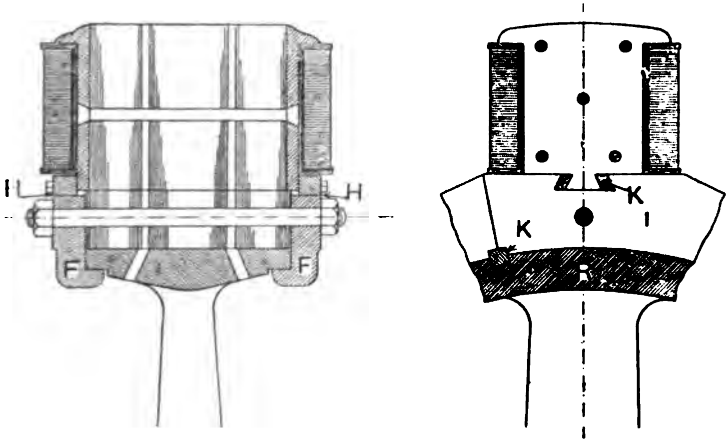


FIG. 135.

the armature frame must be of greater depth and size to give it the necessary rigidity.

Poles and Pole Shoes.—Owing to the requirements of speed and frequency the poles are generally of greater dimensions in the direction parallel to the shaft than in a direction tangential to the rim. For frequencies between 25 and 60 cycles per second the pole pitch at the periphery ranges from 20 to 30 centimetres, about 65 per cent. of which is occupied by the pole shoe arc. The breadth of core at right angles to shaft is always less than the polar arc, and ranges between 10 and 30 centimetres. For this reason pole cores are, with very few exceptions, of rectangular cross-section, and constructed of either cast or laminated steel.

Figs. 134 to 136 illustrate three types of construction all of which are in extensive use. The pole of Fig. 134 is of cast-steel and secured to the rim of the magnet wheel by bolts. The pole shoes may either be solid or laminated, depending upon whether the slots of the stator are semi-closed or open. When solid, the pole and shoe can be cast in

one. With laminated shoes the stampings, about 1 mm. thick, are usually dovetailed on to the pole-piece and riveted together between end-plates. Instead of employing solid pole cores and laminated shoes, the

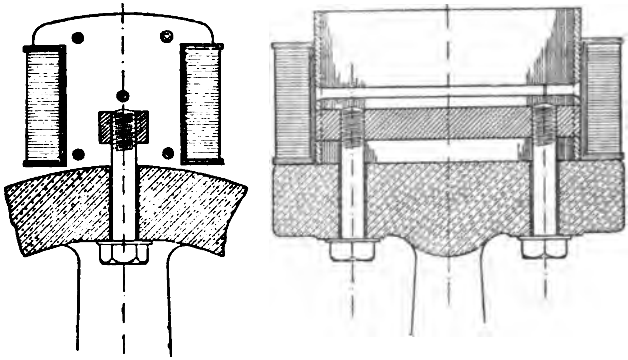


FIG. 136.

poles are in a large number of designs laminated throughout as in Fig. 136. The laminations are T-shaped, so that, besides increasing the pole face area, the projecting parts serve to retain the field coils in position against the action of centrifugal force. Each pole is secured to the rim by two bolts, which are screwed into a solid iron bar passing right through the laminated pole core. A second design with laminated poles is shown in Fig. 135. The poles fit into dovetails on the rim, and are secured by the keys K_1, lateral movement being prevented by the end rings H, H.

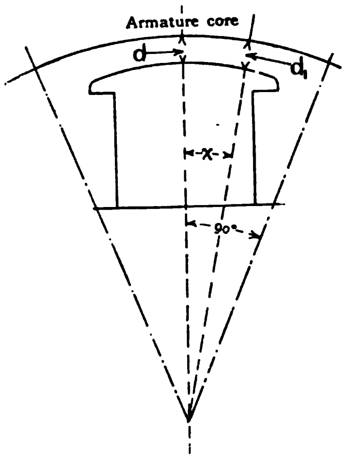


FIG. 137.

Form of Pole Shoes. — The distribution of the magnetic flux on different parts of the pole face is largely determined by the length of the air-gap at any point, to which the magnetic flux is almost inversely proportional. If with open slots the strength of the field be not graded near the pole edges, pronounced harmonics are introduced into the wave-form. There are several methods of shaping the pole face in order to get the desired sine wave. Fig. 137 gives the outline of a laminated pole, one pole pitch corresponding to 180 electrical degrees. To obtain a theoretically perfect sine wave the air-gap is laid out thus:—Let *d* denote the length of air-gap under

the centre of the pole face, then the length d of air-gap at any other point at an angular distance from the centre line equal to x electrical degrees is expressed by—

$$d_1 = \frac{d}{\cos x}$$

Fig. 138 shows two approximate methods, each of which gives a flux distribution agreeing very closely with the theoretical curve.

Field Coils.—With revolving field alternators the field coils are generally of rectangular copper strip wound on edge with paper insulation between the turns and the external surface of the winding, protected merely by a coating of insulating varnish. If ordinary wire be employed, the turns, owing to the high centrifugal force, would tend to roll over each other and thereby damage the insulation by chafing. With strip wound on edge the effect of centrifugal force is simply to compress the turns towards the periphery without chafing the insulation. Besides being mechanically strong, this type of winding has the further advantage that the heat generated in the coil is quickly dissipated, thereby tending to maintain the coil at a more uniform temperature.

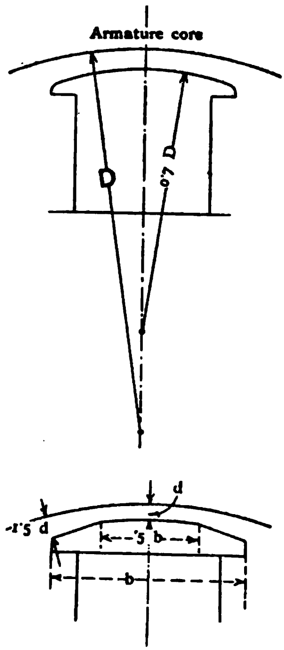


FIG. 138.

The copper strip is wound on a spool of metal or compressed paper, fitting tightly on the pole core. When of metal, the spool requires to be insulated with presspahn, manila paper, or empire cloth, bound on with tape and impregnated with varnish.

The coils are held in place by the overhanging edges of the pole shoes (see Figure 136) or by special gun-metal bridges (Figure 134) which are secured to pole. The bridge pieces also serve as damping coils for maintaining steady running when alternators are operated in parallel.

Slip-rings and Brushes.—The slip-rings, of cast-iron or gun-metal, are generally supported from a cast-iron spider keyed or shrunk on to the shaft. A good design for slip-rings is shown in Figure 139. The rings, cast with lugs L, are bolted to the spider B and insulated therefrom by the vulcanite bushes and washers V. The sectional area of the rings should be designed to give the required mechanical strength, which, as a rule, is more than ample for current-carrying purposes.

Connection to the field winding is made by copper strip or cable clamped by insulating blocks to one of the flywheel arms and attached to the rings as shown at D.

Copper brushes were at one time largely used, but this led to a rapid deterioration of both brushes and slip-rings, with the result that in nearly every case they have been abandoned for carbon ones. This is also an advantage from the manufacturer's point of view, in that carbon brushes may then be standardised and made suitable for both direct- and alternating-current machines. In medium-sized alternators the spindles S, which support the brushes, are fixed to arms projecting from a cast iron-quadrant Q, the latter being bolted to a step cast with the bearing cap. In very large alternators the brushes are supported from an overhanging bracket arm, as in Plate X.

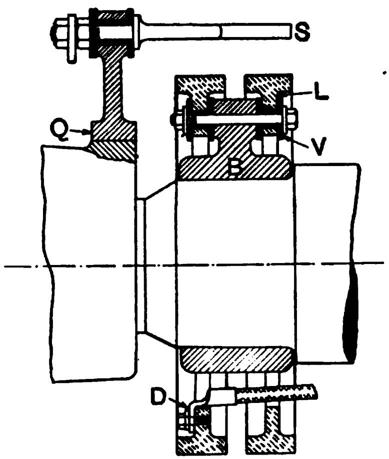


FIG. 139.—Slip-rings.

Exciters.—The power required for exciting the field magnets is small, ranging from 1 to 3 per cent. of the rated output of the alternator. In many cases a small direct-current shunt-wound generator, known as an exciter, provides the exciting current, the exciter armature being either direct coupled to the alternator shaft or driven from it through gearing. With this arrangement each generating set is self-contained, and the exciting current can readily be altered by means of a rheostat connected in series with the exciter's field winding. An inherent disadvantage with this method of excitation is that any change in the speed of the prime mover affects the exciter's voltage, and therefore the current through the field coils of the alternator. From this point of view it is better to have the exciter driven by a separate prime mover. This arrangement is usually adopted in large generating stations, where one dynamo will excite several alternators, each alternator having its own field regulating resistance.

TURBO-ALTERNATORS

Stators.—In high-speed alternators, constructed by different firms, there is comparatively little difference in the design of the stationary or high tension element. This follows slow-speed practice fairly closely, though the external appearance is widely different, owing

to the fewer poles, smaller diameter, and greater length of high-speed machines.

It has been found necessary to introduce a special system of brackets for clamping the end-connections to the stator frame. This is because of the enormous mechanical forces developed between the coils of the various phases when the alternator is suddenly short circuited or paralleled with other machines before it is properly synchronised. These sudden stresses will also occur in slow and medium speed alternators, but owing to the short span of the coils across the pole pitch the end-connections will generally have sufficient mechanical strength to prevent any appreciable displacement or distortion. Again, with turbo-generators the magnetic leakage across the slots and the end-connections bears a much smaller ratio to the total flux per pole than it does in the case of engine-type alternators. The current on short circuit will therefore be considerably greater in the turbo-alternator, and stresses as high as 1500 kgs. per metre length of end-connections have frequently been met with in practice.

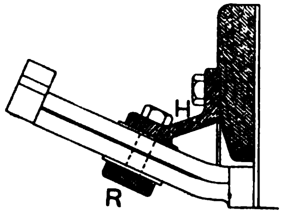


FIG. 140.—Clamping arrangement for turbo-alternator.

The old plan of tying coils together with torpedo twine and securing them with wooden blocks is wholly inadequate, and very strong metal clamps must be used. This necessitates protecting the winding outside the slots with an insulation which is not only strong enough to withstand the whole testing pressure, but is of such a good mechanical nature that it will not be crushed under the pressure of the clamps.

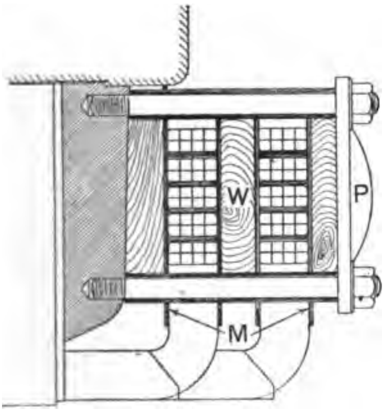


FIG. 141.—End clamp for turbo-alternator windings.

Designs of clamping arrangements are shown in Figures 140 and 141. The former is suitable for barrel winding in which all the coils are of the same shape and fit against one another forming a continuous lattice-work (Figure 152). The end-connections are supported by an internal gun-metal ring R, which is well insulated and bolted to a series of brackets H supported from the end flanges of the core. For concentric coil winding laid up in two or more ranges an arrangement similar to that shown in Figure 141 must be adopted, where the supports are formed by screwing brass or bronze bolts into the end-

flanges of the core. The windings are clamped between the latter and metal plates P, and the various ranges separated by mica strips M and hard wood blocks W. The leakage lines surrounding the end-connections also become linked with the clamping bolts and supporting ring and induce eddy currents therein. It is therefore advisable to construct the clamps entirely of non-magnetic material, as otherwise

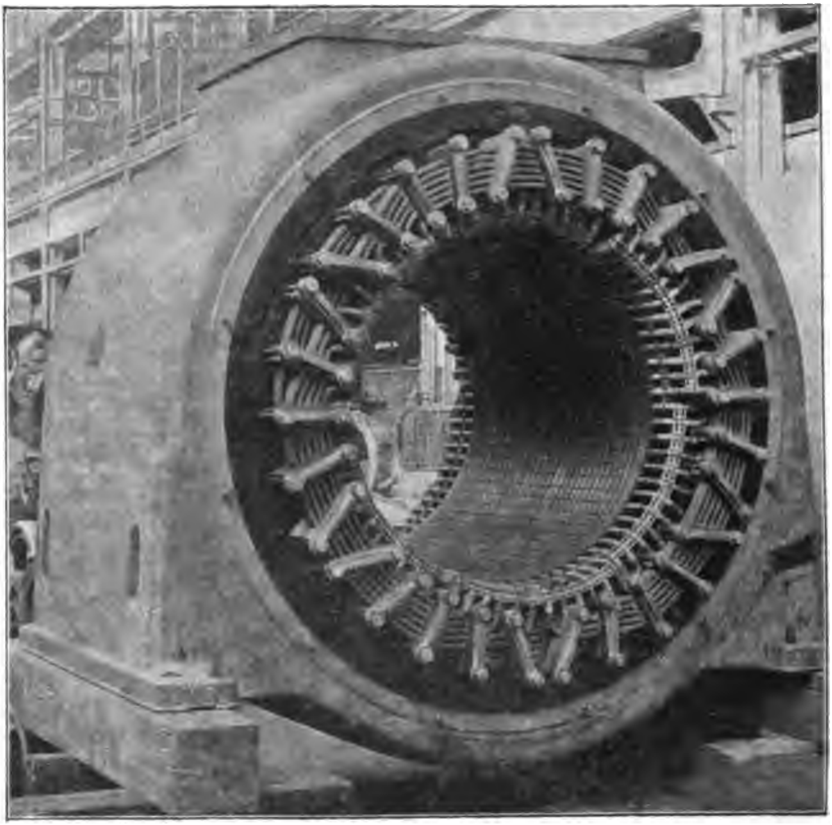


FIG. 142.—Winding of Westinghouse 6000-K.V.A. turbo-alternator.

they are liable to become hot owing to the greater eddy current loss resulting from the increased leakage. Figure 142 shows the windings and clamping arrangements on a 6000-K.V.A. 11,000-volt 3-phase generator built by the British Westinghouse Company.

Rotors.—The design of high-speed rotors is essentially a mechanical problem. For the usual frequencies, between 25 and 60 cycles per second, the number of poles is usually four, but seldom exceeds six. The volume of iron and copper per pole must therefore

be relatively large, and at the high peripheral speed prevailing enormous stresses will be set up, due to centrifugal force. For example, in the case of a 1000-K.W. rotor having a diameter of 80 cms. and revolving at 1500 revolutions per minute, each kilogramme of matter at the periphery tends to fly outwards with a force $= 11.2 \times 10^{-6} \frac{D}{2} . R^2 = 11.2 \times 10^{-6} . 40 . 1500^2 = 1000$ kilogrammes where $\frac{D}{2} =$ radius of rotor in cms. and $R =$ revolutions per minute. Another difficulty associated with turbo-machinery is the violent vibrations which may be set up by the slightest out-of-balance of the rotor.

The materials used should therefore be as homogeneous as possible, and their distribution absolutely symmetrical. Rotors are generally constructed of forged or cast steel of the best quality. When of the latter the steel should be cast under pressure, because otherwise there is a danger from blow-holes, which not only affect the strength of the structure but throw the rotor considerably out of balance.

In some cases the balancing difficulties may not occur for some considerable time after the machine has been erected. When in continual use the insulation dries and becomes compressed under the action of centrifugal force of the conductors. This often results in a displacement of the field windings and the consequent upsetting of the rotor balance. Considerable attention must therefore be given to the design of the windings and insulation, so that the field coils remain rigid under all conditions of service.

Excessive vibrations are also produced when the normal running speed coincides with, or approaches to, the "critical speed." This latter is reached when the natural period of the transverse vibration of the shaft, loaded with the rotating element, corresponds to the impulses given to the shaft by the centrifugal forces resulting from any want of balance or from curvature of the shaft. At critical speed steady operation of the revolving mass is impossible, though at speeds somewhat above or below this the vibrations are almost eliminated. When possible, the normal running speed should be below critical speed, but if otherwise the normal speed should be reasonably higher, so that the critical speed, except when starting up or shutting down, is never reached in ordinary operation.

High-speed rotors are constructed for peripheral speeds of between 65 and 100 metres per second, and are of two distinct types—

(1) Those having definite poles, with bobbins and coil windings.

(2) Those having a cylindrical core, with the field windings distributed in slots.

These are known as the "definite" or "salient" pole and "cylindrical" types respectively.

Definite Pole Type.—The construction of definite or salient pole rotors is illustrated in Figures 143 and 144. In the case of the

former the rotor body is forged with four projecting poles, and is bored out at each end to receive the shafts S, S. These are of special high tensile steel, and enter the rotor to about one-quarter of its length. The solid pole shoes P are made to slide over T-shaped projections on the rotor body, and are cast with a large number of ribs R, which become highly saturated and diminish the field distortion on load. Each magnet coil M is wound in four sections with copper strip, and supported on a metal spool with heavily insulated flanges.

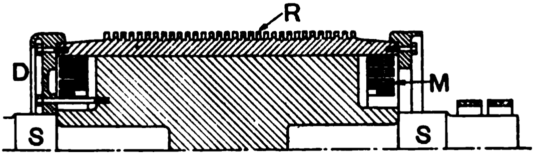


FIG. 143A.—Definite pole rotor for turbo-alternator.

To guard against centrifugal force, which would tend to spread the windings, eight V-shaped phosphor-bronze clamps C are inserted between adjacent coils and pressed against them by bolts screwed into the main forging. Special care must be observed in the fixing of these clamps, for should any of the holes be a little bit out, then on tightening up a very serious bending stress may be put on some of the bolts. In the design under consideration this straining of the bolts is avoided by making the underneath part of the bolt head or nut spherical, so as to give a ball-and-socket action.

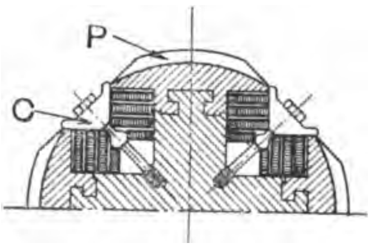


FIG. 143B.

The end caps D, also of phosphor bronze, are bolted both to the rotor body and the pole shoes, and prevent any longitudinal movement of the latter. They also act as a protection to the winding.

In the second construction (Figure 144) the rotor body is built up of a central solid steel casting, which is machined and bored out to a diameter larger than the shaft, and two cross-shaped pieces D fitted at each end to carry the rotor on the shaft. The pole shoes E are laminated and dovetailed into the poles of the steel casting. The phosphor-bronze end pieces F, dovetailed to D, retain the pole shoes in position by bolts passing lengthwise through the laminations. They also help to keep the spools in position against the action of centrifugal force. The field coils are wound on copper bobbins provided with insulated flanges, and held down against the horizontal component of the centrifugal force by gun-metal wedges H. As will be seen from the cross-section, the latter are not bolted to the

centre casting, but rock on the pivot K and so accommodate themselves to variations in the field coil instead of imposing a bending strain at the neck of the support.

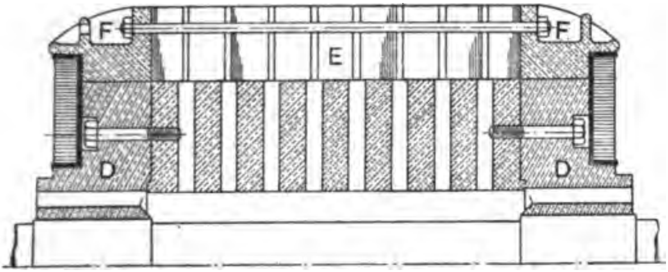


FIG. 144A.—Definite pole rotor for turbo-alternator.

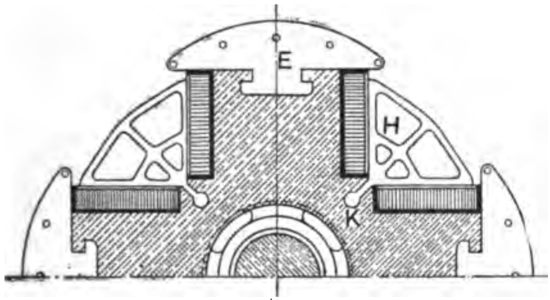


FIG. 144B.

Cylindrical Type.—Figure 145 illustrates the cylindrical type of construction. The rotor body consists of sheet steel laminations which are mounted direct on a cast-steel shaft and clamped between steel end flanges E. The laminations are separated at intervals of from 6 to 10 cms., so as to form radial air ducts for the ventilation of the core. The field winding is embedded in a number of slots milled out along the core on either side of the poles, the position of the latter being indicated by the letters N and S. The winding for each pole consists, in this case, of three former-wound coils of copper strip laid on the flat, the slot portions of the coil being enveloped with micanite or press-spahn tubes. The coils are retained in their slots by wedges G of phosphor bronze, and the external surfaces are finished to give a perfectly smooth exterior.

The end connections of the winding are held down against the action of centrifugal force by phosphor-bronze covers H bolted to the flanges F. Before fitting the end covers the ends of the rotor windings are enveloped by press-spahn sheets held together with compound and subjected to pressure by binding wire, thus making the

insulating cover as solid as possible. The press-spahn is afterwards turned in a lathe to exactly fit the end shield. The ventilation of the rotor is assisted by a fan J fixed to the shaft, the path of the air currents being indicated by the arrows.

At one time the definite-pole design was exclusively used in this country, but manufacturers are now adopting the cylindrical construction first introduced by Brown, Boveri & Co. of Baden. The cylindrical type is superior for the following reasons:—

(1) A winding distributed in slots offers more cooling surface than the coils on salient poles.

(2) A smooth cylindrical surface will give better security as to

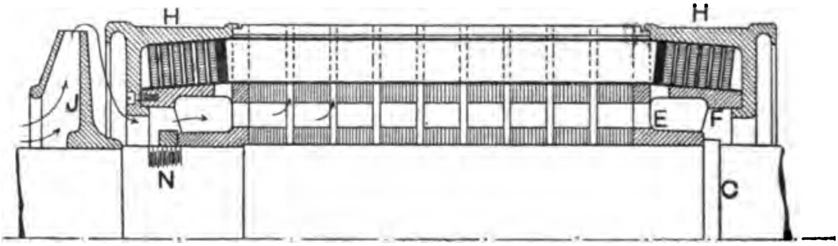


FIG. 145A.—Cylindrical rotor for turbo-alternator.

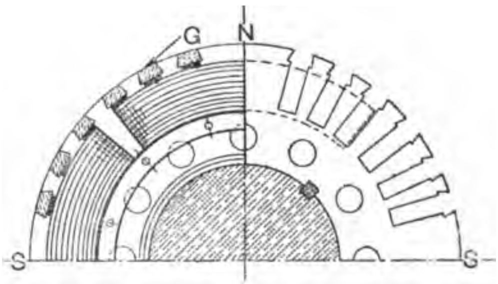


FIG. 145B.

noiseless running than where sudden variations in the sections of the air currents are caused by projections or irregularities.

(3) A permanent balance is more certain to be secured with a solid cylindrical structure than when there are projecting poles and external field bobbins.

In very large machines the second consideration is of great importance, for any approach to a noiseless operation is extremely difficult with salient pole designs.

ARMATURE WINDINGS

The armatures of alternating current generators are, with very few exceptions, drum wound. Since all connections from conductor to

conductor must be made across the flanges of the core, it follows that the conductors forming the respective sides of a turn must be situated in fields of opposite polarity, so that the E.M.F. shall act in the same direction through the winding.

The most convenient mode of studying drum windings is to represent the armatures and poles as laid out flat, and this procedure will be adopted in the following diagrams.

Single-Phase.—The winding of a 6-pole single-phase alternator having one conductor and slot per pole is developed in Figure 146;

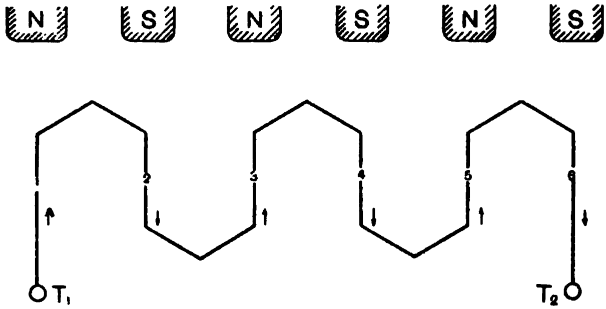


FIG. 146.

the conductors being numbered 1 to 6. If the field magnets rotate from right to left, then the direction of the E.M.F. induced in each conductor as it is cut by the respective fields will be as indicated by the arrow heads. The six conductors are connected together, so that

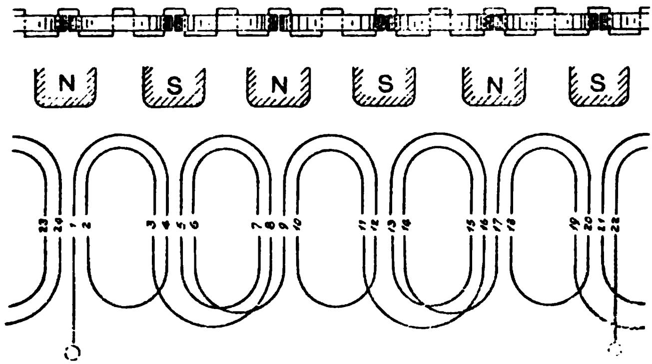


FIG. 147.—Whole-coiled single-phase winding.

when the winding is traversed from terminal T_1 to terminal T_2 the E.M.F.'s act in the same direction round the circuit, the P.D. between the terminals being six times the voltage induced in each conductor.

Next, suppose the same armature with one slot per pole to have four conductors located in each slot. The conductors can now be

connected up by two different methods shown in Figures 147 and 148 respectively. In the former the circuit through the armature from terminal to terminal will be along the conductors in the following order:—1, 4, 2, 3, 8, 5, 7, 6, 9, 12, 10, 11, 16, 13, 15, 14, 17, 20, 18, 19, 24, 21, 23, and 22; whereas in Figure 148 the order of the con-

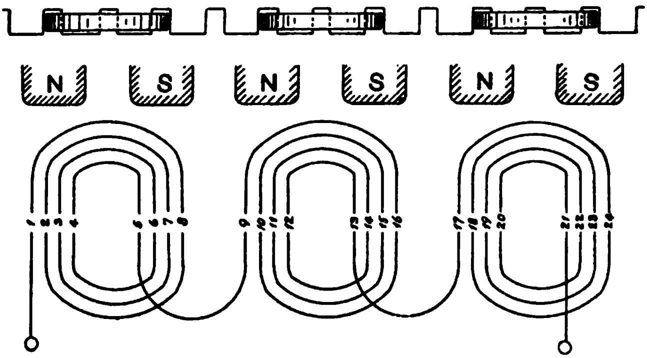


FIG. 148.—Half-coiled single-phase winding.

ductors is:—1, 8, 2, 7, 3, 6, 4, 5, 9, 16, 10 . . . . . 19, 22, 20, 21. So far as the magnitude and wave-form of the E.M.F. is concerned these two windings are identical, the difference between them being in the method of making the end-connections.

This difference is clearly brought out in the respective figures, where there is also shown the back ends of the armatures as they would

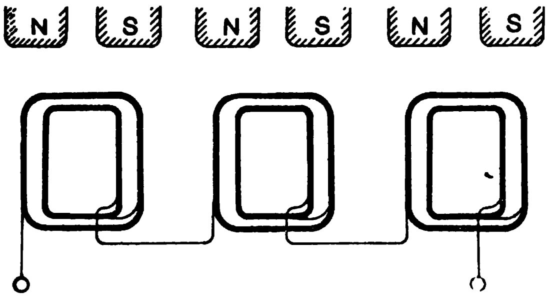


FIG. 149.—Half-coil single-phase winding.

actually appear with their windings in place. In Figure 147 the end-connections are spread over the entire circumference, whereas, with the second winding only half the circumference is so utilised, alternate parts being devoid of end-connections.

It should also be noted in connection with these two styles of windings that the first consists of six similar coils connected in series,—*i.e.* there is *one coil per pole*; and the second of three coils in series,—

i.e. *one coil per pair of poles.* These windings are designated *ordinary or whole-coiled* and *hemitropic or half-coiled* respectively.

With the former the whole of the poles are subtended by coils, whereas with hemitropic windings the coils subtend alternate poles only. In the whole-coiled winding the current circulates in a clockwise direction in one-half the coils, and counter-clock in the other; but with a half-coiled winding the current traverses the entire winding in the one direction. This difference is due to the situation of the coils relative to the poles. In Figure 147 any two adjacent coils are situated under poles of opposite polarity, whereas in Figure 148 they are under poles of the same polarity.

A hemitropic winding, by reason of its greater mean length per turn, requires more copper than the corresponding whole-coiled winding; but, on the other hand, it gives a less complicated arrangement of end-connections, and approximately one-half the self-induction. With ordinary single-phase windings the additional end-connections are

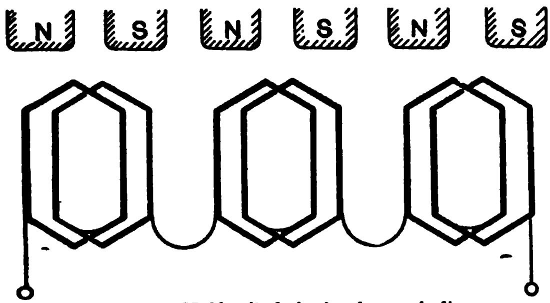


FIG. 150.—Half-coiled single-phase winding.

not of very great moment, so that either style of winding may be adopted.

With polyphase alternators, however, the end-connections of an ordinary winding would be somewhat complicated, and in order to have as simple an arrangement as possible hemitropic windings are the more frequent. For high-voltage alternators the windings would be made up of coils consisting of from, say, 4 to 30 turns, and wound either by hand or on formers.

Coil windings are usually represented, not by a number of conductors concentrated together, but by a single heavy line, the connections between the various coils being indicated by fine lines.

The single-phase windings so far examined have had their conductors concentrated in one slot per pole, but the more usual arrangement is to distribute the sides of a coil over two or more slots. Such windings are said to be of the *multiple-coil* type, to distinguish them from *single-coil* or concentrated windings. The development of a 6-pole double-coil hemitropic winding is shown in Figures 149 and 150.

In the former the coils are of rectangular shape and individual elements of unequal pitch, whereas in Figure 150 the elements of a coil are all of equal pitch but with evolute end-connections. The winding of Figure 151 is that of a 4-coil ordinary winding with two coils of unequal pitch per pole.

In low-voltage alternators for large output the number of conductors

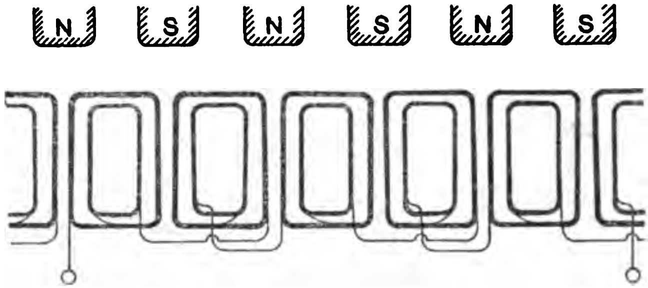


FIG. 151.

per pole is generally limited to 1 or 2, the end-connections being made so as to form either a wave or a lap winding. Figure 152 gives the development of an 8-pole hemitropic winding with 24 conductors. There are three slots per pole with one conductor in each, and the ends of the winding terminate at T_1 and T_2. In tracing out the circuit

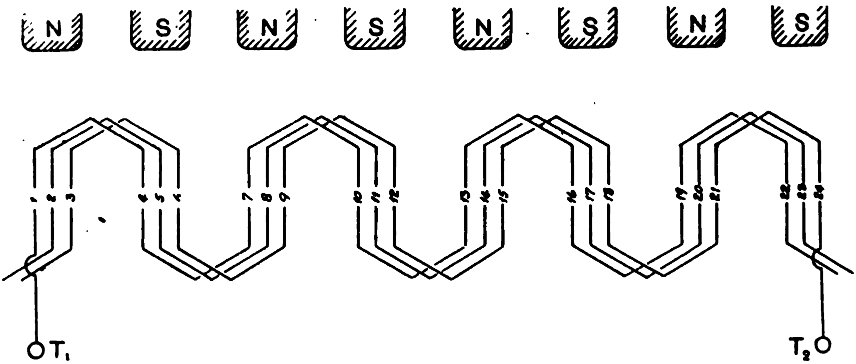


FIG. 152.

from T_1, a tour of the armature is made by conductors 1, 4, 7, 10, 13, 16, 19, and 22; starting off again from 22, a second and third tour will be made by 2, 5, 8, 11, 14, 17, 20, and 23; and 3, 6, 9, 12, 15, 18, 21, and 24.

The development of the same armature when the conductors are connected up to form a lap winding is shown in Figure 153.

With both lap and wave windings the turns are of equal pitch. The

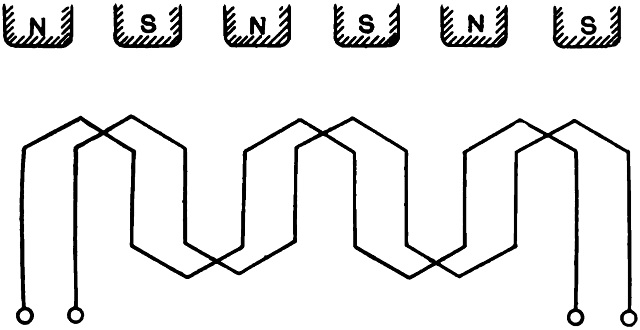


FIG. 154.—Two-phase winding.

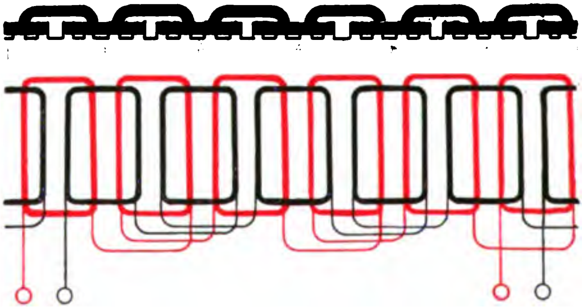


FIG. 155.

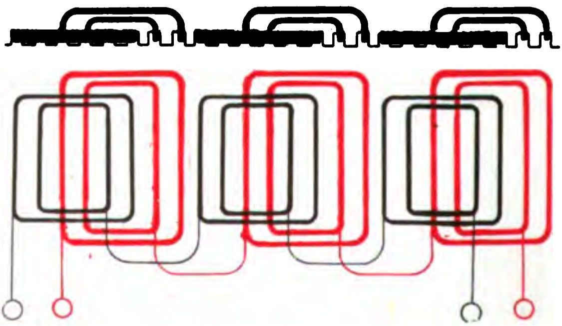


FIG. 156.

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E.M.F. generated in the two windings would be equal in value and of the same wave form, the choice between the two methods of connection being purely a question of mechanical convenience and cost.

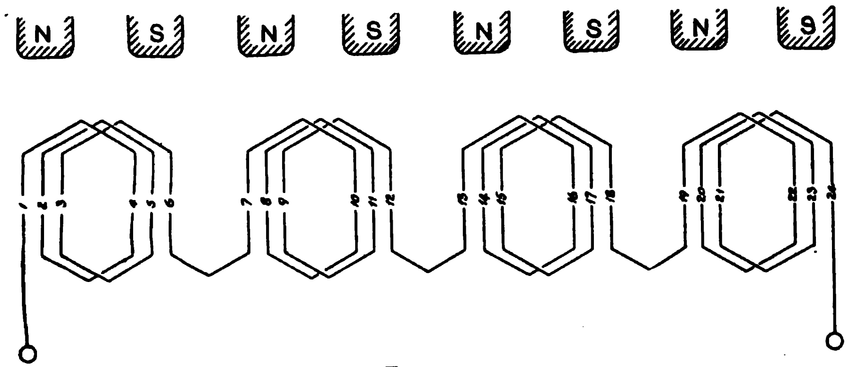

FIG. 153.

Two-Phase.—A 2-phase armature corresponds to a double single-phase armature, and, as previously stated, any single-phase generator may be converted into a 2-phaser by superposing on the original set of coils a second set exactly similar, but displaced through an angle corresponding to half the pole pitch. Hence, to convert the stator represented in Figure 146 into a 2-phaser it would be necessary to mill out six additional slots intermediate between the original six and wind in them the coils of the second phase.

The 2-phase equivalent is shown in Figure 154, and to distinguish the second phase from the first, the former is shown in red.

With 2-phase alternators, either whole-coil or half-coil windings may be employed. The former are, however, limited to armatures where the number of slots per pole per phase is an even integer.

If an armature for a 2-phase winding have 6 slots per pole—*i.e.* 3 slots per pole per phase, then the winding must be hemitropic. Of course, when the sides of the coils of each phase are distributed over 4 slots per pole, either style of winding may be adopted.

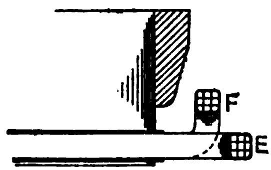

FIG. 157.

Figures 155 and 156 respectively show the development of ordinary and hemitropic windings for a 6-pole stator having spiral-wound coils distributed over two slots per pole.

Owing to their overlapping, the end-connections of each winding of a 2-phase generator must be bent into two different planes,—*i.e.* laid up in two ranges, as shown in Figure 157. With the set of coils marked E the end-connections are brought out straight, so as to lie on a

cylindrical surface; whereas, in the second set F, the coil-ends after clearing the slots are bent up at right angles.

The end bends must, of course, be designed so that the mean length per turn is as nearly as possible the same for both phases, otherwise the two circuits would be of unequal resistance.

Short-coil Windings.—To obviate the overlapping of the coils in polyphase windings it has been proposed to use coils having a breadth equal to half the pole-pitch. A winding composed of such coils is termed a *short-coil winding*, and the diagram of one for a 4-pole stator is shown in Figure 158.

In these windings there are two coils—one for each phase—within the pole-pitch, and the coils, all of the same size and shape, are assembled on the armature side by side, thus eliminating any crossing of the end-connections. Comparing with the 2-phase winding of Figure 154, it will be noted that the coils are much narrower, hence the term *short coils.*

With this winding the mean length of turn per coil is reduced, and consequently the total length of wire per phase. On the other hand, the pole-face arc must not exceed the width of a coil, and, in consequence of this, the armature periphery is not so well utilised. For this reason, together with the fact that narrow coils give a wave form of E.M.F. which is highly peaked, short-coil windings are very rarely employed.

Three-Phase.—If three such windings as are shown in Figure

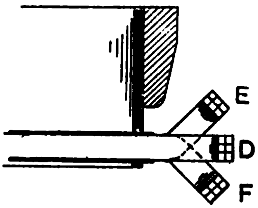

FIG. 160.

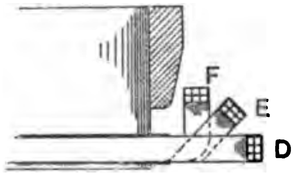

FIG. 161.

146 be superposed and spaced out symmetrically over the armature periphery, then the 3-phase winding of Figure 159 will be obtained. The stator must now have 24 slots, each side of a coil being concentrated in a single slot.

The use of whole-coiled windings for 3-phase alternators has an inherent disadvantage, in that the overlapping end-connections must be laid up in three ranges as in Figure 160. The ends D of the first phase are brought out straight, while with the second and third phases the most convenient arrangement would be to bend their ends in opposite directions, as indicated at E and F respectively.

The end-connections of the second phase protrude in front of the poles of the rotor, and thus prevent it being withdrawn sideways. This

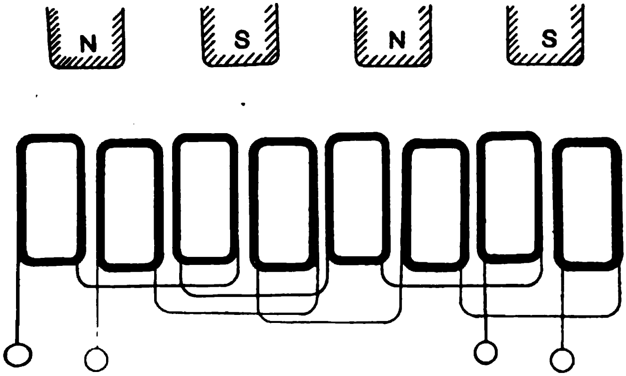

FIG. 158.

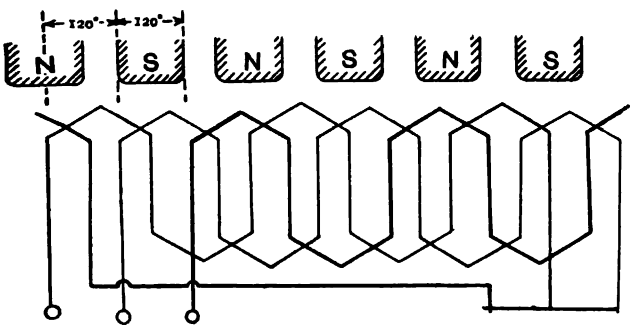

FIG. 159.

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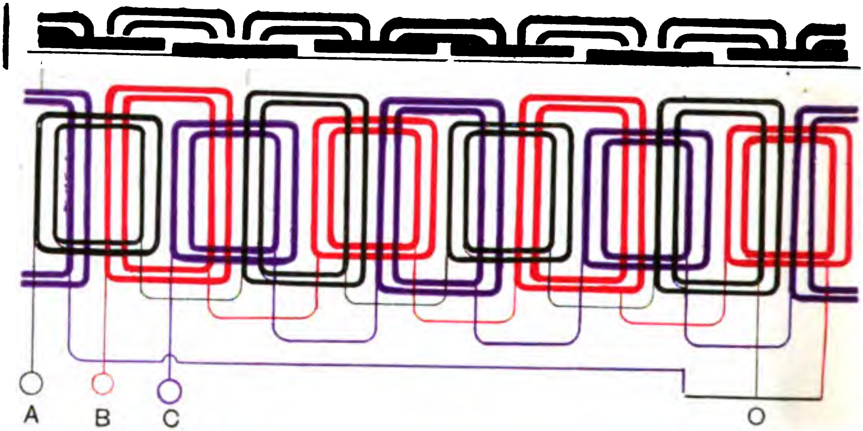

FIG. 162.

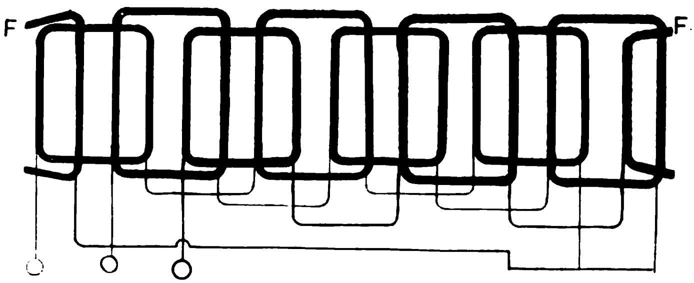

FIG. 163.

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objection could be overcome by bending the end-connections backwards into the position shown in Figure 161, but this entails too complicated an arrangement of end-connections. Whole-coiled windings are therefore very seldom used for 3-phase machines, hemitropic windings rendering a much simpler arrangement of end-connections.

Figure 162 is for an 8-pole hemitropic 2-coil winding where the coils of each phase have a relative displacement of 120° electrical. The windings are shown star-connected, the various phases starting from the terminals A, B, and C, and terminating at the neutral point O. The end-connections can now be laid up in two ranges.

The coils belonging to any one phase occupy alternate front and back ranges, the coils in the different ranges being represented by long and short coils respectively. It will be noted that each phase consists of two long coils and two short coils,—*i.e.* two in the front range and two in the back. The three phases are therefore exactly similar, and should the length of the mean turn of the front range coils differ from that of the back range coils, each phase will still have the same total length of conductor, and be consequently of the same resistance and quite balanced.

This symmetry only occurs when the number of poles is a multiple of four, and should this type of winding be employed in an armature wound for 6, 10, 14, etc. poles one of the coils must necessarily be askew when passing from a back to a front range. This is indicated at FF in Figure 163, which shows the development of a single-coil winding for a 6-pole machine.

In the case of alternators of large diameter the arrangement of 3-phase coils in two ranges has one disadvantage: since the coils overlap one another at their ends, the top half of a wound stator cannot be removed without first unwinding those coils whose end-connections lie across the joints in the stator frame.

However, by adopting the arrangement of winding shown in Figure 164 the necessity of unwinding any of the coils is done away with. In this winding the three sets of coils have a relative displacement of 60 electrical degrees, and their end-connections are laid up in three ranges, all the coils belonging to one phase being of the same range. The different ranges are here also distinguished by coils of different lengths; but it must be remembered that the actual lengths of wire forming the coils of each phase must be approximately equal, otherwise the phases will be unbalanced. At the planes marked *dd*, which should be made to coincide with the joints in the stator frame, the core may be divided without interfering with the coils themselves, it only being necessary to break the connections between the coils. Since the coils of the various phases are displaced by 60 degrees, the above winding is really a 6-phaser, and to convert it into a true 3-phase arrangement

the red phase winding must have its connections reversed. Hence for star grouping the beginning of the red phase is connected to the neutral point. The one fault with this winding is that, owing to the coils being in three ranges, the crossing of the various phases involves a rather complicated arrangement of end-connections.

Figure 165 shows the development of a 6-pole lap-wound armature, having three slots per pole per phase and one conductor per slot, the elements of all the coils being of the same breadth. This winding is half-coiled, but whole-coiled windings are equally suited for 3-phase bar windings.

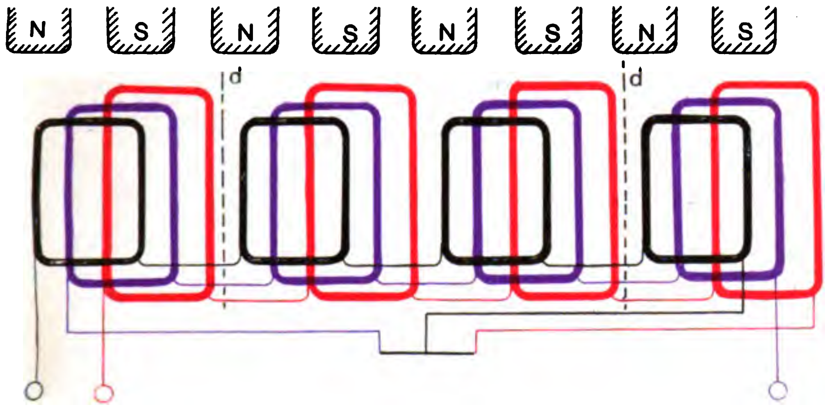

FIG. 164.

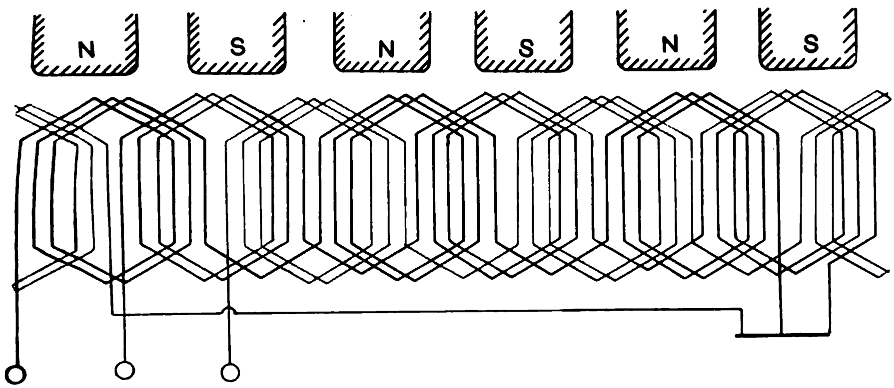

FIG. 165.

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CHAPTER VII

ALTERNATORS:—E.M.F. EQUATION—HARMONICS IN E.M.F. WAVE DUE TO TEETH—MAGNETIC CIRCUIT CALCULATIONS

In Figure 166, representing the distribution of flux density along the air-gap of an alternator, let x denote the distance of a conductor C from a fixed point O at which the strength of magnetic field is zero, l the length of active conductor parallel to the shaft, B the flux density at C,

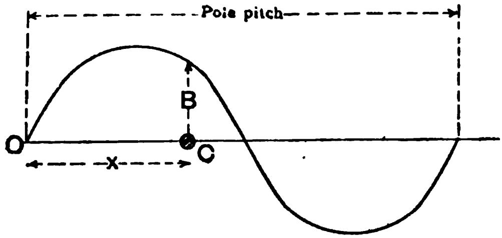


Fig. 166.

and t the time taken to pass from O to C; then the E.M.F. generated in the conductor is $e = l \cdot B\frac{dx}{dt}10^{-8}$ volts, where $\frac{dx}{dt}$ denotes the velocity of the field poles relative to the armature. If the former rotate with a constant angular velocity, $\frac{dx}{dt}$ is constant and *the shape of the curve of E.M.F. induced in each conductor will be the same as the shape of the curve of flux distribution in the air-gap.* Further, if τ denote the pole pitch and T_o the periodic time of the electromotive force,

$$\int_0^{\frac{T_o}{2}} e\,dt = \int_0^{\frac{\tau}{2}} Bl\,dx\,10^{-8} = Bl\frac{\tau}{2} \times 10^{-8} = \Phi \times 10^{-8}$$

$$i.e. \quad E_{av.}\frac{T_o}{2} = \Phi \times 10^{-8}$$

where $E_{av.}$ = the average value of the induced E.M.F.

Φ = total flux entering or leaving the armature per pole.

Now $T_o = \frac{1}{\sim}$, where $\sim$ denotes the frequency of the induced E.M.F. Hence $E_{av.} = 2 \sim \Phi 10^{-8}$ volts per conductor.

For a concentrated winding, the E.M.F.'s induced in the various turns belonging to one phase attain their zero and maximum values at the same instant, so that, if there be T turns in series, the average induced E.M.F. is

$$E_{av} = 4T \cdot \sim \cdot \Phi \cdot 10^{-8} \text{ volts}$$

where $\sim = \frac{Rp}{60}$, R being the revolutions per minute and p the number of pairs of poles.

Form Factor.—To determine the effective or R.M.S. value E of the induced electromotive force, the shape of the E.M.F. wave must be known. Let k_1 denote the ratio E/E_{av}, then the effective value of the E.M.F. for a concentrated winding is

$$E = k_1 \cdot 4T \cdot \sim \cdot \Phi \cdot 10^{-8} \text{ volts}$$

The coefficient k_1 is termed the *form factor*, its value for a given alternator depending on the ratio of pole arc to pole pitch, and the extent to which the pole shoe edges are bevelled.

In the case of an alternator having a pole arc ψ equal to 50 per cent. of the pole pitch, and a uniform length of air-gap over the pole face, the curve of instantaneous E.M.F. for one half-period would be a rectangle, the fringing of magnetic field at the pole tips being neglected. If E_1 denote the constant value of the induced E.M.F. when the conductors are under the pole face, then the effective value

$$E = \sqrt{\frac{E_1^2 \times \psi/2}{\psi}} = 0.707 E_1$$

Next suppose that the pole arc is reduced to 25 per cent. of the pole pitch, while the flux per pole remains the same. The curve of induced E.M.F. will again be a rectangle, but will have a maximum value double that in the first case. The R.M.S. value of the E.M.F. now $= E = \sqrt{\frac{(2E_1)^2 \cdot \psi/4}{\psi}} = E_1$. Since the total flux per pole is assumed to have the same value for each ratio of pole arc to pole pitch, the average value of the E.M.F. $= 0.5$, E_1 remains unaltered. The form factors for $\psi = 0.5$ and $\psi = 0.25$ will therefore be 1.41 and 2.0 respectively, thus showing that for a given flux per pole the effective value of the E.M.F. is increased by reducing the ratio of pole arc to pole pitch.

The assumption of sharp rectangular polar edges does not in general apply to practical cases, as the edges of the pole shoes are always more or less rounded. If, retaining the same value of the magnetic flux, the tips of the poles be rounded off there will be a greater concentration of the flux under the centre of the pole. This will cause the curve of instantaneous E.M.F. to become more peaked and thus increase the value of the form factor. Hence with rectangular poles the value of k_1 for the same magnetic flux per pole and the same

ratio of pole-arc/pole-pitch, will be less than is the case when the pole tips are bevelled. In modern alternators the contour of the pole face is generally shaped (Figure 138) so that the flux distribution in the air-gap, and consequently the E.M.F. induced in each conductor, approximates to a simple sine curve. When such is the case

$$E = \frac{E_{max}}{\sqrt{2}}, \quad E_{av} = \frac{2}{\pi}E_{max}, \quad \text{and} \quad k_1 = \frac{\pi}{2\sqrt{2}} = 1.11$$

With a concentrated winding the value of the effective E.M.F. per phase will therefore be expressed by

$$E = 4.44 \,.\, T \,.\, \sim \,.\, \Phi \,.\, 10^{-8} \text{ volts}$$

Breadth Factor.—The above formula for the induced E.M.F. was derived on the assumption that the width of one side of a coil was

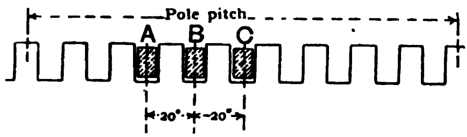


FIG. 167A.

negligible, so that the E.M.F.'s generated in the individual turns are in phase with one another and can be added arithmetically. With a distributed winding the E.M.F. induced in the various elements of a coil will be in different phases. For instance, in the single-phase winding of Figure 153 the E.M.F. induced in each of the three elements of a coil will have the same effective values, but differ in phase by 1/6 of 180 degrees—*i.e.* 30 degrees. If the vectors OD, DE, and EF (Figure 167B) represent the E.M.F. induced in each element respectively, then their vector sum OF gives the effective value of the E.M.F. between the terminals of the coil, which, as will be obvious from the diagram, is somewhat less than the arithmetical sum of the E.M.F.'s induced in the various elements. With a distributed winding the pressure at the terminals of each phase will, therefore, for the same values of T, $\sim$, Φ, and ψ, be less than for a concentrated winding.

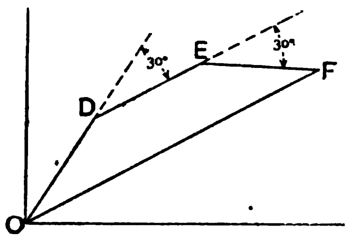


FIG. 167B.

The general expression for the induced E.M.F. per phase becomes

$$E = 4.44 \,.\, k_2 \,.\, T \,.\, \sim \,.\, \Phi \,.\, 10^{-8} \text{ volts} \quad . \quad . \quad . \quad . \quad (55)$$

The coefficient k_2 is generally referred to as the *breadth factor*, and its value for a distributed winding will always be less than unity. To determine the breadth factor for various arrangements of coils some

assumption must be made as to the distribution of magnetic flux along the air-gap, and in what follows it will be assumed that the flux varies according to the simple sine law. As an example, the value of k_2 will be calculated for a 3-phase armature having 3 slots per pole per phase. There will be 9 slots per pole as shown in Figure 167A, the slots having an angular pitch of 20 degrees. Suppose the conductors of slots A, B, and C to be in series, then if $E_{max} \sin \theta$ denotes the E.M.F. generated in the middle slot that generated in the other two will be $E_{max} \sin (\theta + 20)$ and $E_{max} \sin (\theta - 20)$. The maximum value of the E.M.F. at the terminals of the coil will then be

$$E_{max} \sin \theta + E_{max} \sin (\theta + 20) + E_{max} \sin (\theta - 20)$$
$$= E_{max} \sin \theta (1 + 2 \cos 20) = 2.879 \, E_{max} \sin \theta$$

If the conductors had been concentrated in a single slot the maximum E.M.F. for the same number of turns per coil would have been $3 \, E_{max} \sin \theta$. The ratio

$$\frac{2.879 \, E_{max} \sin \theta}{3 \, E_{max} \sin \theta} = 0.96$$

is the value of the breadth factor for a 3-phase winding having 3 slots per pole per phase. The values of k_2 for other arrangements of windings, derived in a similar manner, are set forth in Table XVI. For a single-phase wound on a core so that half the slots are left empty the value of the breadth factor will be the same as for a 2-phase winding having the same number of slots per pole per phase.

TABLE XVI.—VALUES OF THE BREADTH FACTOR k_2.

Winding.	Slots per Pole per Phase.					
	1	2	3	4	5	∞
Three-phase	1.0	0.966	0.960	0.958	0.957	0.956
Two-phase . . . . . . Single-phase (50 per cent. of slots without coil)	1.0	0.924	0.910	0.906	0.904	0.903
Single-phase (slots all wound) . .	1.0	0.707	0.667	0.654	0.647	0.637

From the above table it will be noted that as the winding becomes more distributed the value of k_2 decreases, and therefore also the value of the induced E.M.F. for an alternator of given constants. On the other hand, the spreading of the coils over several slots per pole diminishes the inductance of the winding and utilises to a better extent the available winding space, thus ensuring a more uniform heating of the armature. Another consideration in favour of distributed coils is that the armature reactions, for a given number of armature ampère-

turns, are considerably less than when the winding is concentrated. Hence it is, in most cases, advantageous to employ a distributed winding, and in modern alternators the standard practice is to distribute the sides of a coil over from two to five slots, five and six being the usual number of slots per pole per phase for turbo-alternators.

Example.—From the following data of a 440-K.V.A. 3-phase alternator calculate the flux per pole required at no-load. The pole shoes are shaped so as to give a sine distribution of flux.

Terminal volts = 600	Number of slots = 240
Speed in R.P.M. = 375	Conductors per slot = 1
Number of poles = 16	Winding star connected

$$\text{Flux per pole} = \Phi = \frac{E \times 10^8}{4.44 \,.\, k_2 \,.\, T \,.\, \sim}$$

$$\text{Volts per phase} = E = \frac{600}{\sqrt{3}} = 346$$

$$\text{Slots per pole per phase} = \frac{240}{16 \times 3} = 5$$

Breadth factor $k_2 = 0.957$

Turns per phase $= T = 40$

$$\text{Frequency} = \sim = \frac{375 \times 8}{60} = 50$$

Substituting these values in the above equation, the flux per pole at no-load

$$\Phi = \frac{346 \times 10^8}{4.44 \times 0.957 \times 40 \times 50} = 4.1 \times 10^6 \text{ lines}$$

Harmonics in E.M.F. Waves due to Toothed Armatures

In order that the electromotive force wave on open circuit may be a sine curve, the flux density in the air-gap must vary according to the simple harmonic law. With smooth-core armatures—*i.e.* armatures having totally closed slots—this is attained by shaping the pole shoes so that the reluctance of the air-gap at any point is inversely proportional to the cosine of the electrical angle between that point and the centre of the pole. When the armature has open slots the magnetic reluctance at different parts along the air-gap will vary according to the position of the armature teeth relative to that part. Consequently, the movement of the poles past the armature will cause variations in the magnitude and distribution of the main field, and thereby introduce harmonics into the E.M.F. wave.

The variation to which the main field is subjected may be rendered clear by considering two typical cases: (1) when the polar arc is a multiple of the tooth pitch; and (2) when the polar arc is a multiple plus one-half the tooth pitch. These two cases are illustrated for an alternator

having nine slots per pole, in Figures 168 and 169 respectively, the width of a tooth in each case being equal to the width of a slot.

When the polar arc is a multiple of the tooth pitch the two extreme positions of the armature relative to the poles are when (*a*) two slots and (*b*) two teeth are under the pole tips; these positions are shown at A and B respectively (Figure 168). In position A, since there is very little fringing, owing to the uniform air-gap, the greater portion of the flux will enter the armature by those teeth actually under the pole shoe, and hence the boundary lines of the flux will be approximately as shown at *a* and *b*. When the poles move into the position B, the flux will spread out towards the outer edges of the teeth, so that it now enters the armature by way of seven teeth instead of six teeth as previously.

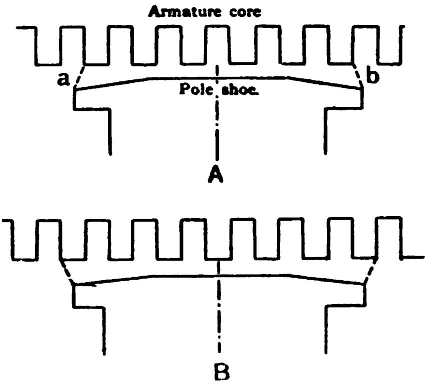


FIG. 168.

$\frac{\text{Pole arc}}{\text{Tooth pitch}} = 6.$

Hence as the poles move past the armature the reluctance of the main magnetic circuit is subject to a periodic variation in magnitude which causes the magnitude of the flux to vary periodically also. This magnetic oscillation is usually referred to as "flux pulsation." When the polar arc is a multiple plus one-half of the tooth pitch (Figure 169) the two extreme positions are when (*a*) a slot is under the trailing tip and a tooth under the leading tip, and (*b*) a tooth is under the trailing tip and a slot under the leading tip. These two positions are shown at A and B respectively, the boundary lines of the flux in the two cases being approximately as indicated. From the latter diagrams it will be seen that the number of teeth by which the flux enters the armature, and consequently also the magnitude of the main flux, remains constant, but that the line of symmetry of the field is subject to periodic variation relative to the centre line of the poles.

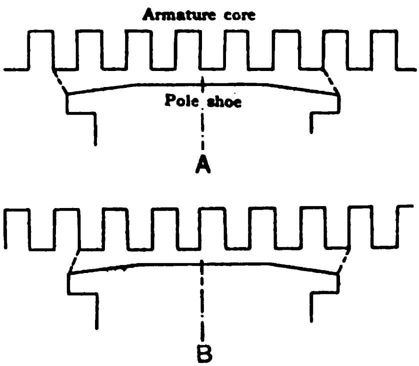


FIG. 169.

$\frac{\text{Pole arc}}{\text{Tooth pitch}} = 5.5.$

This gives rise to a to-and-fro oscillation of the main flux, which may be termed a "flux swing." Although the principal variations to which the main flux is subject in the two cases is as described, yet, owing to there being no definite boundary line between the fields, the magnetic flux is also subject to a slight "swing" in the first case and to a slight "pulsation" in the second. When the proportions of polar arc to tooth pitch lie between the values given above, the oscillations which occur will include both the types described, but their magnitudes will be considerably less. In either case the frequency of oscillation will be equal to the number of teeth passing a pole per second—*i.e.* $= 2n\sim$, where n = number of slots per pole pitch and $\sim$ = frequency of the generator.

When the flux is subject to pulsations the E.M.F. ripples will be proportional to the flux linked with a coil. The flux linked is a minimum when the centre of the coil coincides with the mid-pole position, and is a maximum when it coincides with the middle of the pole. Hence the E.M.F. ripples will be a minimum in the former position and a maximum in the latter. On the other hand, the main E.M.F. generated in a coil due to the rotation of the poles or armature will be a maximum in the former position and a minimum in the latter. Hence for the case shown in Figure 168 the ripples in the E.M.F. wave will be a maximum as the wave crosses the zero line and a minimum at the crest of the wave.

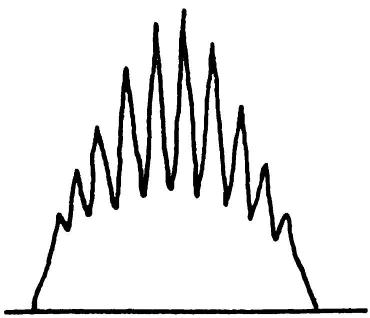


FIG. 170.

$\dfrac{\text{Pole arc}}{\text{Tooth pitch}} = 5.5.$

When the flux is subject to a swing it is evident from Figure 169 that no harmonics will be generated in a coil so long as the sides of the latter are in the interpolar space. As soon, however, as they come under cover of the pole shoes, ripples in the E.M.F. wave will appear. Hence when the polar arc is a multiple plus one-half of the tooth pitch the ripples in the E.M.F. wave will be a maximum on the crest and a minimum as the wave crosses the zero line. This is clearly shown in Figure 170, which gives the wave form of an alternator where $\dfrac{\text{pole arc}}{\text{tooth pitch}} = 5.5$, the alternator being connected to a number of unloaded cables.

In a general case the ripples on the crest of an E.M.F. wave will be due to swaying of the flux, whereas those near the zero portion of the wave will be due to flux pulsation. The latter are always less apparent than the former, owing to the fact that the harmonics do not show so much on the sloping sides of the curve, and also because any pulsation

of the main flux is bound to be damped out to some extent by the eddy currents in the iron.

Frequency of the Harmonics.—Since the frequency of oscillation of the main magnetic flux $= 2n$ times the frequency of the alternator, and therefore must be an even multiple of the machine frequency, it would appear as if it gave rise to an even harmonic in the E.M.F. wave. This, however, is not the case; for, owing to the "flux pulsation" and "flux swing" not each existing throughout the whole period, the ripples will exhibit a periodic discontinuity of phase. Owing to these discontinuities the positive and negative half-waves of electromotive force will be identical, in spite of the fact that the frequency of any train of ripples is an even multiple of the fundamental frequency.

When there are n teeth per pole the magnetic flux in the air-gap of an alternator is equivalent to an alternating flux of frequency $2n$ superposed upon a main sinusoidal flux of unity frequency and having the same average value as that of the irregular flux curve. If Φ denote the flux per pole, the main flux linked with a coil at any displacement θ of coil from centre of pole $= \phi_1 = \Phi \cos \theta$. If the main flux had a constant value all over the air-gap the alternating flux of frequency $2n$ would be of constant amplitude. Assuming the oscillations of flux to follow a simple sine law, the value of the high-frequency component of the flux linked with a coil would be

$$\phi_2 = \epsilon_1 \cos (2n\theta - a)$$

But the amplitude ϵ_1 varies with the magnitude of the main flux; hence

$$\phi_2 = \epsilon\Phi \cos \theta \cos (2n\theta - a)$$

where ϵ is the amplitude factor, the value of which is always less then unity. The instantaneous value of the total flux linked with an armature coil is therefore

$$\phi = \phi_1 + \phi_2 = \Phi \cos \theta\{1 + \epsilon \cos (2n\theta - a)\}$$

and the E.M.F. induced thereby is

$$e = -\mathrm{T}\frac{d\phi}{dt} = -\mathrm{T}\frac{d\phi}{d\theta}\frac{d\theta}{dt} = -2\pi \sim \mathrm{T}\frac{d\phi}{d\theta}$$

$$= -2\pi \,.\, \mathrm{T} \,.\sim.\, \Phi \,.\, \frac{d}{d\theta}[\cos \theta\{1 + \epsilon \cos (2n\theta - a)\}]$$

$$= 2\pi \,.\, \mathrm{T} \,.\sim.\, \Phi[\sin \theta + \epsilon \,.\, \frac{2n-1}{2} \,.\, \sin \{(2n-1)\theta - a\}$$

$$+ \epsilon \,.\, \frac{2n+1}{2} \,.\, \sin \{(2n+1)\theta - a\}]$$

Hence the oscillation of magnetic flux with a frequency $2n$, as due to n slots per pole, introduces into the E.M.F. wave two harmonics of the orders $(2n-1)$ and $(2n+1)$. For example, in a 3-phase alternator having two slots per pole per phase, $n = 6$ and

$$e = 2\pi \,.\, \mathrm{T} \,.\sim.\, \Phi\{\sin \theta + \frac{11\epsilon}{2} \sin (11\theta - a) + \frac{13\epsilon}{2} \sin (13\theta - a)\}$$

The lowest harmonics introduced by the action of the armature slots will be the eleventh and thirteenth respectively. With three slots per pole per phase the lowest harmonics would be the seventeenth and nineteenth.

This conclusion as to the order of the most important harmonics can be explained as follows. An oscillation of flux due to $2n$ teeth per pair of poles is equivalent to an alternating flux of frequency $2n$ superposed on a constant flux. Now the former, as explained on page 266, may be resolved into two components, each of one-half the amplitude, rotating forward and backwards with $2n$ times the velocity of the field system. If the speed of the main field be taken as unity, the speeds of the forward and backward rotating fields relative to the armature will be $(2n+1)$ and $(2n-1)$ respectively; hence the appearance of the $(2n+1)^{\text{th}}$ and $(2n-1)^{\text{th}}$ harmonics in the E.M.F. wave.

Harmonics due to armature teeth should as far as is practical be eliminated from the E.M.F. waves of alternating current generators, for there may possibly arise conditions, similar to those shown in Figure 14, where these high-frequency harmonics resonate with the external circuit, in the event of which the pressure of the system may rise considerably above its normal value. With open slot armatures the elimination of the ripples in the E.M.F. wave is greatly helped by rounding off the pole edges. This prevents any sudden changes in flux as the pole-edge leaves a tooth, and if there be a large number of teeth per pole, so that the flux swing is small, the ripples produced will be of exceedingly small magnitude. With semi-closed slots the slot openings will have very little effect upon the reluctance of the air-gap, and with suitably shaped pole shoes it is possible to obtain an E.M.F. wave which deviates by little from a pure sine curve. Another method of preventing open slots from affecting the air-gap reluctance is to make the pole edges slightly askew (Figure 171), so that, if the edge of the pole shoe be projected on the armature, this projection will be parallel to the line joining the middle point of a slot directly under the side-edge of a pole piece to the middle point of an adjacent slot directly under the opposite edge. With this design of pole shoe the reluctance of the magnetic circuit will remain constant for all positions of the poles relative to the armature.

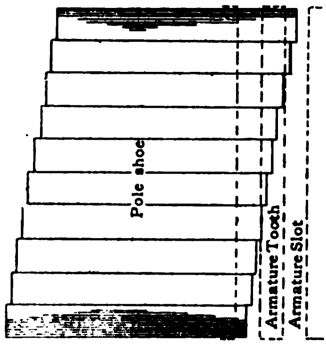


FIG. 171.—Askew pole shoes consisting of ten packets of laminæ.

With a concentrated winding the shape of the E.M.F. wave will be similar to that of the flux curve, and, if the latter exhibits irregularities

due to the presence of open slots, corresponding ripples will appear in the E.M.F. wave. When the same winding is distributed over several slots per pole per phase, not only will the amplitude of the harmonics be less owing to the smaller slot openings, but the ripples in the E.M.F. curve for the whole coil will not be pronounced as in the electromotive force wave of an individual element. For example,

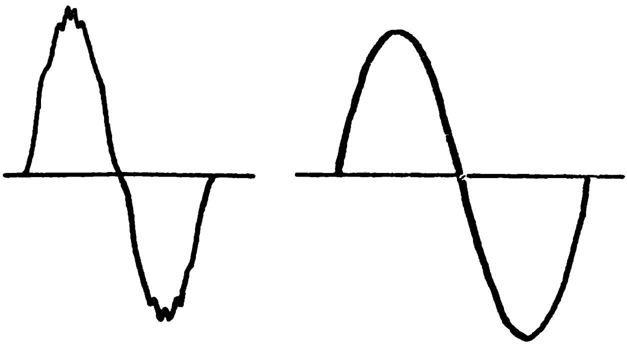


FIG. 172.

consider a 3-phase alternator having three slots per pole per phase, and suppose that the E.M.F.'s induced in the various elements of a coil may be represented by a non-sinusoidal curve A (Figure 172), each curve being displaced from another by a distance equal to the slot pitch, *i.e.* 20 degrees electrical. The resultant wave of the E.M.F. for the whole coil, obtained by adding the ordinates of the three curves together, is represented by the second curve B drawn to a smaller scale. The

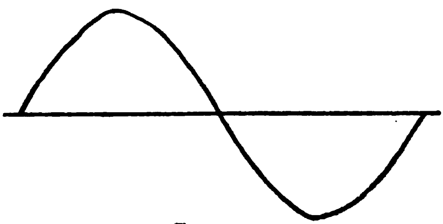


FIG. 173A.
(1) No-load.

latter is more regular than the component curve, thus showing that the distribution of the winding results in a nearer approximation to the sine wave of electromotive force.

From these considerations it will be seen that, even with slotted armatures, an alternator can be designed to give an E.M.F. curve which at no-load is approximately sinusoidal. It is, however, impossible with salient pole field magnets to retain this shape under all conditions of load. For, when current flows in the armature

winding a cross magnetomotive force is set up which distorts the magnetic flux and increases the flux density in the air-gap under the trailing horn of the pole introducing a pronounced third or fifth harmonic into the E.M.F. wave. This cross magnetic flux, which is a maximum on non-inductive load and decreases as the power factor is diminished, can only be annulled by distributing the field winding in several partially or totally closed slots, as is the case with rotors of the cylindrical type. The possibility of obtaining a sine wave of E.M.F. at all loads is therefore another consideration in favour of cylindrical rotors for turbo-alternators. The effect of armature reaction on the electromotive force wave form of a 600-K.W. 3-phase star-connected alternator is clearly shown by the curves in

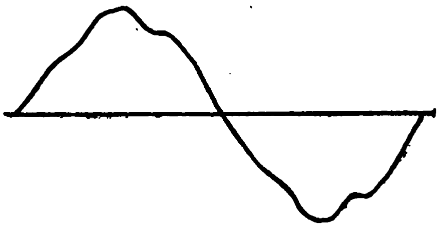


FIG. 173B.

(2) Full-load at 0.74 power factor.

Figure 173, which give the wave forms for (1) no-load, and (2) full-load at 0.74 power factor.

Harmonics in Three-phase Alternators.—In a star-connected generator all harmonics whose frequency is a multiple of three times the fundamental, though present in the induced E.M.F. curve for each phase, are entirely eliminated in the terminal E.M.F. curve taken across the lines. The reason for this will be made clear by considering the general equations for the E.M.F.'s induced in the various phases. The expressions for the E.M.F.'s are as follows (see p. 7):—

$$
\begin{aligned}
e_1 &= E_1\sin\theta + E_3\sin(3\theta+\phi_3) + E_5\sin(5\theta+\phi_5) + \ldots \\
e_2 &= E_1\sin(\theta-120) + E_3\sin\{3(\theta-120)+\phi_3\} + E_5\sin\{5(\theta-120)+\phi_5\} + \ldots \\
&= E_1\sin(\theta-120) + E_3\sin(3\theta+\phi_3) + E_5\sin(5\theta-240+\phi_5) + \ldots \\
e_3 &= E_1\sin(\theta-240) + E_3\sin\{3(\theta-240)+\phi_3\} + E_5\sin\{5(\theta-240)+\phi_5\} \\
&= E_1\sin(\theta-240) + E_3\sin(3\theta+\phi_3) + E_5\sin(5\theta-120+\phi_5)
\end{aligned}
$$

From the above it will be seen that the third harmonic is in phase at any instant in all the three windings, and so cancels out in the E.M.F. wave taken across the line. This conclusion also applies to the ninth, fifteenth, and twenty-first harmonics. Hence no triple-frequency currents can flow in the line wires unless the star point of the generator is connected to the neutral point of the load either through a fourth wire or via earth. The elimination of the third harmonic from the

E.M.F. wave of a star-connected alternator is clearly indicated by the curves in Figure 174. The first curve, having a pronounced third harmonic, shows the E.M.F. across one phase. The second curve is the line E.M.F., and shows no indication of a third harmonic. With a mesh-connected armature the triple-frequency harmonics act in the

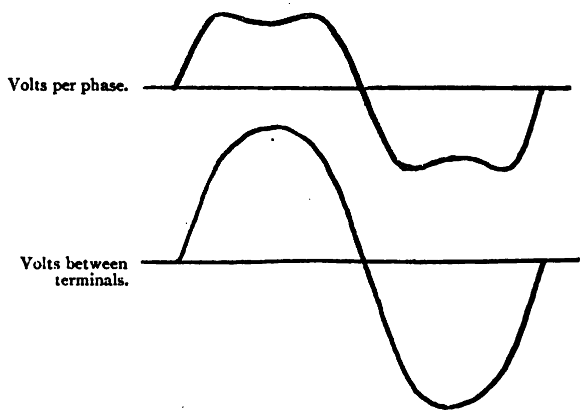


Fig. 174.—E.M.F. wave of star-connected alternator.

same direction round the mesh, and thereby tend to circulate currents internally. The increased I^2R loss and heating due to these local currents may, under favourable circumstances, be considerable, so that the star connection is to be preferred.

Magnetic Circuit Calculations

From the E.M.F. equation given in a previous section of this chapter it will be seen that for a winding having a given number of turns the induced voltage is a function of the magnetic flux. In order that this flux may become linked with the armature turns the field winding must produce a certain excitation. If the magnetic flux entering the armature per pole be plotted as a function of the field ampère-turns, then the curve so obtained gives the magnetisation curve of the alternator under consideration. For a given speed the total E.M.F. induced in the armature winding, when on open circuit, is expressed by

$$E = 4.44\, k_2 \,.\, T \,.\, \sim \,.\, \Phi \,.\, 10^{-8} = C\Phi \text{ volts}$$

where C is a constant. Hence the no-load saturation curve may be computed from the magnetisation curve by marking off along the ordinate reference axis the corresponding values of induced E.M.F. (see Figure 175). This curve is obtained experimentally by driving the alternator at normal speed and observing the terminal voltage

corresponding to various values of the exciting currents. For excitations between zero and normal—*i.e.* from O to A, the curve is approximately a straight line, because for low inductions the permeability of the iron is approximately constant, thus permitting the magnetic flux and consequently the induced E.M.F. to increase in the same proportion as the excitation. The less the radial depth of air-gap the steeper will be the slope of the straight line portion of the curve. When the excitation is increased beyond that corresponding to the point A, the curve bends over to the right, and on further increasing the exciting current would tend to become nearly horizontal, due to the saturation of the magnetic circuit.

To obtain the magnetisation curve when working out the design of

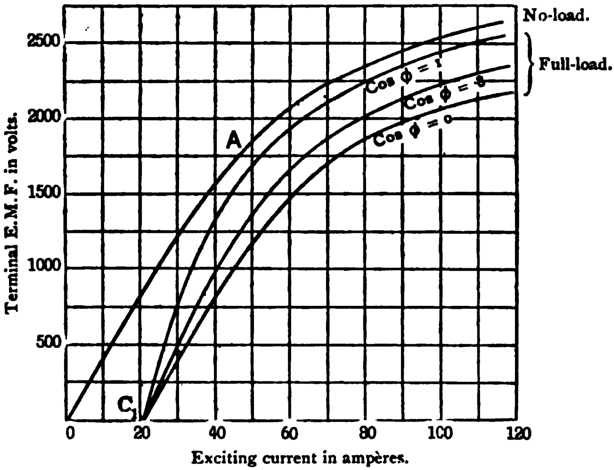


FIG. 175.—Saturation curves of an alternator.

an alternator it is necessary to calculate the excitation corresponding to various values of the magnetic flux. In a composite magnetic circuit, such as occurs in electrical machines, the magnetomotive force of the exciting coils is expressed by

$$0.4\,\pi\,.\,AT = \int H\,.\,dl$$

$$= H_1L_1 + H_2L_2 + H_3L_3 + \ldots\ldots$$

$$\text{or} \qquad AT = 0.8\ H_1L_1 + 0.8\ H_2L_2 + 0.8\ H_3L_3 + \ldots\ldots$$

where AT denotes the ampère-turns for the field coil; H_1L_1, H_2L_2, H_3L_3, etc. the fall of magnetic potential over the various parts of lengths L_1, L_2, L_3, etc. and permeability μ_1, μ_2, μ_3, etc.

Now, since

$$H = \frac{B}{\mu} = \frac{\Phi}{A_1\mu}$$

where B = flux density, A the cross-sectional area of magnetic circuit, and Φ the total magnetic flux,

$$AT = \frac{0.8\ \Phi_1}{A_1\mu_1} . L_1 + \frac{0.8\ \Phi_2}{A_2\mu_2} . L_2 + \ldots\ldots$$
$$= \frac{0.8\ B_1}{\mu_1} . L_1 + \frac{0.8\ B_2}{\mu_2} . L_2 + \ldots\ldots$$

Now $\frac{0.8\ B}{\mu}$ denotes the ampère-turns for each centimetre of length, and as its value corresponding to any flux density B can be obtained from

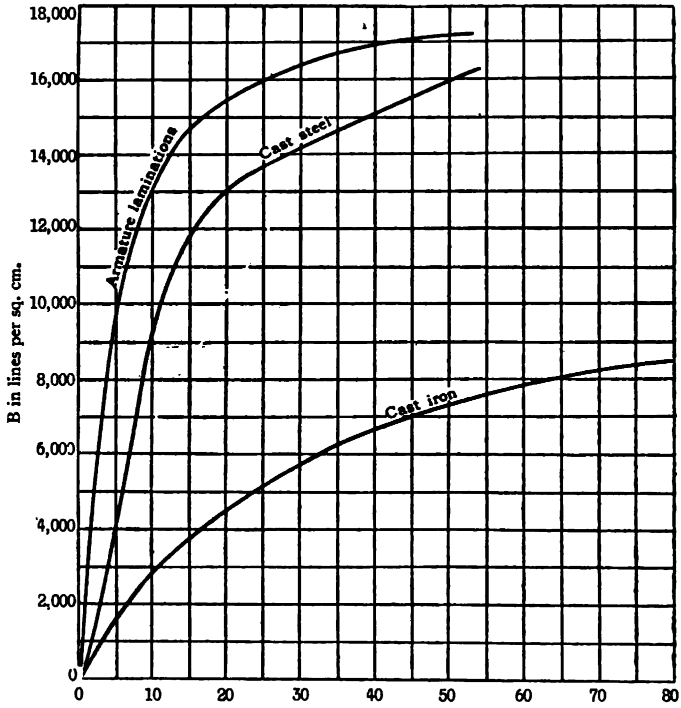


FIG. 176.—Magnetisation curves.

the curves of Figure 176, the equation for ampère-turns may be written thus,

$$AT = at_1 . L_1 + at_2 . L_2 + at_3 . L_3 + \ldots.$$

where at_1, at_2, at_3, etc. denote the ampère-turns per centimetre of length for the various parts of the magnetic circuit. From the above equations it will be seen that in order to calculate the ampère-turns of field excitation corresponding to various values of armature flux it is essential to know (1) the dimensions of the magnetic circuit, and (2) magnetisation or B/H curves of the iron. The latter have, in Figure 176, been

plotted for armature laminations, cast steel, and cast iron: a separate curve for the high densities occurring in armature teeth is given on page 235.

Magnetic Leakage and Leakage Coefficient

The total flux generated in each magnet core of an alternator does not pass through the armature core, but a certain percentage leaks across from one pole to another by way of the intervening air-paths as shown in Figure 177. The ratio of the whole flux generated in each unit of the magnetic circuit, to the useful flux which crosses the air-gap and enters the armature, is termed the *coefficient of magnetic leakage*, and is denoted by

$$\sigma = \frac{\text{Number of lines generated in field poles}}{\text{Number of lines in armature}} = \frac{\Phi_m}{\Phi_a}$$

σ is sometimes referred to as the *leakage factor* or *dispersion coefficient.*

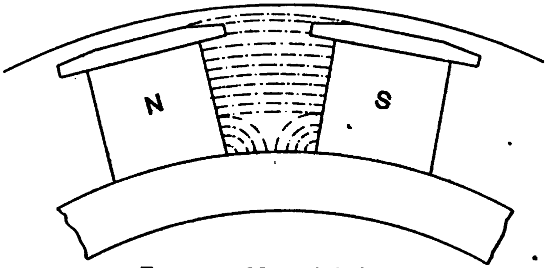


FIG. 177.—Magnetic leakage.

Since the total flux generated in each magnet core = useful flux (Φ_a) + leakage flux (Φ_l), the leakage factor can also be expressed by

$$\sigma = \frac{\text{Useful flux} + \text{leakage flux}}{\text{Useful flux}} = \frac{\Phi_a + \Phi_l}{\Phi_a} = 1 + \frac{\Phi_l}{\Phi_a}$$

For a given magnetomotive force the stray flux is proportional to the permeance of the paths through which leakage can take place; whereas the useful flux is approximately proportional to the permeance of the air-gap and teeth which are effective in carrying the main flux. Hence

$$\sigma = 1 + \frac{\text{Permeance of leakage paths}}{\text{Permeance of teeth and air-gap}} = 1 + \frac{P_l}{P_a}$$

The value of the ratio $\frac{P_l}{P_a}$ will depend upon the shape of the pole cores and the pole shoes, the length of the interpole arc, radial depth of the air-gap, the flux density in the pole core, and the extent of the armature reaction.

The flux through any cross-section of the magnetic circuit is a

maximum in the magnet ring, and gradually diminishes as the air-gap is approached, until at the armature surface it is reduced to the useful flux which becomes linked with the winding. In practical calculations it is convenient to assume that the useful flux only passes through the air-gap and that the leakage lines leave the main magnetic circuit at the pole face, thus allowing the total flux generated to pass through the magnet core. This assumption considerably simplifies calculations, and the results obtained are of sufficient accuracy for ordinary work.

Calculation of Ampère-turns.—In multipolar machines each pair of poles along with the corresponding portion of the magnet ring, air-gap, and armature will be treated as a unit magnetic circuit (see Figure 186A).

For convenience in determining the ampère-turns of excitation required per pair of poles the following parts of the magnetic circuit will be considered separately: (1) armature core, (2) armature teeth, (3) air-gap, (4) pole core and pole shoes, and (5) the magnet yoke, the symbols used for the various parts being as follows:—

Part.	Length in Cms. per Pair of Poles.	Cross-sectional Area in Cms.²	Flux Density in Lines per Cm.²	Ampère-turns per Pair of Poles.
Armature core .	L_c	A_c	B_c	AT_c
Armature teeth .	L_t	A_t	B_t	AT_t
Air-gap . .	2δ	A_g	B_g	AT_g
Magnet poles .	L_p	A_p	B_p	AT_p
Magnet yoke .	L_y	A_y	B_y	AT_y

Armature Core.—After crossing the air-gap and entering the armature by way of the teeth and the slots, the flux from any N-pole passes along two diverging paths towards adjacent S-poles. The flux crossing any radial cross-section of the core = 0.5 Φ_a, so that the induction density in the core is

$$B_c = \frac{\Phi_a}{2\ A_c} = \frac{\Phi_a}{2\ L_n\ .\ d}$$

where L_n = net length of armature iron parallel to shaft
d = radial depth of iron below teeth.

Since the armature laminations are separated from each other by paper or varnish, and radial air-ducts are formed in the core to facilitate ventilation, the nett length of the core L_n will be less than the gross length L_g by an amount varying from 20 to 35 per cent., depending upon the number of ventilating ducts. The thickness of insulation between adjacent plates is generally 10 per cent. of the thickness of the core plates; the nett length of core is thus

$$L_n = 0.9\ (L_g - \text{length taken up by ventilating ducts})$$

The above expression for B_c is based upon the assumption that the flux is distributed uniformly over a cross-section of the core. Now, for reasons to be afterwards discussed, such an assumption is not in accordance with actual facts, but the error so introduced is less than the possible error due to variation in the permeability of the iron. Having determined the value of B_c, the corresponding value of the ampère-turns at_c per cm. of length can be obtained from the curves of Figure 176. The ampère-turns per pair of poles to drive the flux Φ_a through the armature core are therefore

$$AT_c = at_c \cdot L_c$$

Armature Teeth.—If it be assumed that all the useful flux enters the armature core entirely through the teeth, then the flux density in the latter, at any particular section and as determined on this basis, is known as the *fictitious* or *apparent density*. At the high values of the latter—16,000 and upwards—occurring in modern machines, the permeability of the iron will be comparatively low, with the result that the total flux in the core does not enter by way of the teeth only; but a certain proportion enters through the slots, the ventilating ducts, and the insulation between the plates, the latter offering parallel paths of somewhat higher reluctance. The way in which the flux enters the armature through the teeth and slots is clearly shown by the stream line diagrams of Figure 178, originally published by Hele-Shaw, Hay and Powell,* in a paper on the magnetic flux distribution in toothed armature cores. The distribution of the flux was investigated by the aid of an

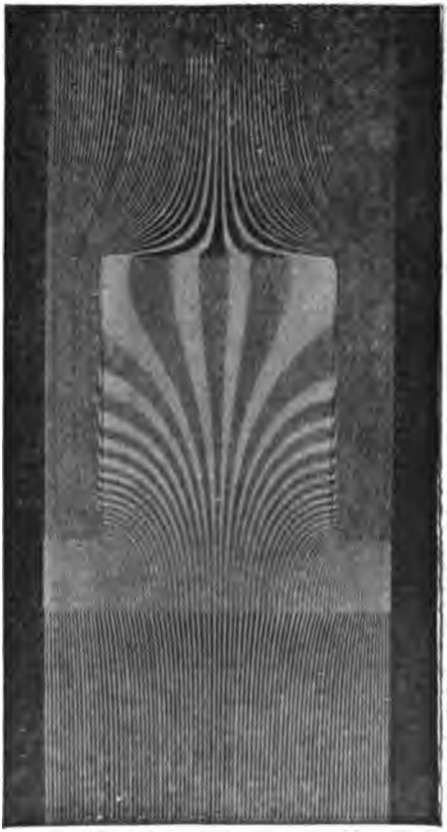


FIG. 178A.

* *Journ. of the Inst. of Elect. Engineers* (1904), vol. xxxiv. p. 21.

hydraulic model in which the stream lines in glycerine forced between two sheets of glass were made to imitate the analogous passage of lines of force through media of different permeability.

In standard alternators with stationary external armatures the flux density in the teeth at the section of minimum thickness seldom exceeds 18,000 or 19,000 lines per cm.[2]

For inductions not exceeding 18,000 the percentage of the flux which enters the armature through the slots will be small, and the excitation required for the teeth can therefore be calculated with sufficient accuracy by assuming that the whole of the flux enters the armature core through the teeth only. If t_p denote the pitch of a tooth at periphery, and b_i (page 239) the ideal pole breadth, then the flux passing down each tooth $= \Phi_a \cdot \frac{t_p}{b_i}$, and the corresponding flux density at any section, of width t, is

$$B_t = \frac{\Phi_a \cdot t_p}{L_n \cdot t \cdot b_i}$$

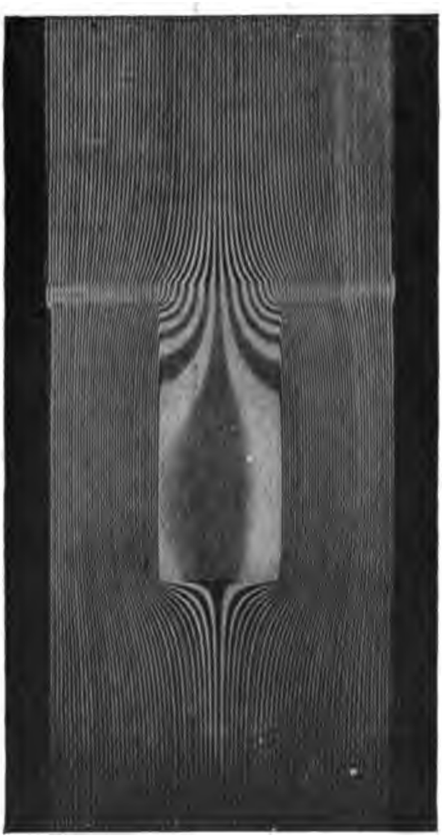


FIG. 178B.

The induction densities B_{max}, B_{mid}, and B_{min} at the top, middle, and root of tooth respectively, are determined from the above equation and the corresponding values of the ampère-turns per cm. derived from Figure 176. If the values of at_{max}, at_{mid}, and at_{min} be plotted as a function of the length of the tooth, a curve such as Figure 179 is obtained. The area of the curve represents the ampère-turns per tooth and is expressed by

$$0.8\int_{h=0}^{h=\frac{L_t}{2}} H \cdot dh = \int_{h=0}^{h=\frac{L_t}{2}} at_t \, dh$$

where dh = small element of radial length of tooth.

The area can, with sufficient accuracy, be determined by Simpson's rule, since the curve resembles a parabola; hence there is obtained the following equation to the ampère-turns for the teeth

$$AT_t = L_t\left(\frac{at_{max} + 4\,at_{mid} + at_{min}}{6}\right)$$

In highly saturated teeth, where the flux density exceeds 18,000 lines per cm.², the lines of force along the edges of the slot curve round

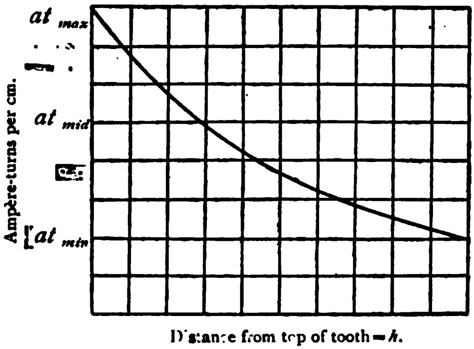


FIG. 179.

and enter the iron through the sides of the teeth as depicted in Figure 178. The iron of the teeth and the non-magnetic material within the slot may be looked upon as two magnetic paths of the same radial depth, in parallel. For any small section of depth h the total flux corresponding to one tooth pitch will divide between the parallel paths of iron, air, etc. in proportion to their respective permeances. When, as is invariably the case, the slots have parallel sides, the iron section of the tooth increases the farther it is from the air-gap periphery. The percentage of flux through the slot therefore diminishes as the tooth root is approached, and at a section about midway down the tooth the magnetic flux will be almost entirely confined to the iron. The ampère-turns for strongly saturated teeth may be derived by the following method, due originally to Parshall and Hobart.*

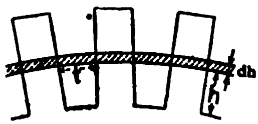


FIG. 180.

Consider a cylindrical section through the teeth and slots as shown by shaded band in Figure 180. Then through this cylindrical surface of width dh and distant h cms. from top of teeth there passes the total armature flux, which divides itself between the teeth and the slots; thus

Total flux = flux through teeth + flux through slot.

* *Engineering*, vol. lxvi. p. 130.

Now, the apparent density in the teeth can be expressed by

$$B_{app} = \frac{\text{Total flux per tooth}}{\text{Iron section per tooth}} = \frac{\text{Iron flux} + \text{air flux}}{\text{Iron section}}$$

$$= \frac{\text{Iron flux}}{\text{Iron section}} + \frac{\text{Air flux}}{\text{Air section}} \times \frac{\text{Air section}}{\text{Iron section}}$$

$$= B_{real} + H_{real} \times \frac{\text{Air section}}{\text{Iron section}}$$

Where, for the section dh,

B_{real} = Actual or real density in the teeth at this section,

and H_{real} = Actual density in air at this section.

Now, if t denote the width of tooth at depth h from circumference, s the

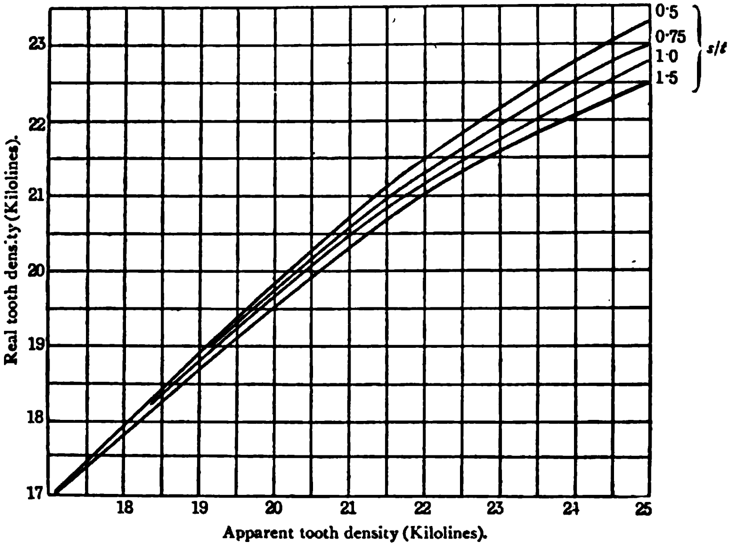


FIG. 181A.

corresponding width of slot, and η the ratio of nett length of iron to gross length of core,—*i.e.* $\eta = \frac{L_n}{L_g}$, then,

Iron section per tooth $= t \,.\, L_n = t \,.\, \eta \,.\, L_g$

and the air section per tooth $= \underbrace{s \,.\, L_g}_{\text{slots}} + \underbrace{t(1-\eta)L_g}_{\text{ducts and insulation}}$

The apparent induction in the teeth can therefore be written

$$B_{app} = B_{real} + H_{real} \frac{L_g\{s + t(1-\eta)\}}{t\eta L_g}$$

$$= B_{real} + 0.4\,\pi \,.\, at_t \,.\, \left\{\frac{\frac{s}{t} + (1-\eta)}{\eta}\right\}$$

The value of at_t corresponding to any given induction B_{real} can be obtained from the magnetisation curve for armature iron (Fig. 181B); and hence, for any given values of η and s/t there can be calculated the corresponding value of B_{app}. In Figure 181A the true iron induction has, for four values of the ratio s/t and a value of $\eta = 0.75$, been plotted as a function of the apparent induction, the latter being expressed by the equation

$$B_{app} = \frac{\Phi_a t_p}{L_n \cdot t \cdot b_i}$$

In order to obtain the ampère-turns for the teeth, the apparent

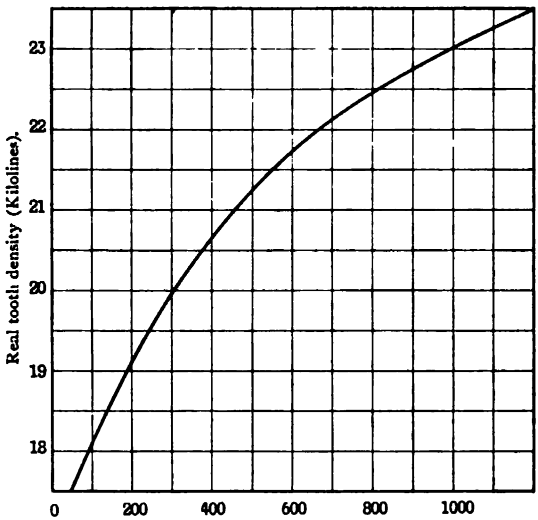


FIG. 181B.

induction at the top, middle, and roots of teeth are calculated from the equations

$$B_{max} = \frac{\Phi_a \cdot t_p}{L_n \cdot t_1 \cdot b_i}; \quad B_{mid} = \frac{\Phi_a \cdot t_p}{L_n \cdot t_2 \cdot b_i}; \quad B_{min} = \frac{\Phi_a \cdot t_p}{L_n \cdot t_3 \cdot b_i}$$

From Figure 181A there are found the corresponding values of the real induction, and hence the ampère-turns at_{max}, at_{mid}, and at_{min}. The total ampère-turns for the teeth are then, according to Simpson's rule, expressed by

$$AT_t = L_t\left(\frac{at_{max} + 4at_{mid} + at_{min}}{6}\right)$$

Air-gap.—Consider, in the first instance, a smooth-core armature —*i.e.* one with totally enclosed slots; then the lines of force, in passing from pole face to armature core, tend to spread out as much as possible, so as to make the reluctance of the air-gap a minimum. If

the induction in the air-gap be plotted as a function of (1) armature periphery and (2) the axial length of armature core, then the curves shown in Figures 182 and 183 are obtained. Since the pole shoes are invariably bevelled the former curve will approximate to a sine wave, whereas the latter will be almost flat, with several hollows corresponding to the number of air-ducts in the core. Suppose that the air-gap area between the pole and armature surfaces be divided into a number of tubes (Fig. 182), then, since the same M.M.F. is impressed upon each tube, the number of lines through any tube will depend upon its permeance. In order to calculate the air-gap flux it is therefore necessary to obtain the sum of the permeance for all flux tubes between two neutral zones. A simpler method of calculation is to reduce all flux tubes to the same length δ, *i.e.* multiply the cross-section of each tube by $\frac{\delta}{\delta_x}$, where δ_x is the real length of tube.

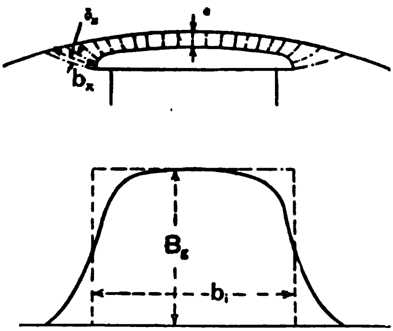


FIG. 182.

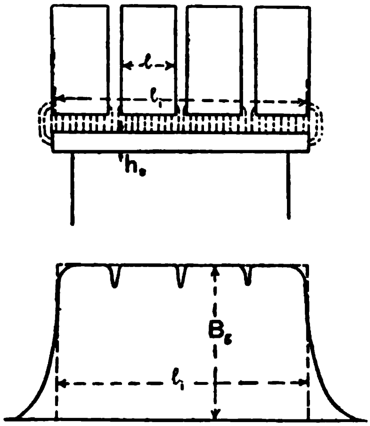


FIG. 183.

Since, in all equivalent tubes, the induction B_g is constant and equal to that under the centre of the pole shoe, the original curve of flux distribution can be replaced by a rectangle of the same area and height $= B_g$. The length b_i of this rectangle will be referred to as the ideal pole breadth. In a similar manner the irregular curve of Figure 183 can be replaced by an equivalent rectangle of height B_g and length l_i, the latter being termed the ideal length. The flux per pole can now be expressed in terms of b_i and l_i, thus

$$B_g \,.\, b_i \,.\, l_i = \Phi_a$$

$$i.e.\ B_g = \frac{\Phi_a}{b_i \,.\, l_i}$$

Hence, with a *smooth-core armature* the ampère-turns for the air-gap are per pair of poles

$$AT_g = 0.8 \,.\, 2\,\delta B_g = 1.6\,\delta B_g$$

In armatures with semi-closed or open slots the presence of the

teeth disturbs the uniformity of the flux distribution, causing a crowding of the lines of force into the regions of the polar surface which are opposite the teeth, and thereby increasing the fall of magnetic potential over the air-gap above the value which it would have if the core were smooth and the flux distribution uniform. The value of AT_g as given by the above equation will therefore be too small for an open-slot armature, the number of ampère-turns necessary to send the flux through the gap being determined by the maximum flux density under the teeth. The average flux density is, however, smaller than the maximum value, and the ratio of the maximum density to the mean density is the ratio of the number of ampère-turns necessary to send the flux through the air-gap with a slotted armature, to the number of ampère-turns necessary to send the same flux through the same air-gap with a smooth armature.

Denoting this ratio or correction coefficient by k_3, the equation to the air-gap ampère-turns for an open slot armature is

$$AT_g = 1.6\,k_3\,.\,\delta\,.\,B_g = 1.6\,k_3\,.\,\delta\,.\,\frac{\Phi_a}{b_t\,.\,l_t} \quad . \quad . \quad . \quad (56)$$

Calculation of k_3.—The coefficient k_3 also represents the ratio

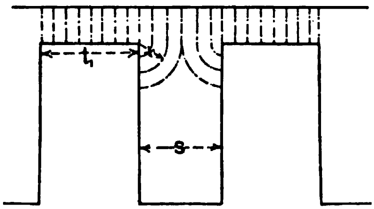


FIG. 184.

of the permeability of the air-gap of a smooth-core armature to that of a slotted armature, and may be determined mathematically as follows. Referring to Figure 184, it will be assumed that the lines of force passing from pole face to crowns of the teeth are straight, and that those lines which fall outside the space immediately above a tooth consist partly of quadrants of circles whose common centre is at the edge of the tooth. For one tooth pitch and 1 cm. length of tooth measured in a direction parallel to the shaft the permeance of air-gap is

$$\frac{t_1}{\delta} + 2\int_{x=0}^{x=\frac{s}{2}} \frac{dx}{\delta + \frac{\pi . x}{2}} = \frac{t_1}{\delta} + \frac{4}{\pi}\log_e\left(\frac{\pi}{4}\,.\,\frac{s}{\delta} + 1\right)$$

Since the corresponding permeance for a smooth-core armature is

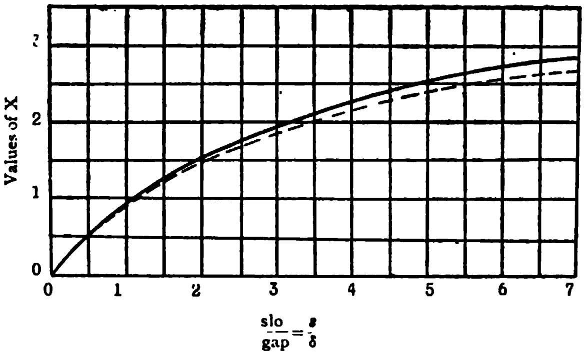
$\frac{s+t_1}{\delta}$, it follows that the correction coefficient is expressed by

$$k_3 = \frac{s+t_1}{\delta\left\{\frac{t_1}{\delta}+\frac{4}{\pi}\log_e\left(\frac{\pi}{4}\cdot\frac{s}{\delta}+1\right)\right\}}$$

$$= \frac{s+t_1}{t_1+2.9\,\delta\log\left(\frac{\pi}{4}\cdot\frac{s}{\delta}+1\right)} = \frac{s+t_1}{t_1+X\delta}$$

where the factor $X = 2.9\log\left(\frac{\pi}{4}\cdot\frac{s}{\delta}+1\right)$

An inspection of the stream line diagrams of Figure 178 will show that the lines of force which enter a tooth by way of adjacent slots extend farther down on the tooth than has been assumed in deriving



Full line curve gives values obtained experimentally. Dotted curve gives values from theoretical formula.

FIG. 185.—Air-gap correction coefficient $k_3 = \frac{s+t_1}{t_1+X\delta}$

the mathematical equation. The fringing permeance will therefore be slightly greater than that given by the above integration, with the result that the value of k_3 will be somewhat too great. Again, the permeance of the teeth has been taken as infinite in comparison with the air-gap and slots, but the effect of tooth saturation will be to squeeze more lines out into the slot and so further reduce the amount of correction necessary. The value of this factor X, as given by $X = 2.9\log\left(\frac{\pi}{4}\cdot\frac{s}{\delta}+1\right)$, will therefore be too small, so that its actual value must be determined experimentally. In this connection the stream line diagrams of Figure 178 are of special interest, as they offer a simple method of determining the numerical values of the factor X for various cases. The correction coefficient k_3 is first derived by measuring the number of stream lines per unit of length in the space well under cover

of a tooth, and dividing this by the mean number of stream lines per unit of length for tooth and slot. All the factors, except X, in the expression $k_3 = \frac{s + t_1}{t_1 + X\,\delta}$ are thus known, and hence the value of X can be determined.

The reluctance of the air-gap in dynamo machines with slotted armatures has recently been investigated experimentally by Thomas F. Wall,* the field in the air-gap between the pole face and the armature core being explored by a narrow search coil. The values of X as deduced from the experiments of Wall have been plotted as the full line curve of Figure 185, the abscissæ giving the values s/δ. In order to effect a comparison there is also plotted to the same reference axis the corresponding values of X as obtained from the above theoretical formula.

Calculation of b_r.—Since the pole shoes of an alternator are usually bevelled, the radial depth of air-gap will not be uniform under the pole face. Hence, in order to arrive at the value of b_r, the flux lines between pole face and armature core must be drawn out as shown in Figure 182, it being assumed that the surface of the core is smooth. Let b_x denote the mean breadth and δ_x the mean length of any flux tube, then the permeance of a tube of width $l_i = \frac{b_x \,.\, l_i}{\delta_x}$. The M.M.F. acting on the tubes under the pole face where the gap is uniform is less than that acting upon the tubes at the tips of the poles where the flux density in the teeth and armature core, and therefore the reluctance, is low. The M.M.F. acting upon the tubes at the fringe will therefore be approximately $= \frac{1}{2}\, 0.4\pi.(AT_g + AT_t + AT_a)$, whilst that acting upon the tubes under the pole face will be $= \frac{1}{2}\, 0.4\,\pi \,.\, AT_g = k_3 \,.\, \delta \,.\, B_g$, where $1.6\, k_3 \,.\, \delta \,.\, B_g$ has been substituted for AT_g.

Let b = length of polar arc under which the air-gap is uniform. The flux under this arc will be $= B_g l_i \,.\, b$.

The M.M.F. acting on the tubes at the fringe will be

$$= \tfrac{1}{2}\, 0.4\,\pi \,.\, AT_g\left(\frac{AT_g + AT_t + AT_a}{AT_g}\right)$$

$$= K \,.\, k_3 \,.\, \delta \,.\, B_g, \quad \text{where } K = 1 + \frac{AT_t + AT_a}{AT_g}$$

The flux per tube = M.M.F. × permeance

$$= K k_3 \delta B_g \,.\, \frac{l_i b_x}{\delta_x}$$

The total flux at both fringes of pole

$$= 2\, K k_3 \delta \,.\, B_g l_i \,.\, \Sigma \frac{b_x}{\delta_x}$$

* *Journ. of the Inst. of Elect. Engineers* (1908), vol. xl. p. 550.

Total air-gap of flux

$$= B_g l_i b + 2\,\delta K k_3 . B_g l_i . \Sigma\frac{b_x}{\delta_x}$$

$$= B_g l_i\left(b + 2\delta K k_3 . \Sigma\frac{b_x}{\delta_x}\right)$$

$$= B_g l_i b_i$$

Hence,
$$b_i = b + 2\,\delta K k_3 . \Sigma\frac{b_x}{\delta_x}$$

With a ratio of pole arc to pole pitch $= 0.65$, and a pole shoe of the proportions shown in Figure 138, a very close approximation to a sine wave of flux distribution is obtained.

For such a design the ideal pole breadth, corresponding to a uniform air-gap of length δ, is approximately equal to the breadth of the pole shoe b; *i.e.*

$$b_i \cong b$$

In turbo-alternators with cylindrical rotors the ideal pole breadth will approximate to $b_i \cong 0.65\,\tau$.

Calculation of l_i.—If l (Figure 183) denote the length of core between adjacent ducts, then the ideal length l_i is obtained by adding to the iron length $4\,l$, a certain allowance for spreading at the air-ducts and outer flanks of the pole shoes. The influence of the ducts is determined in a similar manner to that already explained in connection with the slot fringing. If the number of ducts be denoted by n_d, then the width of each duct is

$$w = \frac{L_g - (n_d + 1)l}{n_d}$$

and if ln is the length which must be added to allow for spreading at the flanks of the pole shoes the total permeance is

$$\frac{b_i\,l_i}{\delta} = \frac{b_i}{\delta}\{(n_d + 1)l + n_d X'\delta + l_x\}$$

the value of X′ being obtained from the full line curve of Figure 185. The spreading of the lines of force at the air-ducts is therefore equivalent to increasing the length of the core from $(n_d + 1)l$ to $(n_d + 1)l + n_d X'\delta$.

If the height of the pole shoe be denoted by h_s, then the permeance of the path taken by the flux spreading out from the flanks is

$$2b_i\int_{x=o}^{x=h_s}\frac{dx}{\delta + \pi x} = \frac{4.6}{\pi}\cdot b_i \log\left(\frac{\pi h_s + \delta}{\delta}\right) = \frac{b_i\,.\,l_x}{\delta}$$

$$\text{Hence, } l_x = 1.46\,\delta \log\left(\frac{\pi h_s + \delta}{\delta}\right)$$

$$\text{and } l_i = (n_d + 1)l + n_d X'\delta + 1.46\,\delta \log\left(\frac{\pi h_s + \delta}{\delta}\right) \quad . \quad . \quad . \quad . \quad (57)$$

Owing to the eddy currents induced in the end-flanges damping out the fluxes at the flanks, the ideal armature length as thus calculated will be somewhat too large. The effect of the damping will, however, be neglected. In order to eliminate this eddy current loss as much as possible, the length of the armature core is in some designs made

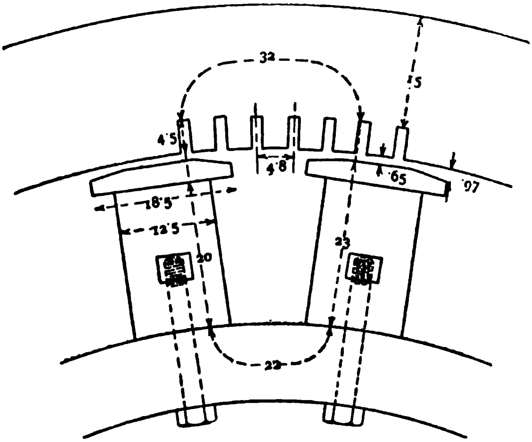


FIG. 186A.—Magnetic circuit of 600-K.V.A. alternator.

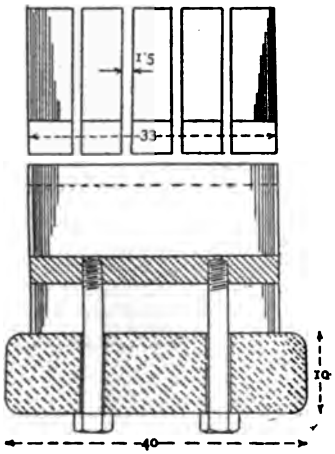


FIG. 186B

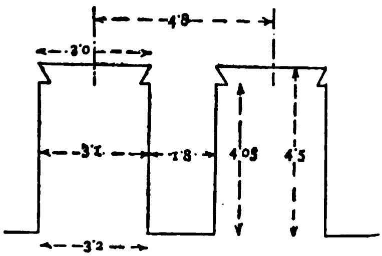


FIG. 186C.

somewhat greater than the width of the poles. If a denote the distance by which the armature core extends on each side beyond the flanks of the pole shoes, then

$$l_i = (n_d + 1)\, l + n_d X'\delta + 2.95 \log \left(\frac{\frac{\pi a}{2} + \delta}{\delta} \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

Pole Core and Yoke.—Let σ denote the leakage factor of the machine, then the flux densities in the pole core and yoke are

$$B_p = \frac{\Phi_a \sigma}{A_p}, \text{ and } B_y = \frac{\Phi_a \sigma}{A_y}$$

Now, if at_p and at_y denote the corresponding values of the ampère-turns per cm. of length as obtained from Figure 176, then

$$AT_p = at_p \,.\, L_p$$
$$\text{and } AT_y = at_y \,.\, L_y$$

The length L_p for the pole core also includes the length of the shoe. In most cases the pole shoe density will be much lower than that of the pole core, but the error involved in assuming both parts to be worked at the same density will be exceedingly small.

Example—In Figure 186 is given a dimensioned drawing of the magnetic circuit of a 600-K.V.A. 50 ~, 3-phase alternator having 24 poles. If the flux per pole at full-load and 0.8 power factor be 4.9×10^6 lines, calculate the ampère-turns to drive the flux through the magnetic circuit, assuming a leakage factor $\sigma = 1.25$.

Armature Core—

Nett length of iron in core $= L_n = \{33 - (4 \times 1.5)\}0.9 = 24$ cms.

Flux density $= B_c = \dfrac{\Phi_a}{2\, L_n d} = \dfrac{4.9 \times 10^6}{2 \times 24 \times 15} = 7000.$

Mean length of path $= L_c = 32$.

From Figure 176 ampère-turns per cm. of length $= 2.5$.

Ampère-turns for core $= AT_c = 32 \times 2.5 = 80$.

Armature teeth—

Breadth across pole tips $= b = 18.5$ cms.

Ideal pole breadth $= b_i = b = 18.5$ cms. (*vide* page 240).

The apparent density, real density, and ampère-turns per cm. for top, middle, and root of teeth are obtained thus:—

$B_{app} = \dfrac{\Phi_a \,.\, t_p}{L_n \,.\, t \,.\, b_i}$	B_{real}.	Ampère-turns per Cm. from Figure 176.
$B_{max} = \dfrac{4.9 \times 10^6 \times 4.8}{24 \times 3.0 \times 18.5} = 17{,}600$ . . .	17,600	80
$B_{mid} = \dfrac{4.9 \times 10^6 \times 4.8}{24 \times 3.1 \times 18.5} = 17{,}000$ . . .	17,000	50
$B_{min} = \dfrac{4.9 \times 10^6 \times 4.8}{24 \times 3.2 \times 18.5} = 16{,}500$ . . .	16,000	30

Length of teeth per pair of poles $= L_t = 2 \times 4.5 = 9$ cms.

Hence ampère-turns for teeth

$$= AT_t = \frac{9(80 + 200 + 30)}{6} = 470$$

Air-gap—

$$\frac{\text{Width of duct}}{\text{Depth of air gap}} = \frac{w}{\delta} = \frac{1.5}{0.65} = 2.3\text{; hence } X' = 1.6. \quad \text{(Figure 185)}$$

$$\text{Ideal pole length} = l_i = (n_d + 1)l + n_d X'\delta + 1.46\ \delta \log\left(\frac{\pi h_s + \delta}{\delta}\right)$$

$$= 5 \times 5.4 + 4 \times 1.6 \times .65 + 1.46 \times 0.65 \times \log\left(\frac{\pi \cdot 3 + 0.65}{0.65}\right)$$

$$= 32 \text{ cms.}$$

$$\text{Flux density} = B_g = \frac{\Phi_a}{b_t \cdot l_i} = \frac{4.9 \times 10^6}{18.5 \times 32} = 8500.$$

$$\frac{\text{Width of slot}}{\text{Depth of air-gap}} = \frac{s}{\delta} = \frac{1.8}{0.65} = 2.8.$$

From Figure 185, $X = 1.8$.

$$\text{Air-gap correction coefficient} = k_3 = \frac{s + t_1}{t_1 + X\delta} = \frac{1.8 + 3.0}{3.0 + 1.8 \times 0.65} = 1.14.$$

Radial depth of air-gap $= \delta = 0.65$ cm.

Ampère-turns for air-gap

$$= AT_g = 1.6\, k_3 \,.\, \delta \,.\, B_g = 1.6 \times 1.14 \times 0.65 \times 8500 = 10{,}000.$$

Pole Core (cast steel)—

Allowing 5 per cent. for insulation between stampings, the sectional area of pole core $= A_p = 12.5 \times 33 \times 0.95 = 400$ cms.^{2}

Length of poles (including shoes) $= L_p = 2 \times 23 = 46$ cms.

$$\text{Flux density in core} = B_p = \frac{\Phi_a \sigma}{A_p} = \frac{4.9 \times 10^6 \times 1.25}{400}$$

$$= 15{,}000 \text{ lines per cm.}^2$$

From Figure 176, ampère-turns per cm. $= 16$.

Ampère-turns for pole core and shoe $= 16 \times 46 = 750$.

Magnet Yoke (cast steel)—

Cross-sectional area $= A_y = 40 \times 10 = 400$ cms.^{2}

Length of magnetic path $= L_y = 22$ cms.

$$\text{Flux density in core} = B_y = \frac{4.9 \times 10^6 \times 1.25}{2 \times 400} = 7600.$$

From Figure 176, ampère-turns per cm. $= 7$.

Ampère-turns for magnet yoke $= 7 \times 22 = 150$.

Total Ampère-turns per Pair of Poles—

$$= 80 + 470 + 10{,}000 + 750 + 150 = 11{,}450 = 11.500.$$

Ampère-turns per Pole $= 5750$.

Calculation of the Leakage Coefficient σ

On page 229 it was shown that the leakage coefficient of an alternator can be expressed by

$$\sigma = \frac{\Phi_a + \Phi_l}{\Phi_a} = 1 + \frac{\Phi_l}{\Phi_a}$$

where Φ_a and Φ_l denote the useful flux and leakage flux respectively. The value of the latter can be determined from the equation.

Number of lines of force = M.M.F. × permeance

$$= \frac{4\pi}{10} \text{ ampère-turns} \times \text{permeance.}$$

In calculating the permeance of the leakage paths, the tubes of force are assumed to follow certain directions in the air according to the situation, and distances apart, of the surfaces from which leakage takes place. Since the combined permeance is due to several paths in parallel, the most convenient method of dealing with the problem is to consider the leakage of each path separately and obtain the total leakage by adding together the individual leakages. Now,

$$\text{Permeance} = \frac{1}{\text{Reluctance}} = \frac{1}{\frac{l}{a} \cdot \frac{1}{\mu}} = \frac{a\mu}{l}$$

where,

a = area of cross-section of path in cms.[2],
l = length of path in cms.,
μ = permeability;

and since the leakage flux is entirely through air,

$$\text{Permeance} = \frac{\text{area of cross-section}}{\text{length}}$$

Case 1.—The method of determining the leakage factor σ will first be investigated for a revolving field alternator in which the poles are inclined to one another at a very small angle. (See Figure 187.) The poles are of rectangular cross-section, and the symbols representing the various dimensions are as indicated in the figure. For the case under consideration the leakage flux *for one interpolar gap* may be treated as made up of four parts, as follows:—

(1) The leakage Φ_{s1} between inner planes of pole shoes.
(2) The leakage Φ_{s2} between flanks of pole shoes.
(3) The leakage Φ_{c1} between inside planes of pole cores.
(4) The leakage Φ_{c2} between flanks of pole cores.

Since AT_g, AT_t, and AT_a denote the ampère-turns for the air-gap, teeth, and armature core per pair of poles, the magnetomotive force between adjacent pole shoes is

$$0.4\,\pi\,(AT_g + AT_t + AT_a) = 0.4\,\pi\,AT_l = 1.25\,AT_l$$

If it be assumed that the lines of force in the interpolar gap are straight, then

$$\Phi_{s1} = 1.25\ \mathrm{AT}_l \cdot \frac{l_s \cdot h_s}{b_1}$$

In the case of the leakage from the flanks of the pole shoe, it will be convenient to assume that the length of each line of force is made up of two quadrants of radius x and a straight portion of length b_1. The leakage flux between the flanks of adjacent pole shoes is therefore

$$\Phi_{s2} = 2\left\{0.4\,\pi \cdot \mathrm{AT}_l \cdot \int_{x=0}^{x=\frac{b_s}{2}} \frac{h_s \cdot dx}{\pi x + b_1}\right\}$$

$$= 2\left\{0.4\,\pi \mathrm{AT}_l \frac{h_s}{\pi} \cdot \log_e \frac{\frac{\pi b_s}{2} + b_1}{b_1}\right\}$$

$$= 1.82 \cdot \mathrm{AT}_l \cdot h_s \cdot \log\left(1 + \frac{\pi b_s}{2b_1}\right)$$

The factor 2 is introduced to allow for leakage from both flanks.

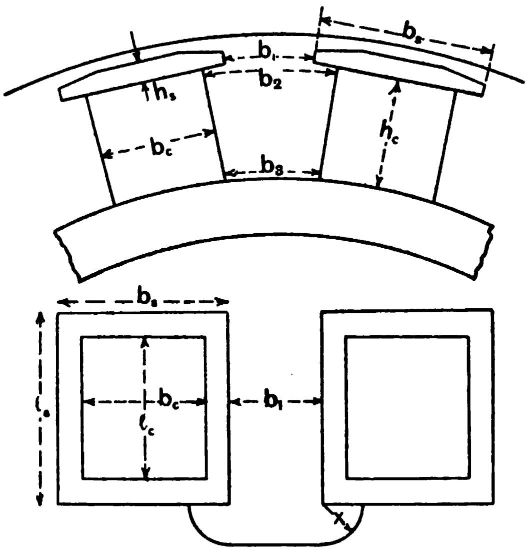


FIG. 187.

In calculating the number of leakage lines between the pole core surfaces, it will be assumed that the magnetic potential of the magnet ring is zero, and that the M.M.F. along the pole increases from a value = 0 at the magnet ring to a value = 0.4 $\pi \mathrm{AT}_l$. The leakage fluxes from the pole core surfaces can be expressed thus—

$$\Phi_{c1} = 0.2\ \pi \mathrm{AT}_l \, . \, \frac{l_c \, . \, h_c}{\frac{b_2 + b_3}{2}} = 1.25\ \mathrm{AT}_l \, . \, \frac{l_c \, . \, h_c}{b_2 + b_3}$$

$$\text{and } \Phi_{c2} = 2 \left\{ 0.2\ \pi\ \mathrm{AT}_l \, . \, \frac{h_c}{\pi} \, . \, \log_e \left(1 + \frac{\pi b_c}{b_2 + b_3} \right) \right\}$$

$$= 0.91\ \mathrm{AT}_l \, . \, h_c \, . \, \log \left(1 + \frac{\pi b_c}{b_2 + b_3} \right) \Big\}$$

Since the above equations for Φ_{s1}, Φ_{s2}, Φ_{c1}, and Φ_{c2} give the leakage flux for one interpolar gap only, the total leakage flux Φ_l, corresponding to a flux Φ_a per pole through the armature, will be

$$\Phi_l = 2\ (\Phi_{s1} + \Phi_{s2} + \Phi_{c1} + \Phi_{c2})$$

The leakage factor σ is therefore given by the equations

$$\sigma = 1 + \frac{\Phi_l}{\Phi_a} = 1 + \frac{\mathrm{AT}_l \, . \, 2\Delta}{\Phi_a} = 1 + \frac{(\mathrm{AT}_g + \mathrm{AT}_t + \mathrm{AT}_a)\ 2\Delta}{\Phi_a} \quad . \ . \ . \ (59)$$

where

$$\Delta = h_s \left\{ 1.25 \frac{l_s}{b_1} + 1.82 \log \left(1 + \frac{\pi b_s}{2 b_1} \right) \right\}$$
$$+ h_c \left\{ 1.25 \frac{l_c}{b_2 + b_3} + 0.91 \log \left(1 + \frac{\pi b_c}{b_2 + b_3} \right) \right\} \quad . \ . \ . \ (60)$$

In machines with circular pole cores the formula for the exact calculation of the leakage fluxes Φ_{c1} and Φ_{c2} would be somewhat more involved than that for rectangular poles. The value of σ can, however, be determined with sufficient accuracy if it be assumed that the circular poles are replaced by square poles of the same cross-sectional area. If d_c denote the diameter of the round pole and l_c the length per side of the equivalent square pole, then $l_c = 0.89\ d_c$.

Case 2.—The above method of determining σ is only applicable to alternators having a large number of poles inclined to one another at a very small angle. In machines with few poles,—*e.g.* turbo-alternators—the angle of inclination between the poles will be large, so that the value of the leakage fluxes must be deduced by some other method. The best procedure, when such cases arise, is to draw out the probable paths taken by the tubes of force, and estimate from the drawing the permeance of the various paths.

In drawing out the tubes of force it is a considerable help to remember that those tubes for which the path $\frac{b_2}{2}$ is greater than $\frac{b_3}{2}$ pass directly into the yoke, as shown in Figure 188A.

This is also the case on the flanks of the poles, as here the flux not only leaks from pole to pole but also from pole to yoke.

In the field system outlined in Figure 188A the leakage flux for each interpolar space may be split up into six component parts as follows: (1) Leakage Φ_{s1} between inner planes of pole shoes; (2) leakage Φ_{s2} between flanks of pole shoes; (3) leakage Φ_{c1} between

planes of pole cores; (4) leakage Φ_{c2} between flanks of pole cores; (5) leakage Φ_{y1} between yoke and inner planes of pole cores; (6) leakage Φ_{y2} between yoke and flanks of pole cores. Adopting the notation shown in the figure, the equations for fluxes Φ_{s1}, Φ_{s2}, Φ_{c1}, and Φ_{c2} are as follows:—

$$\Phi_{s1} = 1.25\ \mathrm{AT}_l \,.\, \frac{l_s \,.\, a_1}{b_1}$$

$$\Phi_{s2} = 1.82\ \mathrm{AT}_l \,.\, h_s \,.\, \log\left(1 + \frac{\pi b_s}{2b_1}\right)$$

$$\Phi_{c1} = 1.25\ \mathrm{AT}_l \,.\, \frac{h_1}{h_c} \,.\, \frac{l_c \,.\, a_2}{b_2}$$

$$\Phi_{c2} = 1.82\ \mathrm{AT}_l \,.\, \frac{h_1}{h_c} \,.\, a_2 \,.\, \log\left(1 + \frac{\pi b_c}{2b_2}\right)$$

The number of leakage lines passing from the inside planes of

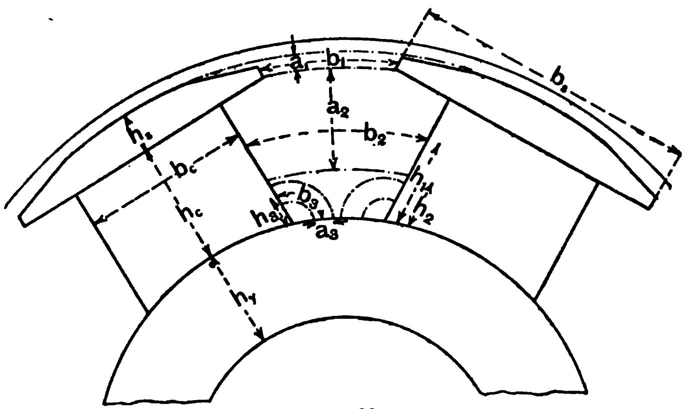


FIG. 188A.

the pole core into the yoke can be approximately determined by dividing the path taken by the flux into two or three tubes and adding together the permeances obtained from the mean cross-section and length of each tube. If the permeance of this part of the leakage path be denoted by $\Sigma\frac{l_c a_3}{b_3}$, then the corresponding value of the leakage flux will be

$$\Phi_{y1} = 1.25\ \mathrm{AT}_l \,.\, \Sigma\frac{l_c a_3}{b_3} \,.\, \frac{h_3}{h_c}$$

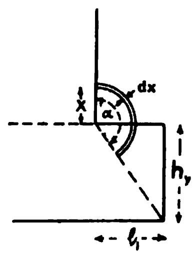


FIG. 188B.

In calculating the remaining portion of the leakage—*i.e.* Φ_{y2}, the area of the pole flank will be regarded as repeated on the end of the magnet ring, which latter will, of course, project some distance beyond the pole. Since the permeance of a small

strip of air of width dx (Fig. 188B), and stretching across one-half the pole face $= \frac{b_c}{2} \cdot \frac{dx}{\pi x} \cdot \frac{180}{a}$, and the M.M.F. acting on the strip $= 1.25\ AT' \frac{x}{h}$, the leakage flux Φ_{y2} can be expressed by the equation

$$\Phi_{y2} = 1.25\ AT_l \cdot \frac{b_c}{2\,\pi h_c} \cdot \frac{180}{a} \int_{x=0}^{x=h_2} dx$$

$$= 0.2\ AT_l \cdot \frac{b_c}{h_c} \cdot \frac{180}{a} \cdot h_2$$

where $a = 90° + \sin^{-1} \frac{l_1}{\sqrt{h_y^2 + l_1^2}}$

The equation to the leakage factor for the field system shown in Figure 188A can now be written as

$$\sigma = 1 + \frac{(AT_g + AT_t + AT_a)\ 2\Delta}{\Phi_a}$$

where

$$\Delta = 1.25 \frac{a_1 l_s}{b_1} + 1.82\ h_s \log\left(1 + \frac{\pi b_s}{2b_1}\right)$$
$$+ \frac{h_1}{h_c} \cdot a_2 \left\{1.25 \frac{l_c}{b_2} + 1.82 \log\left(1 + \frac{\pi b_c}{2b_2}\right)\right\}$$
$$+ 1.25\ \Sigma \frac{l_c \cdot a_3}{b_3} \cdot \frac{h_3}{h_c} + 0.2 \frac{b_c}{h_c} \cdot \frac{180}{a} \cdot h_2 \quad . \quad . \quad . \quad . \quad (61)$$

Value of σ.—Since the leakage permeance is due to the air spaces between adjacent poles, its value will be approximately constant. The magnitude of the leakage flux will therefore vary in direct proportion to AT_l. Now, in order to compensate for the voltage drop in the windings, the flux Φ_a through the core of the armature must be augmented as the load comes on the machine. An increased flux through the armature means an increase in the ampère-turns AT_h, and a corresponding increase in the leakage flux Φ_l. But the latter increases faster than Φ_a, for as the teeth become more and more saturated the same increase in the ampère-turns expended over the armature will produce less and less increase of Φ_a, whereas the leakage flux continues to increase almost directly as the ampère-turns AT_l. The value of the leakage factor σ will therefore be greater at full-load than at no-load.

For alternators with rectangular pole cores, having an axial length not exceeding twice the breadth, the full-load value of σ ranges from 1.15 in large machines to 1.25 in machines of about 100 K.V.A. If the pole height exceed 20 centimetres or thereabouts, and the axial length be greater than that mentioned above, then the value of σ might be as high as 1.3. Since the exact calculation of the leakage factor can only be made after the machine dimensions are finally fixed, an assumed value of σ must be taken, in order to determine a trial value of either the cross-section of the pole or magnetic flux per pole. On page 401 the full-load value of σ has been tabulated for five alternators of various outputs, and when getting out

a new design these values should be a help in obtaining a first approximation to the leakage factor.

Example.—Figure 189 gives a dimensioned drawing of a 600-K.V.A., 24-pole, 3-phase alternator. From the following data it is proposed to calculate the leakage factor σ at full-load.

Full-load flux $= \Phi_a = 4.9 \times 10^6$ lines.
Ampère-turns for air-gap $= AT_g = 10000$.
Ampère-turns for teeth $= AT_t = 470$.
Ampère-turns for armature core $= AT_c = 80$.

Since the angle of inclination between the poles is small, the

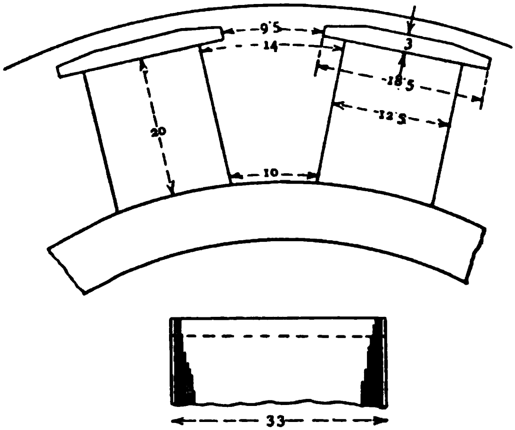


FIG. 189.

value of the leakage factor may be calculated as for Case 1. From the drawing there is obtained the following dimensions:—

$h_s = 3$ cms. $\quad h_c = 20$ cms.
$l_s = 33$ cms. $\quad l_c = 33$ cms.
$b_s = 18.5$ cms. $\quad b_c = 12.5$ cms.
$b_1 = 9.5$ cms. $\quad b_2 = 14$ cms. $\quad b_3 = 10$ cms.

$$\Delta = h_s\left\{1.25\frac{l_s}{b_1} + 1.82\log\left(1 + \frac{\pi b_s}{2b_1}\right)\right\}$$
$$+ h_c\left\{1.25\frac{l_c}{b_2 + b_3} + 0.91\log\left(1 + \frac{\pi b_c}{b_2 + b_3}\right)\right\}$$
$$= 3\left\{1.25 \times \frac{33}{9.5} + 1.82\log\left(1 + \frac{\pi \times 18.5}{2 \times 9.5}\right)\right\}$$
$$+ 20\left\{1.25 \times \frac{33}{14 + 10} + 0.91\log\left(1 + \frac{\pi \times 12.5}{14 + 10}\right)\right\} = 16.5 + 42 = 58.5$$

$$\text{Leakage factor } \sigma = 1 + \frac{(AT_g + AT_t + AT_c)2\Delta}{\Phi_a}$$
$$= 1 + \frac{(10000 + 470 + 80) \times 2 \times 58.5}{4.9 \times 10^6} = 1.25$$

CHAPTER VIII

ALTERNATORS:—THEORY AND CALCULATION OF ARMATURE REACTION AND REGULATION

WHEN an alternator supplies current to an inductive load the terminal pressure is, for the same speed and excitation, less than the E.M.F. on open circuit, the fall in terminal pressure being attributable to

(1) Volts lost in armature resistance, and the screening effect of eddy currents induced by the armature flux.

(2) Volts drop due to the inductance of the armature winding.

(3) Weakening of main field by the demagnetising action of a lagging armature current.

The drop in voltage resulting from the latter is, as will be afterwards explained, proportional to sin ϕ_1, where ϕ_1 is the phase angle between the current and the induced E.M.F. The excitation necessary to maintain the normal terminal pressure therefore varies for the same armature current, according to the power factor of the load. For a loaded alternator the curve showing the variation of terminal pressure with excitation is termed the load characteristic, or load saturation curve, corresponding to a given armature current and power factor. In Figure 175, page 227, the saturation curves are also plotted for full-load current at the following power factors: cos $\phi = 1$, cos $\phi = 0.80$, and cos $\phi = 0$. Each of the load curves crosses the abscissa reference axis at the same point C_1, and the value of the excitation at this point is the ampère-turns required to send full-load current through the armature when short-circuited. The importance of the load curves is that they indicate (1) the point at which the machine is normally excited, and (2) the change in terminal pressure which occurs when full-load current is thrown on or off.

Regulation.—In the majority of alternating current supply systems the generators have to supply power at approximately constant terminal pressure, the load on the machines varying considerably in magnitude and power factor. The excellence of an alternator therefore largely depends upon the degree to which it regulates for constant terminal voltage. Should the drop in terminal voltage between no-load and full-load exceed a certain amount, fixed according to the work for which the alternator is designed, the output will have exceeded the limit set by

armature reaction, even although the heating may be within the permissible limit. The output of an alternator is thus limited by temperature rise and armature reaction. The same considerations limit the output of direct-current generators, the only difference between the two cases being that the limit set by armature reaction in an alternator does not arise from the question of commutation, but from the point of view of regulation. The problem of alternator regulation is complicated by the fact that the reactions taking place are not determined by the output of the machine alone, but also by the phase relation of the current and voltage, and the saturation of the magnetic circuit.

The pressure regulation of an alternator may be specified by either of the following methods:—

(1) The percentage *drop* in terminal voltage between no-load and full-load; or

(2) The percentage *rise* in terminal voltage when full-load is switched off.

In both cases the exciting current and speed are supposed to remain constant while the load is switched *on* or *off*. Owing to the greater degree of saturation, the second definition gives a somewhat better regulation than the first. The change in pressure depends greatly on the power factor of the load,—the smaller the power factor the greater will be the change in voltage. For lighting distribution with fully loaded transformers, ϕ – the angle of lag of the current—will be about 10 degrees; hence the power factor cos ϕ is approximately unity. For power distribution with fully loaded induction motors, ϕ has an average value of about 35 degrees; this corresponds to a power factor of 0.85. In ordinary use, however, the motors of an installation will seldom be all fully loaded, and the average power factor may for long periods not exceed about 0.7. The regulation is therefore generally specified for (1) full-load at unity power factor, and (2) full-load at 0.85 or 0.8 power factor. The two values are termed respectively the "inherent regulation at unity power factor" and "the inherent regulation at 0.85 or 0.8 power factor." The full-load current at other than unity power factor is to be taken as the value corresponding to unity power factor, *i.e.* the kilovolt-ampères (K.V.A.), and not the kilowatts, is the basis on which alternators are rated. The usual values of inherent regulation range from 4.0 to 6.0 per cent. for unity power factor, and from 15 per cent. to 20 per cent. for power factors between 0.85 and 0.8.

The methods of estimating the various reactions which affect the regulation of an alternator will now be discussed. That component of the voltage drop resulting from the resistances of the stator winding and the effects of eddy currents induced in the conductors is given by the equation

$$e_r = I_a r_a \text{ volts per phase}$$

where I_a is the armature current per phase and r_a (see equation 11 page 102) is the effective resistance per phase.

Induction of an Alternator Armature.—When an alternator is loaded the magnetomotive force due to current in the armature winding sets up a flux the greater part of which traverses the main magnetic circuit, causing an alteration in (1) the total number of lines entering the armature, and (2) the distribution of the main flux over the pole face. This part of the reaction is known as the demagnetising and distorting M.M.F. of the armature, and its effect upon the regulation is treated in a succeeding section of this chapter. The other portion of the armature flux becomes linked with the active conductors and end-connections without passing through the main magnetic circuit. Strictly speaking, this local flux, except that portion linked with the end-connections and those conductors which lie between the poles, does not exist separately in closed circles, but shows itself in a distortion of the main flux, which, however, is only local and does not extend over the pole face. This second portion of the armature flux may be regarded as independent of the position of the coils relatively to the poles, and will therefore have an approximately constant value, whatever the position of the poles.

Since the armature current is alternating, the local or leakage flux, which does not become linked with the main field, will be continually altering in magnitude and direction, so that there is set up a self-induced E.M.F. proportional to the leakage flux of each phase and lagging 90 degrees behind the current. The armature winding of an alternator will therefore possess a certain amount of inductance, so that if L, expressed in henries, denotes the coefficient of self-induction per phase and $\sim$ the frequency of the current, then the reactance per phase

$$= 2\pi \sim L \text{ ohms}$$

Further, let I_a denote the armature current per phase, then, assuming a sinusoidal rate of change, the equation to the self-induced E.M.F., or *reactance voltage*, is

$$e_x = 2\pi \sim LI_a \text{ volts}$$

To calculate e_x, it is necessary in the first instance to estimate the value of L.

The exact value of the self-induction of an armature winding is somewhat difficult to determine, its magnitude depending upon the reluctance of the paths taken by the leakage flux. Since the former is partly of iron, the inductance will depend upon the saturation of the teeth, and therefore to some extent varies for different excitations, and different values of armature current. Even in the light of these uncertain factors it is possible to derive, from general principles, formulæ for the leakage flux, and the methods set forth herein have been found to give

results which agree very closely with those afterwards obtained experimentally.*

Calculation of Inductance and Reactance Voltage.†—The inductance of a coil in absolute units is equal to the number of times the component turns become linked with the lines of force threading through the former when the current carried is 1 C.G.S. unit. Consider an armature coil having C conductors per side, *i.e.* C turns per coil, and suppose it to be traversed by a current of 1 ampère, then the M.M.F. per coil side $=\frac{4\pi}{10}C$. If P_m denote the permeance of the magnetic circuit upon which this M.M.F. acts, then the flux produced by the coil

$$= \text{M.M.F.} \times \text{permeance} = \frac{4\pi}{10} . C . P_m$$

As this flux becomes linked with C conductors the general expression for the inductance of any group of conductors is

$$L = 0.4\,\pi . C^2 . P_m\, 10^{-8} \text{ henries}$$

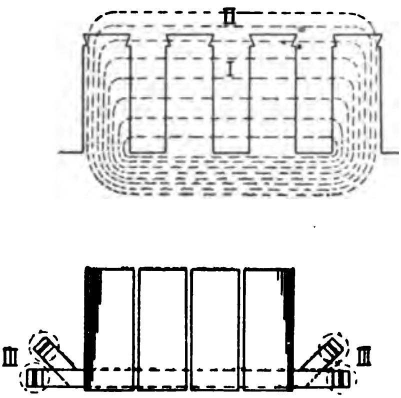


FIG. 190.

For the purpose of calculation, the leakage flux producing armature self-induction may be considered as made up of three parts: (1) the flux within the slot; (2) flux issuing from crowns of teeth, and passing along the air-gap; and (3) the flux linked with the end-connections. These three fluxes are indicated in Figure 190 by the symbols I., II., III., for a coil occupying three slots. The self-induction of an armature coil therefore consists of three parts as follow:—

L_s = inductance due to slot leakage.
L_a = inductance due to tooth-head or air-gap leakage.
L_e = inductance due to leakage round end-connections.

* J. Rezelman, *Electrician* (1909), vol. lxiii. pp. 742, 796.
† Arnold, *Die Wechselstromtechnik*, vol. iv.

Inductance due to Slot Leakage.—Figure 191 represents three slots in which are placed conductors belonging to the same phase. The armature M.M.F. tends to establish round each slot an altering magnetic flux the path of which is as indicated by the faint dotted lines. The tendency is for the teeth *b* and *c* to be traversed by two magnetic fluxes; but since these at any instant are equal and opposite to one another, the flux passing through these teeth is really zero. The greater portion of the lines of force forming the slot leakage therefore passes direct from *a* to *d*, as indicated by the heavy line. Of course, such a distribution of flux is based upon the assumption that the iron is of infinite permeance; but should the teeth *a* and *d* become saturated, a small portion of the slot leakage will pass along intermediate teeth.

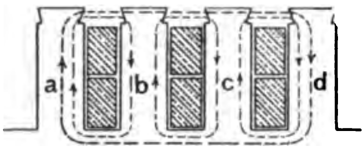


FIG. 191.

Open Slots.—The expression for the inductance due to leakage within the slot will first be derived for a coil wound in open slots of the type outlined in Figure 192. The symbols used in the formula will have the following significance:—

l_{in} = length of embedded conductor per turn.
d = depth of winding.
d_1 = depth of slot from air-gap to top of winding.
s = width of slot.

The slot leakage may be divided into portions: (1) the flux across the winding space, and (2) the flux above the winding. Considering a small strip of the winding space having a width dx, and distant x centimetres from the bottom of the winding, the tube of force passing through this strip will take the path indicated by the chain dotted line. Assuming the permeance of the iron portion of the path to be infinite as compared with the path across the slot, the permeance of the path taken by the tube of force $= \frac{l_{in} dx}{qs}$, where q is the number of slots per pole per phase. Let A_x $(= q \cdot s \cdot x)$ denote the area of cross-section of the winding space up to the height x, and A $(= q \cdot s \cdot d)$ the *total* area of cross-section of winding space, then the M.M.F. acting across the strip dx when one ampère flows in each conductor is

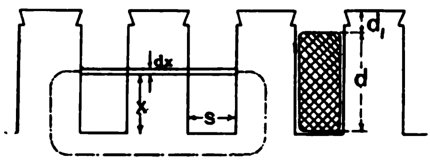


FIG. 192.

$$\text{M.M.F.} = 0.4\,\pi \cdot \text{C} \cdot \frac{\text{A}_x}{\text{A}}$$

where C = number of conductors in q slots.

For the flux passing across the strip dx

$$=0.4\,.\,\pi\,.\,C\,\frac{q.s.x}{q.s.d}\,.\,\frac{l_{in}dx}{qs}=0.4\,.\,\pi\,.\,C\,\frac{x}{d}\,.\,\frac{l_i dx}{qs}$$

the number of linkages is

$$=0.4\,\pi\,.\,C\,.\,\frac{x}{d}\,.\,\frac{l_{in}dx}{q\,.\,s}\,.\,C\,\frac{x}{d}$$

Hence for the winding space the total interlinkage of flux with C conductors in q slots is

$$L'_s=0.4\,\pi\,\frac{C^2}{q}\,.\,\frac{l_{in}}{d^2s}\int_o^d x^2dx=0.4\,\pi\,\frac{C^2}{q}\,.\,\frac{l_{in}d}{3s}$$

Above the winding space the M.M.F. over the entire cross-section $=\frac{4}{10}\pi C$, and if the influence of the grooves in the sides of the teeth be neglected the permeance of this portion of the leakage path $=\frac{l_{in}d_1}{q.s}$. The inductance of the embedded conductors due to the leakage above the winding space

$$=L''_s=0.4\,\pi\,.\,C^2\,\frac{l_{in}d_1}{q\,.\,s}$$

Hence for open slots the inductance per coil due to slot leakage is expressed by the equation

$$L_s=L'_s+L''_s=0.4\,\pi C^2\,\frac{l_{in}}{q}\left(\frac{d}{3s}+\frac{d_1}{s}\right)\times 10^{-8}\text{ henries} \quad . \quad . \quad . \quad (62)$$

The expression $\frac{l_{in}}{q}\left(\frac{d}{3s}+\frac{d_1}{s}\right)$ denotes the equivalent permeance of the slot leakage paths.

Semi-closed Slots.—Semi-closed slots are generally proportioned as shown in Figure 193, the tooth horns being either rounded or straight. As the shape of the horns and the magnitude of the angle x do not influence the leakage to any great extent, the following formula, though derived for a slot with straight horns, is also applicable to oval slots. The additional symbols for closed slots are :—

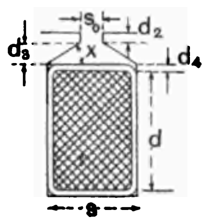


FIG. 193.

s_o = width of slot opening.
d_2 = thickness of lip of tooth.
d_3 = depth of sloped portion of slot.
d_4 = thickness of insulation between top of winding and slope.

The leakage inductance L_s, derived in a similar manner to equation 62, is expressed by the equation

$$L_s=0.4\,\pi\,\frac{C^2 l_{in}}{q}\left(\frac{d}{3s}+\frac{d_4}{s}+\frac{2\,d_3}{s+s_o}+\frac{d_2}{s_o}\right)\times 10^{-8}\text{ henries} \quad . \quad . \quad . \quad (63)$$

The third term within the brackets is the permeance per centimetre length for the path across the sloped portion of the slot, and in arriving at the value the length of path has been taken as $\frac{s+s_o}{2}$.

Inductance due to Tooth Head Leakage.—In the case of alternators, which generally have comparatively large air-gaps, this component of the leakage flux will not be materially different when the rotor is absent from that when it is present. The permeance of the paths taken by the leakage flux under consideration will therefore be calculated on the assumption that the rotor is removed and that the tubes of force are as in Figure 194. The permeance of the path taken by the tooth head leakage will be integrated over a coil span, that is, for a distance equal to about one pole pitch.

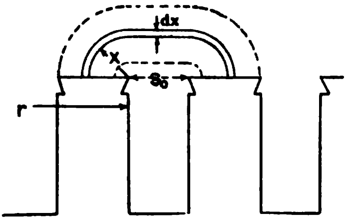


FIG. 194.

In the case of a 3-phase machine having one slot per pole per phase, the permeance of the path taken by the leakage across the tooth heads will be

$$P_a = l_{in}\int_{x=o}^{x=r}\frac{dx}{s_o+\pi x} = \frac{l_{in}}{\pi}\log_\epsilon\frac{\pi r+s_o}{s_o} = \frac{2.3}{\pi}l_{in}\log\left(1+\frac{\pi r}{s_o}\right)$$

$$\simeq \frac{2.3 l_{in}}{\pi}\log\frac{\pi r}{s_o}$$

If p denote the number of pairs of poles and τ the pole pitch then the value of r is approximately expressed by

$$r = \tau\frac{p}{p+1}$$

Hence for a concentrated winding the leakage inductance per coil is

$$L'_a = 0.4\,\pi C^2 . \frac{2.3}{\pi}l_{in}\log\frac{\pi r}{s_o} = 0.92 . C^2 . l_{in}\log\frac{\pi r}{s_o}\times 10^{-8}\text{ henries}$$

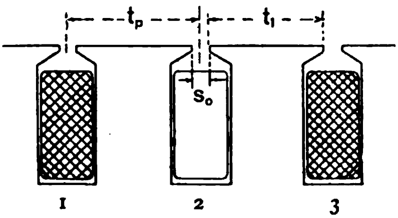


FIG. 195.

The influence of the spread of the coils upon the tooth head leakage will be examined for a winding having three slots per pole per phase.

In Figure 195—

t_p = slot pitch.
t_1 = width of tooth at air-gap.
C_s = conductors per slot.

For a current of one ampère the conductors in slots 1 and 3 each produce the following fluxes :—

$$0.92 \, . \, C_s \, . \, l_{in} \log\left(1 + \frac{\pi t_1}{s_o}\right) \text{ lines}$$

linked with C_s conductors in one slot;

$$C_s . 0.4 \pi l_{in} \int_{x=o}^{x=t_p} \frac{dx}{s_o + \pi (t_1 + x)} = 0.92 \, . \, C_s . \, l_{in} \, . \log\left(1 + \frac{\pi t_p}{s_o + \pi t_1}\right) \text{ lines}$$

linked with $2C_s$ conductors in two slots; and

$$C_s . \, 0.4 \pi l_{in} \int_{x=o}^{x=r} \frac{dx}{s_o + \pi \, (t_1 + t_p + x)} = 0.92 \, . \, C_s \, . \, l_{in} \, . \log \frac{\pi r}{s_o + \pi (t_1 + t_p)} \text{ lines}$$

linked with $3C_s$ conductors in three slots.

Similarly, one ampère flowing in the conductors of slot 2 produces

$0.92 \, . \, C_s . \, l_{in} \, . \log\left(1 + \frac{\pi t_1}{s_o}\right)$ lines linked with C_s conductors

and $0.92 . C_s . \, l_{in} . \log\left(\frac{\pi r}{s_o + \pi \, t_1}\right)$ lines, linked with $3C_s$ conductors

Now, $$1 + \frac{\pi t_p}{s_o + \pi t_1} \cong 2$$

$$\frac{\pi r}{s_o + \pi \, (t_1 + t_p)} \cong \frac{r}{2t_p}$$

and $$\log \frac{\pi r}{s_o + \pi t_1} \cong \log \frac{r}{t_p} = \log \frac{r}{2t_p} + \log 2$$

If the various fluxes be multiplied by the turns with which they are respectively linked, then the sum of these products gives the inductance of the coil due to leakage across the tooth heads, *i.e.*

$L_a = \Sigma$(flux produced × number of turns with which the flux is linked)

$$= 0.92 \, C_s^2 l_{in} \left[3 \log\left(1 + \frac{\pi t_1}{s_o}\right) + 2.1 + 9 \log \frac{r}{2t_p}\right]$$

Since $C_s = \dfrac{C}{q}$, then in general the tooth head leakage may be expressed by

$$L_a = 0.92 \, \frac{C^2}{q} l_{in} \, \Delta' \times 10^{-8} \text{ henries} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

where Δ' denotes the equivalent permeance between tooth heads. The expression for Δ' can be derived for any number of slots per

pole per phase in a similar manner to that set forth above. Up to six slots per pole per phase the expressions for Δ' are as follow:—

$$\left.\begin{array}{ll}
q=1 & \Delta'=\log\left(1+\frac{\pi t_p}{s_o}\right) \\
q=2 & \Delta'=\log\left(1+\frac{\pi t_p}{s_o}\right)+2\log\frac{r}{t_p} \\
q=3 & \Delta'=\log\left(1+\frac{\pi t_p}{s_o}\right)+0.7+3\log\frac{r}{2t_p} \\
q=4 & \Delta'=\log\left(1+\frac{\pi t_p}{s_o}\right)+1.35+4\log\frac{r}{3t_p} \\
q=5 & \Delta'=\log\left(1+\frac{\pi t_p}{s_o}\right)+2.22+5\log\frac{r}{4t_p} \\
q=6 & \Delta'=\log\left(1+\frac{\pi t_p}{s}\right)+3.16+6\log\frac{r}{5t_p}
\end{array}\right\} \quad (65)$$

Of course for open slots $s_o = s$.

Inductance due to End-coil Leakage.—The leakage lines which become linked with the end-connections have their path chiefly in air, for though the end-connections are more or less surrounded by solid iron parts—*e.g.*, core clamping plates and end-shields—yet it is possible to fix the distance between the windings and the iron parts so that only a small proportion of the end-leakage flux enters the iron. Again, the flux surrounds the end-connections

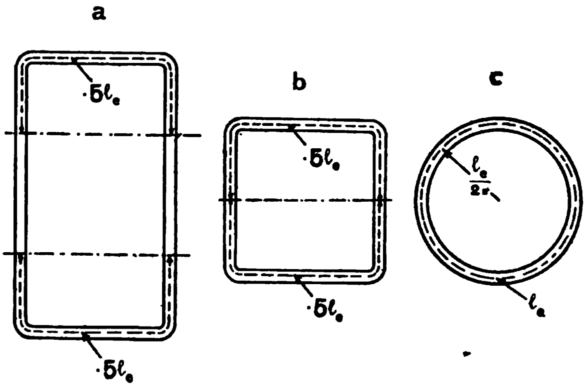


FIG. 196.

in such manner that there is very little mutual inductance between the various phases of a polyphase winding. In the main the end leakage is equal to the flux which would be created by an independent group of coils having a shape similar to the coil formed by putting two projecting ends together. For instance, if the end-connections of the single coil shown in Figure 196*a* be cut away from the conductors

and put together as at b, then the self-induction of the end-connections will be approximately the same as that for the equivalent coil. Since the equivalent coil will in general be nearly square, its self-induction may be taken as equal to that of a circular coil with the same number of turns and a circumference equal to that of the length of the end-connections per armature turn, then the radius of the circular coil $= \frac{l_e}{2\pi}$ (see Figure 196c). The self-induction of a coil consisting of C turns $= 0.4\ \pi\ C^2 \times$ permeance. Now, if d_s denote the diagonal of the rectangular section of the end-connections (see Figure 197), then the permeance of the leakage path is

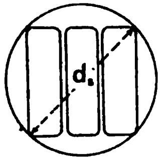


FIG. 197.

$$P_e \cong \int_{x=\frac{d_s}{2}}^{x=\frac{l_e}{2\pi}} \frac{l_e dx}{2\pi x} = \frac{l_e}{2\pi}\left(\log_e \frac{l_e \cdot 2}{2\pi d_s}\right)$$

$$= \frac{2.3 l_e}{2\pi}\left(\log \frac{l_e}{d_s} - \log \pi\right)$$

The self-induction due to the end-leakage flux of each coil is therefore

$$L_e = 0.46\ l_e\ C^2\left(\log \frac{l_e}{d_s} - 0.5\right) \times 10^{-8} \text{ henries} \quad . \quad . \quad . \quad . \quad . \quad (66)$$

Reactance Voltage.—Having determined the equations for the component self-inductions L_s, L_a, and L_e, the self-induction per coil of C turns $= L_s + L_a + L_e$. If there be n_c coils in series, then the self-induction per phase

$$L = n_c\ (L_s + L_a + L_e)$$

Assuming the inductance to be constant, and that the change of current is sinusoidal, the reactance voltage per phase is expressed by the equation

$$e_x = 2\ \pi \sim n_c\ (L_s + L_a + L_e)\ I_a \text{ volts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

where I_a = current per phase.

Though in deriving the above formula various assumptions have been made as to the path taken by the leakage flux and the permeance of the iron parts, yet this method of calculation is found to give results which approximate very closely to the value experimentally obtained for L when the impedance is measured with the rotor removed.

Example.—From the following data relating to a 600-K.V.A., 3300-volt, 50 ~, 3-phase alternator calculate the reactance voltage at full-load. The dimensions of the slots are as in Figure 289, page 393.

Coils in series per phase (n_c) . . .	= 12.
Turns per coil (C)	= 16.
Slots per pole per phase (q) . . .	= 2.
Full-load current per phase (I_a) . .	= 105 ampères.
Embedded length per turn (l_{in}) . .	= 54 cms.
Free length per turn (l_e).	= 116 cms.
Number of poles	= 24.
Pole pitch (τ)	= 28.8 cms.
Tooth pitch at air-gap (t_p)	= 4.8 cms.

Inductance per coil due to slot leakage is

$$L_s = 0.4\,\pi\,C^2\,\frac{l_{in}}{q}\left(\frac{d}{3s}+\frac{d_1}{s}\right)\times 10^{-8}$$

$$= 0.4\,.\,\pi\,.\,16^2\,.\,\frac{54}{2}\left(\frac{3.55}{3\times 1.8}+\frac{0.65}{1.8}\right)\times 10^{-8} = 8.8\times 10^{-5} \text{ henries.}$$

Inductance per coil due to tooth head leakage is

$$L_a = 0.92\,\frac{C^2}{q}\,l_{in}\,\Delta'\,.\,10^{-8}$$

For $q=2$, the value of Δ' is

$$\Delta' = \log\left(1+\frac{\pi t_p}{s_o}\right) + 2\log\frac{r}{t_p}$$

$$= \log\left(1+\frac{\pi\times 4.8}{1.8}\right) + 2\log\frac{26.6}{4.8} = 2.46$$

$$\left[r = \tau\,.\,\frac{p}{p+1} = 28.8\times\frac{12}{13} = 26{\cdot}6\right]$$

Substituting this value of Δ' in the above equation for L_a,

$$L_a = 0.92\,.\,\frac{16^2}{2}\,.\,54\,.\,2.46\,.\,10^{-8} = 15.5\times 10^{-5} \text{ henries.}$$

Inductance per coil due to end-leakage is

$$L_e = 0.46\,l_e C^2\left(\log\frac{l_e}{d_s} - 0.5\right)\times 10^{-8}$$

$$= 0.46\,.\,116\,.\,16^2\left(\log\frac{116}{5} - 0.5\right)\,10^{-8} = 12\times 10^{-5} \text{ henries}$$

Reactance per phase $= r_x = 2\pi\sim .\ n_c\,(L_s + L_a + L_e)$
$= 2\pi\,.\,50\,.\,12\,(10 + 15.5 + 13)\,10^{-5}$
$= 1.37$ ohms.

Reactance voltage per phase

$e_x = 2\pi\sim n_c\,(L_s + L_a + L_e)\,I_a$
$= 1.37\times 105$
$= 144$ volts.

Demagnetising and Cross-magnetising M.M.F. of Armature.—As previously mentioned, the other component of the flux due to the armature magnetomotive force traverses the same path as the flux of the field magnets, and combines with it to form a resultant flux. This part of the effect of the armature M.M.F. represents the "true armature reaction," and its nature for loads of various power factors will now be examined.

Figure 198, represents a portion of a 3-phase alternator the armature of which has a single-coil winding; for the sake of clearness one phase only is shown. When the armature supplies current, each coil has, for every position it occupies relative to the poles, a certain M.M.F. The current if in phase with the induced voltage will attain

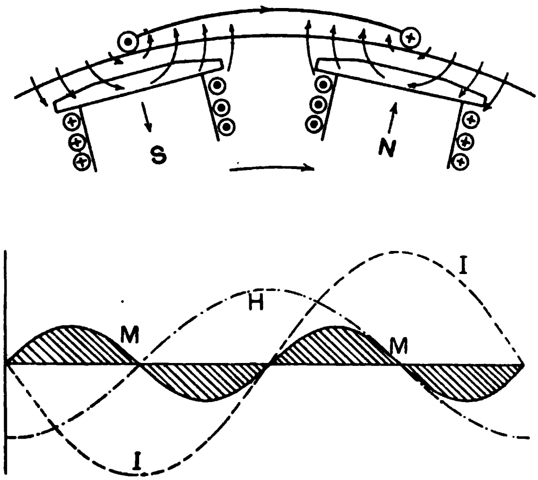


FIG. 198.—Current in phase with induced E.M.F.

its maximum value when the centre of a coil is midway between two poles. In the figure the coils are shown in this position, and the direction of the current indicated by the crossed and dotted conductors, a dot indicating that the current is flowing upwards out of the plane of the paper. The magnetic flux due to the magnetomotive force of each armature coil passes athwart the poles as represented by the arrows. For the instant considered, the armature ampère-turns acts as a cross M.M.F., the effect of which is to increase the flux at the edge of the trailing pole tip and to decrease it at the leading pole tip,—*i.e.*, the curve of flux distribution is merely distorted. When the poles have rotated, from the position shown in Figure 198, through an angular distance corresponding to one pole pitch, the current in the armature coils will have reversed, and so also will the direction of

the main field; hence the armature will have precisely the same effect, namely, entirely cross-magnetising As the poles move away from the slots containing the winding, two magnetic paths are offered to the armature flux, one of which is directly through the main magnetic circuit instead of across the poles. The armature M.M.F. is now divisible into cross and demagnetising ampère-turns; the latter directly affect the strength of the flux, whereas the former simply distort the strengthened or weakened field. The ratio of the demagnetising ampère-turns to the distorting ampère-turns gradually increases, until the sides of a coil are midway between the poles, in which case all the turns act against the field (see Figure 199). In this position, however, the armature current is zero, so that there is no

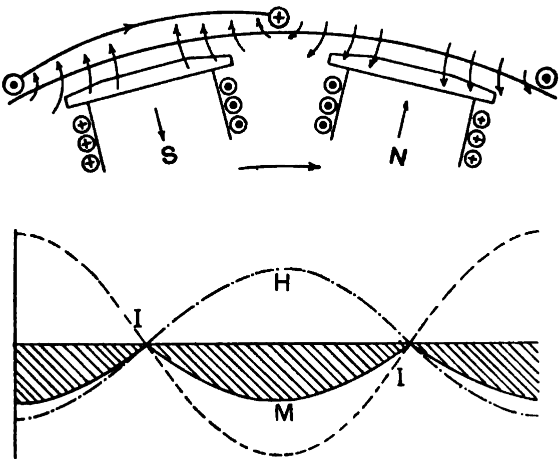


FIG. 199.—Current lagging in quadrature with induced E.M.F.

demagnetising action. Later, as the coils come under the influence of the next poles, the armature turns are again divisible partly into direct- and partly into cross-magnetising turns.

In order to estimate the effect of armature reaction during a whole period, it is necessary to consider both the instantaneous value of the current in a coil for each position of the poles and the magnetising effect that a constant current in the same turns would have for the same position. Still considering a case where the current is in phase with the induced E.M.F., let the curve H in Figure 198 represent for each position the direct magnetising effect of the armature coils when carrying unit current. This curve, which is supposed to vary according to a sine law, will attain its maximum and zero values when the conductors are midway between the poles and directly under the poles respectively. The current wave is indicated

by the curve I and the product of the corresponding ordinates of the two curves gives the third curve M, which shows the variation in the M.M.F. due to the direct ampère-turns, forward M.M.F. being plotted above and back M.M.F. below the abscissa reference axis. The shaded area shows that there is a periodic strengthening and weakening of the main field; but over a complete period the two neutralise each other. Hence, when the current is in phase with the induced E.M.F. and saturation of the teeth is neglected, the main flux is not altered in magnitude, but merely distorted. The distortion is pulsating, and ceases at the moment the current is zero; the period of the pulsation is twice that of the current.

Since the self-induction of an alternator itself causes the current

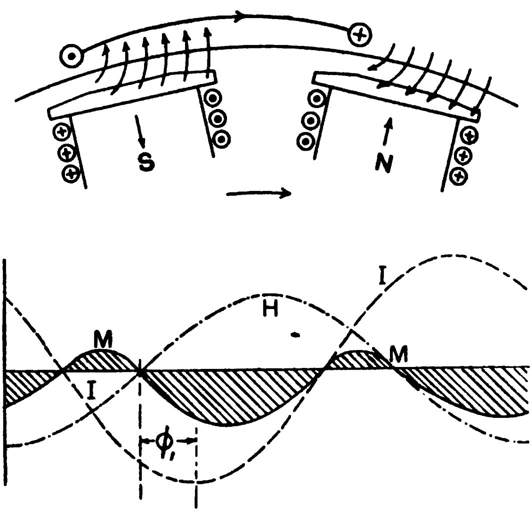


FIG. 200.—Current lagging ϕ_1 degrees behind E.M.F.

to lag, the above condition of current in phase with the induced E.M.F. could exist only in a circuit where the condenser effect of cables, etc., causes the current to lead with respect to the terminal E.M.F. In practice the current generally lags behind the induced E.M.F., and in the extreme case of a lag of 90 degrees the armature current would attain its maximum value when the coils are in the position shown in Figure 199. In this case the armature ampère-turns are directly opposed to those of the field poles, so that the armature M.M.F. is a demagnetising one entirely, the pulsating cross magneto-motive force being zero over one complete period. For intermediate angles of lag the armature reaction, taken over a complete period, will be partly demagnetising and partly distorting. The diagram of Figure 200 is similar to that of Figure 198, except that the current

lags ϕ_1 degrees behind the induced E.M.F., ϕ_1 being about 35 degrees. In this case there is a periodic weakening and strengthening of the main field, but the weakening effect exceeds the strengthening effect. The cross ampère-turns are approximately proportional to the component of the current in phase with the induced E.M.F., whereas the demagnetising ampère-turns vary as the component of the current in quadrature therewith. With a leading current the direct magnetomotive force strengthens the main field, and distorts it in the direction of the rotation of the field.

Calculation of Armature Magnetomotive Force.—In a single-phase alternator, or in one phase of a polyphase machine considered by itself, the magnetomotive force due to current in the armature winding is an alternating one. At any point on the armature periphery it is fixed in direction relative to the poles, although as the latter rotate the armature M.M.F. varies synchronously with the period of the machine. The form of the curve showing the

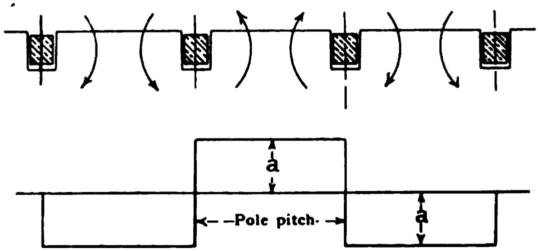


FIG. 201.—M.M.F. wave due to single-coil winding.

variation of M.M.F. along the armature periphery will depend upon the distribution of the winding. Consider a winding the sides of whose coils are concentrated (see Figure 201) in one slot per pole, then if phase A be traversed by a *direct* current the curve of M.M.F. along the air-gap will approximate to a series of rectangles having a width equal to that of the pole pitch and a height equal to one-half the ampère-turns per pole. With a triple-coil 3-phase winding the curve of M.M.F. for one phase when traversed by a direct current will be as shown in Figure 202, the three steps in the curve corresponding to the three coils per element. If, instead of a direct current, an alternating current be sent through the above windings, the magnetomotive force produced becomes periodic in time as well as in place, but at every instant is represented by a right-angle curve like Figures 201 and 202. The horizontal dimensions of the curve remain constant but the vertical dimensions will vary periodically, and in synchronism with the period of the current between certain definite equal positive and negative values. If T_1 denote the turns per pair of poles per phase, and I_a the R.M.S. value of current per phase,

then the amplitude of the ampère-turn curve when the current attains its maximum value is expressed by

$$a = \frac{\sqrt{2} \cdot T_1 \cdot I_a}{2}$$

In order to obtain an expression for the demagnetising and distorting ampère-turns of the armature the above curves of magnetomotive force must be analysed into Fourier's series. Since the M.M.F. curve must be symmetrically positive and negative alternately, it will contain odd harmonics only. The time-period of the magnetomotive force wave is equal to the periodic time T of the current, while the space-period is equal to 2τ, where τ denotes the pole pitch. Let a_1, a_3, a_5, etc., denote the amplitudes of the fundamental wave, and the third, fifth, etc. harmonics respectively, and x the distance along the armature periphery from a point where

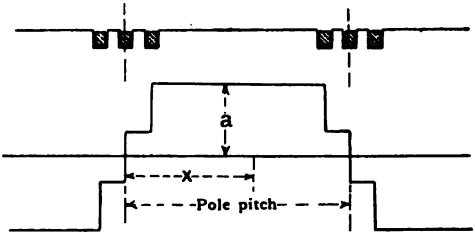


FIG. 202.—M.M.F. wave due to triple-coil winding.

the M.M.F. is zero, then the equation to the armature magnetomotive force at any instant t is

$$y = a_1 \sin \frac{2\pi x}{2\tau} \cdot \sin \frac{2\pi t}{T} + a_3 \sin \frac{6\pi x}{2\tau} \cdot \sin \frac{6\pi t}{T}$$
$$+ a_5 \sin \frac{10\pi x}{2\tau} \cdot \sin \frac{10\pi t}{T}$$

With the distributed windings which are generally employed, the third fifth, etc., harmonics will be almost negligible as compared with the fundamental. Hence it is only necessary to consider the first term of the above series. For a concentrated winding the amplitude a_1 of the fundamental (*vide* page 9)

$$= 2 \times \text{average value of } a \sin \theta = 2a\frac{1}{\pi}\int_0^\pi \sin \theta \, d\theta$$
$$= \frac{4a}{\pi} = \frac{4}{\pi}\sqrt{2}\frac{I_a T_1}{2} = 0.9\ T_1 I_a$$

When the winding is distributed the above equation must be multiplied by the breadth factor k_2; hence the amplitude a_1 is expressed by the general equation

$$a_1 = 0.9 \cdot k_2 \cdot T_1 \cdot I_a$$

the values of k_2 being obtained from Table XVI., page 218.

Single-phase Alternator.—In the case of a single-phase winding the alternating magnetomotive force of amplitude a_1 may, as explained in chapter vii., be replaced by two equal constant waves each half the amplitude of the alternating M.M.F., and revolving in opposite directions at a frequency equal to that of the resultant field. Now, since the winding is moving relative to the main field at the same rate as that at which the component magnetomotive forces are moving with respect to the armature, it follows that one component of the armature M.M.F. remains stationary with respect to the poles, whereas the other component rotates backwards with twice the angular velocity of the main field. The magnetic field produced by each component of the ampère-turns will be referred to as the *synchronous field* and the *inverse field* respectively. The inverse field produces periodic changes in the flux passing through the main magnetic circuit, the action being alternately magnetising and demagnetising. These flux pulsations induce eddy currents in the magnet poles and solid metal parts in the vicinity; but the effect of these is to produce magnetic fields which to a large extent reduce the pulsations to an almost negligible amount. Hence in calculating the armature reaction the effect of the inverse field will be neglected. The sine wave of ampère-turns which remains stationary relative to the main field has an amplitude value

$$=0.45 \cdot k_2 \cdot T_1 \cdot I_a$$

the position it occupies relative to the field poles depending upon the angle of lag of the current behind the induced E.M.F.

Polyphase Alternator.—In a polyphase alternator every one of the phases will have a similar magnetomotive force curve, and the resultant magnetomotive force wave will be obtained by adding the various M.M.Fs. in the proper phase relation as regards time and space, whilst the inverse waves cancel out. In a 2-phase machine the equations for the two fundamental waves are

$$y_{A1}=a_1 \sin \frac{2\pi x}{2\tau} \cdot \sin \frac{2\pi t}{T}$$

$$\text{and } y_{B1}=a_1 \sin \frac{2\pi}{2\tau}\left(x-\frac{\tau}{2}\right)\sin \frac{2\pi}{T}\left(t-\frac{T}{4}\right)$$

When added together

$$y_{A1}+y_{B1}=a_1 \cos 2\pi\left(\frac{x}{2\tau}-\frac{t}{T}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

Now, $\tau=\pi$ and $T=\frac{2\pi}{p}$ where $p=2\pi \sim$, so that when these values are substituted in the above equation

$$y_{A1}+y_{B1}=a_1 \cos (x-pt)$$

Similarly, in a 3-phase alternator

$$y_{A1} = a_1 \sin \frac{2\pi x}{2\tau} \cdot \sin \frac{2\pi t}{T}$$

$$y_{B1} = a_1 \sin \frac{2\pi}{2\tau}\left(x - \frac{2\tau}{3}\right) \sin \frac{2\pi}{T}\left(t - \frac{T}{3}\right)$$

$$y_{C1} = a_1 \sin \frac{2\pi}{2\tau}\left(x - \frac{4\tau}{3}\right) \sin \frac{2\pi}{T}\left(t - \frac{2T}{3}\right)$$

and when added together

$$y_{A1} + y_{B1} + y_{C1} = 1.5\, a_1 \cos (pt - x) \quad . \quad . \quad . \quad (69)$$

Equations 68 and 69 represent a train of waves of wave length 2τ, and amplitudes respectively 1.0 and 1.5 times that of y_{A1} moving forward with the velocity of the field poles. The various fundamentals combine, therefore, to give a series of sine waves of magnetomotive force moving forward synchronously with the field poles, and therefore stationary over the poles. In a similar manner it can be shown that the fifth harmonics combine to give a sine wave of magnetomotive force which at any instant is proportional to $-\cos (pt - 5x)$,—*i.e.* the resultant of the quintuple-frequency harmonics rotates at one-fifth of synchronous speed in the opposite direction to the rotation of the field poles. The seventh harmonic moves with one-seventh the synchronous speed and in the same direction as the field poles, and similarly for the other harmonics. All the harmonics which are multiples of three cancel out. Thus the fundamental, seventh, thirteenth, etc. harmonics revolve at their corresponding speeds in the same direction as the field poles, while the fifth, eleventh, etc., harmonics revolve at their corresponding speeds in the opposite direction to the field poles. Since all the harmonic magnetomotive forces move relative to the poles, they will induce currents in the field coils and pole faces. But, as previously stated, these currents will to a large extent annul any magnetising effect they might otherwise have. Only the resultant of the fundamental waves needs therefore to be taken into account when estimating the armature reaction of a polyphase alternator.

The amplitudes of the resultant ampère-turn wave which moves synchronously with the poles are as follow :—

$$\left.\begin{array}{lll} \text{Two-phase} & A = \dfrac{2a_1}{2} = 0.9\, k_2 T_1 I_a \\[2ex] \text{Three-phase} & A = \dfrac{3a_1}{2} = 1.35\, k_2 T_1 I_a \\[2ex] m\text{-phase} & A = \dfrac{ma_1}{2} = 0.45\, k_2 m T_1 I_a \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

The position which the rotating M.M.F. sine wave occupies relatively to the field poles depends on the power factor of the circuit. When the current is in phase with the induced E.M.F. the rotating

sine wave must have its zero value over the centres of the poles, and in the general case the sine curves will lag or lead from this position by approximately the lag or lead of the current relative to the induced E.M.F. In order to separate the different magnetic effects produced by the synchronous M.M.F. wave the train of sine waves will be resolved into two trains of sine waves, one of which has its maximum ordinates over the pole centres, while the other has its maximum ordinates midway between the poles. If A denote the amplitude of

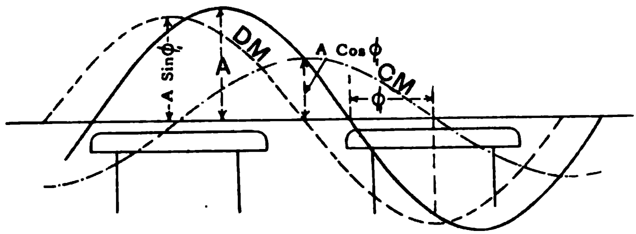


FIG. 203.—Fundamental M.M.F. wave of armature reaction.

the initial sine wave, and ϕ_1 the angle of lag of the current behind the induced E.M.F., then the amplitude of the former component wave will be A sin ϕ_1 and the amplitude of the latter component wave A sin $(\phi_1+90)=$A cos ϕ_1. The waves which have their maximum ordinates over the pole centres are due to the wattless component of the current, and are called the demagnetising waves DM, while the other waves due to the watt component of the current are called the cross-magnetising waves CM; both are shown in Figure 203.

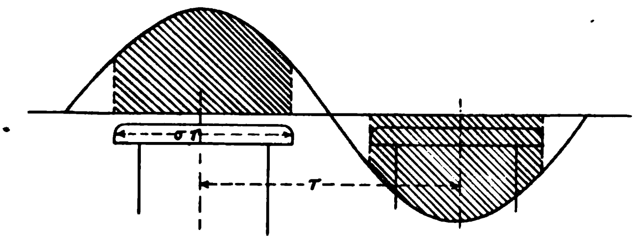


FIG. 204.—Demagnetising component of armature M.M.F.

The set of magnetomotive force waves which have the maximum ordinates over the pole centres will produce a direct magnetising or demagnetising action on the poles, depending upon whether the current is leading or lagging. Since the pole surface of salient pole alternators is not continuous, only that portion of the wave which stands over the pole will have any appreciable effect upon the magnetic flux in the air-gap. It is therefore necessary to determine the average value of the portion of the wave which is shown shaded in Figure 204. If σ denote the ratio of pole arc to pole pitch, then $\sigma\tau$ is the breadth

of the pole face $= \sigma\pi$ radians and the average value of the M.M.F. within the shaded area

$$= \text{maximum ordinate} \times \frac{1}{\sigma\pi} \int_{-\frac{\sigma\pi}{2}}^{+\frac{\sigma\pi}{2}} \cos\theta \,.\, d\theta$$

θ being measured from the centre of the pole

$$= \text{maximum ordinate} \times \frac{\sin \frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma}$$

The demagnetising ampère-turns of armature reaction are therefore given by the general equation

$$AT_{DM} = A \sin\phi_1 \,.\, \frac{\sin \frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma}$$

$$= 0.45 \,.\, k_2 \,.\, m \,.\, T_1 I_a \sin\phi_1 \,.\, \frac{\sin \frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma} \quad . \quad . \quad . \quad (71)$$

The values of $\dfrac{\sin \frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma}$ for various ratios of pole arc to pole pitch are as follow:—

Pole arc ÷ pole pitch (σ) . .	1.0	0.9	0.8	0.75	0.7	0.65	0.6	0.55	0.5
Values of $\dfrac{\sin \frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma}$ . . .	0.63	0.7	0.73	0.78	0.81	0.83	0.85	0.88	0.9

While the demagnetising component of the armature M.M.F. decreases the main field flux as a whole (or increases it when AT_{DM} is negative), and may be reckoned as so many ampère-turns subtracted from (or added to) the excitation of the field magnets, the cross magnetising component shown by the curve CM in Figures 203 and 205 produces neither increase nor decrease of the field excitations as a whole, but increases the M.M.F. acting over one-half of the pole face and diminishes the M.M.F. acting over the other half of the pole face, the increase and decrease being equal in magnitude. The cross ampère-turns thus tend to produce a local flux Φ_{CM} passing through one-half of the air-gap, then along the pole shoe, back through the other

half of the gap, and completing its magnetic circuit through the iron of the armature. This local flux will be constant in magnitude, but will move along with the poles. It will therefore cut the armature conductors and induce an E.M.F. therein. The shaded portion of Figure 205 represents the distribution of the cross magnetomotive force along the pole face. The average value of the M.M.F. acting over one-half the pole face

$$= \text{maximum ordinate} \times \frac{1}{\frac{\sigma\pi}{2}} \int_0^{\frac{\sigma\pi}{2}} \sin\theta\, d\theta$$

$$= A \cos\phi_1 \times \frac{2}{\sigma\pi} \times \left(1 - \cos\frac{\sigma\pi}{2}\right)$$

Since the cross flux is also assisted by the M.M.F. over the other half

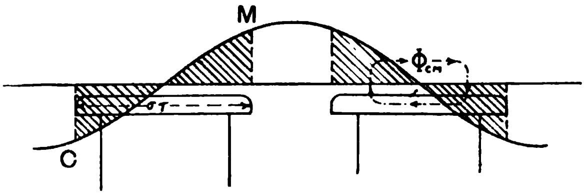


FIG. 205.—Cross-magnetising component of armature M.M.F.

of the pole, the average value of the cross ampère-turns per pole due to the armature is expressed by

$$AT_{CM} = 2\,A\cos\phi_1 \,.\, \frac{2}{\sigma\pi} \,.\, \left(1 - \cos\frac{\sigma\pi}{2}\right)$$

$$= 1.8 \,.\, k_2 \,.\, m \,.\, T_1 \,.\, I_a \,.\, \cos\phi_1 \,.\, \frac{\left(1 - \cos\frac{\sigma\pi}{2}\right)}{\sigma\pi} \quad . \quad . \quad . \quad (72)$$

The values of the factor $\frac{\left(1 - \cos\frac{\sigma\pi}{2}\right)}{\sigma\pi}$ for various ratios of pole arc to pole pitch are as follow:—

Pole arc ÷ pole pitch (σ)	1.0	0.9	0.8	0.75	0.7	0.65	0.6	0.55	0.5
Values of $\frac{\left(1 - \cos\frac{\sigma\pi}{2}\right)}{\sigma\pi}$	0.32	0.3	0.28	0.255	0.25	0.24	0.225	0.21	0.195

As previously stated, the cross magnetomotive force of the armature does not, except in so far as saturation is concerned, affect the average value of the main magnetic flux, but simply alters the distribution of

the latter over the pole face. Now, an exactly similar effect results if the armature induced E.M.F. is obtained by assuming the main flux to remain undistorted, and by superposing on it a cross-flux Φ_{CM} the path of which is indicated in Figure 205. This local flux must be of such a magnitude and distribution as to reproduce the actual distribution when compounded with the main flux. Since the reluctance of the iron in the path of the cross flux is small compared with the reluctance of the path in air, the magnitude of the cross flux

$$\Phi_{CM} = 0.4\,\pi\,.\,AT_{CM} \div \text{the reluctance of two half air-gaps}$$

$$= 0.4\,\pi AT_{CM} \div \frac{2k_3\delta}{\frac{b_i}{2}\,.\,l_i} = 0.31 AT_{CM}\,.\,\frac{b_i l_i}{k_3\delta} \quad . \quad . \quad . \quad . \quad . \quad (73)$$

where δ = radial depth of the air-gap,
l_i = ideal pole length,
b_i = ideal pole breadth.

The electromotive force induced in each phase of the armature winding by the transverse flux is

$$E_{CM} = 4\,.\,k_{1c}\,.\,k_{2c}\,.\,T\,.\sim.\,2\,.\,\Phi_{CM} \times 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad (74)$$

T = turns in series per phase,
k_{1c} = form factor,
k_{2c} = breadth factor.

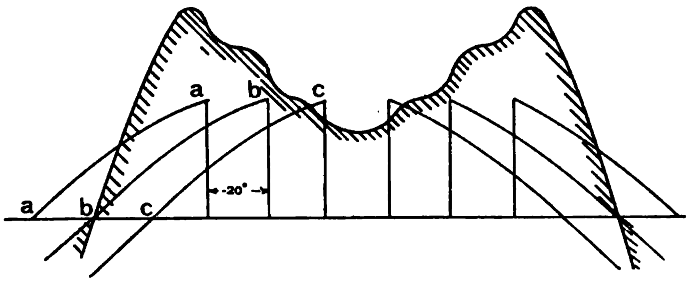


FIG. 206.—Curves of E.M.F. induced in armature winding by the flux of cross-magnetisation.

In order to determine the value of the factors k_{1c} and k_{2c} it is necessary to know the wave form of E.M.F. for each element and for each coil.

If in Figure 205 the air-gap be of uniform width, then by altering the scale the curve CM will approximately represent the distribution of the cross flux along the air-gap. Considering a 3-phase armature having three slots per pole per phase, the E.M.Fs. which the transverse flux induces in the coils of one phase of the armature are represented by the curves *aa*, *bb*, and *cc* shown in Figure 206. These three portions of sine curves are spaced out 20 degrees apart, and when the ordinates of the various curves are added together the resultant E.M.F.

curve is obtained. To allow for the gradual change in the magnitude of the flux, which takes place at the edges of the pole shoe, the resultant curve of E.M.F. has been rounded off at all the sharp corners, thus producing the E.M.F. curve shown shaded. On comparing the latter with Figure 5, page 4, it is evident that the cross electromotive force contains a strongly pronounced third harmonic. In a 3-phase star-connected winding all triple-frequency harmonics are eliminated from the terminal E.M.F.; consequently, when the current is nearly in phase with the E.M.F.—*i.e.* when the cross flux is a maximum, the star voltage will in many cases exceed $\sqrt{3}$ times the phase voltage. In all salient pole machines the effect of the armature cross magnetomotive force is to introduce a third harmonic into the E.M.F. of each phase, and from an inspection of Figure 206 it will be evident that this harmonic is less pronounced the greater the number of slots per pole per phase, and the greater the ratio of pole arc ÷ pole pitch. In cylindrical rotors for turbo-dynamos $\sigma = 1$, and the cross flux is represented by an unbroken sine curve. Hence if the flux distribution at no-load be sinusoidal it will retain this form under all conditions of load, though the point of zero flux density will be shifted backwards opposite to the direction of rotation.

If, in Figure 206, the half-wave of cross E.M.F. be divided into a suitable number of equal parts n, and the ordinates at each point be denoted by $e_1, e_2, e_3, \ldots e_n$, then the form factor of the cross E.M.F. wave is

$$k_{1c} = \frac{\text{R.M.S. value of ordinates}}{\text{Mean value of ordinates}} = \frac{\sqrt{\frac{\Sigma e^2}{n}}}{\frac{\Sigma e}{n}}$$

The breadth factor can also be derived from the figure under consideration, and

$$= k_{2c} = \frac{\text{Area of resultant E.M.F. curve}}{\text{Area of single E.M.F. curve} \times \text{number of slots per pole per phase.}}$$

In a 3-phase alternator having three slots per pole per phase and a pole arc equal to 0.65 times the pole pitch the values of k_{1c} and k_{2c} are 1.04 and 0.96 respectively. The following table gives the values of the form and breadth factors for the usual ratios of pole arc to pole pitch and number of slots per pole per phase. These factors have been derived on the assumption that the depth of the air-gap is uniform over the pole face. If the values are used in connection with alternators having a graded air-gap, the error introduced will be exceedingly small.

[TABLE

TABLE XVII.

VALUES OF k_{1c}— Pole arc ÷ pole pitch	1.0	0.70	0.65	0.55
Form factor k_{1c}	1.11	1.06	1.04	1.01

VALUES OF k_{2c}— Slots per pole per phase $= q$ . . .	1	2	3	4	5
Breadth factor $= k_{2c}$	1	0.966	0.96	0.958	0.957

Combining equations 72, 73, and 74, and substituting for T_1 the value $\frac{T}{p}$ where p is the number of pairs of poles, the expression for the E.M.F. due to the cross magnetomotive force of the armature is

$$E_{CM} = 4.5 \times 10^{-8} k_2 \,.\, k_{1c} \,.\, k_{2c} \,.\, \frac{\left(1 - \cos \frac{\sigma\pi}{2}\right)}{\sigma\pi} \,.\, m \,.\, \frac{T^2}{p} \,.\, \sim \times \frac{b_i l_i}{k_3 \delta} \times I_a \cos \phi_1$$

$$= k_o \times 10^{-8} \,.\, m \,.\, \frac{T^2}{p} \,.\, \sim \,.\, \frac{b_i l_i}{k_3 \delta} \,.\, I_a \cos \phi_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

where

m = number of phases.
T = turns in series per phase.
b_i = ideal pole breadth in centimetres.
l_i = ideal pole length in centimetres.
δ = radial depth of air-gap.
I_a = current per phase.
ϕ_1 = angle of lag of current behind the centre of the pole.
= internal phase angle.

The values of the factor $k_0 = 4.5\ k_2 \,.\, k_{1c} \,.\, k_{2c} \frac{\left(1 - \cos \frac{\sigma\pi}{2}\right)}{\sigma\pi}$ have been calculated for the usual values of σ and number of slots per pole per phase, and are tabulated in Table XVIII.

TABLE XVIII.

Pole arc/Pole pitch (σ)	1.0	0.9	0.8	0.75	0.65	0.6	0.55	0.5
Values of k_o . . .	1.47	1.37	1.28	1.17	1.04	0.96	0.9	0.81

Value of Internal Phase Angle ϕ_1.—From equation 71 it will be seen that the armature demagnetising ampère-turns AT_{DM} is a function of the internal phase angle ϕ_1, the value of which must be known before AT_{DM} can be calculated. In order to obtain an expression for the angle ϕ_1 it is necessary to construct the vector diagram of Figure 207 showing the phase relation of the various E.M.Fs. involved. The terminal voltage E is represented by the vector OA drawn ϕ degrees in advance of the current vector OI. The drop in voltage e_r due to ohmic resistance is represented by AB drawn parallel to OI. The vector BC represents the voltage e_x consumed in armature self-induction, and since it must be in quadrature with the current is drawn at right angles to AB. AC then gives the voltage drop due to armature impedance, while OC, the resultant of OA, AB, and BC, represents the electromotive force E_i virtually induced in each phase of the armature winding. The voltage E_i can be considered as the resultant of two separate electromotive forces E_{CM} and E_o induced by the flux Φ_{CM} of cross magnetisation and main-field flux respectively. Since these two fluxes are in quadrature, the voltages E_{CM} and E_o will always be at right angles, and the diagrammatic representation of the three voltages E_o, E_{CM}, and E_i must always make up a right-angled triangle. The point P must therefore lie on the semicircle OPC. Suppose for a moment that the point P is determined in position, then if OP be produced to meet BC produced in D, it is evident that the angle PCD $= \phi_1$ and that CD $= E_{CM}/\cos\phi_1$. But the angle IOP also $= \phi_1$; hence if CB be produced to meet OI in G, the expression for the angle ϕ_1 is obtained thus

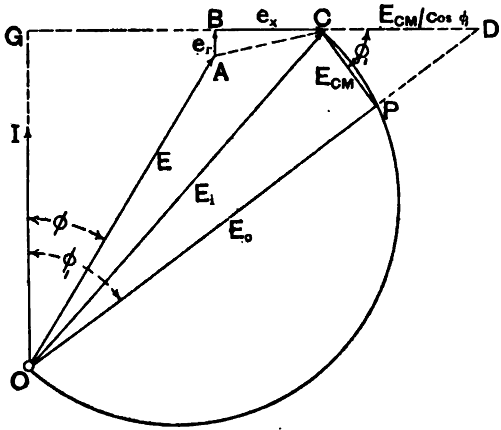


FIG. 207.

$$\tan\phi_1 = \frac{DG}{OG} = \frac{GB + BC + CD}{OG}$$

$$= \frac{E\sin\phi + e_x + \dfrac{E_{CM}}{\cos\phi_1}}{E\cos\phi + e_r}$$

$$i.e.,\ \phi_1 = \tan^{-1}\frac{\left(E\sin\phi + e_x + \dfrac{E_{CM}}{\cos\phi_1}\right)}{E\cos\phi + e_r} \quad . \quad . \quad . \quad . \quad . \quad (76)$$

The value of $\frac{E_{CM}}{\cos\phi_1}$ can be calculated from equation 75, all the factors being known except $\cos\phi_1$. From the above vector diagram it will be observed that the cross E.M.F., so far from having any direct effect in causing a fall of terminal voltage under load, slightly increases the voltage, the distortion of the main flux being accompanied by an increase in the form factor. In subsequent calculations this slight increase in terminal voltage due to E_{CM} will be neglected. The cross E.M.F. has, however, a most important indirect effect, inasmuch as it partly determines the magnitude of the internal phase angle ϕ_1, upon the sine of which depends the value of the armature demagnetising ampère-turns. Hence to predetermine the latter with any degree of accuracy the transverse electromotive force must be taken into account in calculating the angle ϕ_1.

Example.—From the following data relating to a 600-K.V.A., 3300-volt, 50 ~, 3-phase alternator calculate the demagnetising ampère-turns per pole and the cross E.M.F. for a power factor $\cos\phi = 0.8$.

Armature current per phase (I_a) . . .	= 105 ampères.
Terminal volts per phase (E)	= 1900.
IR volts per phase (e_r)	= 30.
Reactance voltage per phase (e_s) . . .	= 144.
Turns in series per phase (T)	= 192.
Slots per pole per phase (q)	= 2.
Number of pairs of poles (p)	= 12.
Radial depth of air-gap (δ)	= 0.65 cm.
Ideal pole breadth (b_i)	= 18.5 cms.
Ideal pole length (l_i)	= 32 cms.
Pole arc ÷ pole pitch (σ)	= 0.65.
k_8	= 1.14

From equation 75,

$$\frac{E_{CM}}{\cos\phi_1} = k_o \cdot 10^{-8} \cdot m \cdot \frac{T^2}{p} \cdot \sim \cdot \frac{b_i \cdot l_i}{k_8 \delta} \cdot I_a$$

$$= 1.04 \times 10^{-8} \times 3 \times \frac{192^2}{12} \times 50 \times \frac{18.5}{1.14} \times \frac{32}{0.65} \times 105 = 400$$

[$k_o = 1.04$ from Table XVIII. page 273]

For $\cos\phi = 0.8$, $\sin\phi = 0.6$, so that internal phase angle

$$\phi_1 = \tan^{-1}\left(\frac{E\sin\phi + e_x + \dfrac{E_{CM}}{\cos\phi_1}}{E\cos\phi + e_r}\right)$$

$$= \tan^{-1}\left(\frac{1900 \times 0.6 + 144 + 400}{1900 \times 0.8 + 30}\right) = \tan^{-1} 1.05 = 46 \text{ degrees}$$

Demagnetising ampère-turns per pole are

$$AT_{DM} = 0.45 \cdot k_2 \cdot m \cdot T_1 \cdot I_a \sin \phi_1 \cdot \frac{\sin \frac{\pi}{2}\sigma}{\frac{\pi\sigma}{2}}$$

$$k_2 = 0.96 \,;\; T_1 = \frac{T}{p} = \frac{192}{12} = 16 \,;$$

$$\sin \phi_1 = \sin 46 = 0.72$$

$$\text{and } \frac{\sin \frac{\pi\sigma}{2}}{\frac{\pi\sigma}{2}} = 0.83$$

Hence

$$AT_{DM} = 0.45 \times 0.966 \times 3 \times 16 \times 105 \times 0.720 \times 0.83$$
$$= 1350 \text{ ampère-turns.}$$

E.M.F. due to cross flux

$$= E_{CM} = \frac{E_{CM}}{\cos \phi_1} \times \cos \phi_1 = 400 \times 0.69 = 275 \text{ volts.}$$

Determination of Regulation when Reactions are known.—When the Ir drop, the reactance voltage, and the demagnetising ampère-turns are known the pressure regulation of an alternator at any given power factor may be derived as follows. Referring to Figure 208,

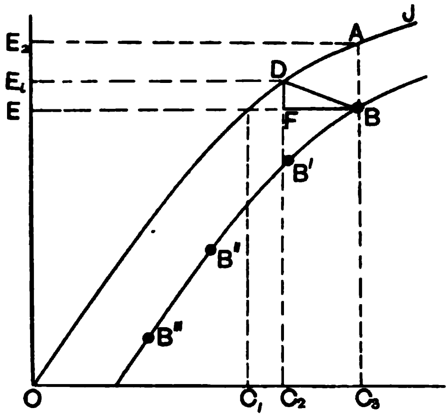


FIG. 208.

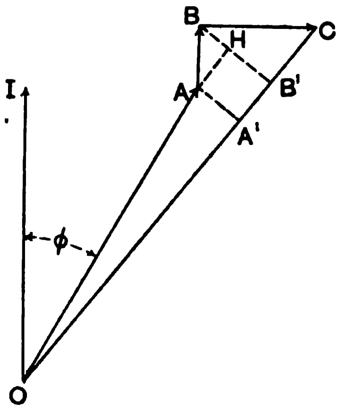


FIG. 209.

the field ampère-turns OC necessary to produce the no-load terminal voltage OE can be obtained from the open-circuit characteristic OJ, the data necessary to plot the curve being easily calculated when the dimensions of the machine are known. When on load the internal voltage E_i to be generated by the flux crossing the air-gap can be obtained from the vector diagram in Figure 209, and in order to

generate the electromotive force, OC_2 ampère-turns are necessary. The ordinate $FD = EE_i$ represents the E.M.F. consumed in armature impedance. When the alternator is fully loaded, let C_2C_3 represent the excitation required to overcome the demagnetising M.M.F. of the armature, then OC_3 ampère-turns are required at full-load to produce the same flux as OC_2 at no-load, and therefore induce an internal voltage OE_i giving a terminal P.D. of OE volts at full-load. Hence, if at full-load an excitation of OC_3 ampère-turns are necessary to maintain a terminal P.D. of OE volts, then when the load is thrown off the terminal volts will rise to OE_2. The rise of pressure AB ($= EE_2$) when expressed as a percentage of the normal terminal volts gives the regulation of the machine according to the second method of specification (p. 251), B being a point on the full-load saturation curve. In a similar manner the points B′, B″, and B‴ corresponding to other excitations can be determined and the curve B‴, B″, B′, B formed by joining up the points gives the load characteristic for a power factor $= \cos\phi$. The equation to the induced E.M.F. per phase can be approximately determined from the vector diagram of Figure 209, where OA, AB, BC, and OC have the same meaning as in Figure 207, and AA′ and BB′ are drawn perpendicular and AH parallel to OC. The phase angle between OA and OC will be very small, and hence can be neglected. The induced $E.M.F_i$ per phase is

$$E_i = OA' + A'B' + B'C$$
$$= OA + AB \cos HAB + BC \sin B'BC$$

Now, $\cos HAB = \cos\phi$

and $\sin B'BC = \sin\phi$

Hence the induced E.M.F. per phase is approximately expressed by

$$E_i = E + I_a r_a \cos\phi + e_r \sin\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

CHAPTER IX

ALTERNATORS:—SYNCHRONOUS IMPEDANCE—REGULATION TESTS—COMPOUNDING AND SUDDEN SHORT-CIRCUITS

Synchronous Impedance.—In the vector diagram of Figure 207 the E.M.F. due to armature self-induction has been considered as combining with the induced E.M.F. in the proper phase relation. Strictly speaking, they do not in reality combine, but their respective magnetic fluxes combine in the armature core. The component E.M.Fs. are therefore imaginary, though their resultant is real.

Referring to Figure 210, let the vectors OE and OI represent the terminal E.M.F. and armature current respectively, the current vector lagging behind OE by an angle ϕ. The E.M.F. of self-induction is in quadrature with the current, and is represented by Oe_x. With a lagging current the reactance voltage will be more than 90 degrees behind the vector of terminal voltage, and has therefore a component Oe tending to diminish the terminal pressure. With a leading current the component of the leakage E.M.F. in phase with the induced voltage acts in the same direction as the latter, and therefore increases the terminal voltage. The E.M.F. of self-induction has therefore the same effect as the "true armature reaction," and for this reason they may be treated collectively and referred to as the *synchronous reactance* of the armature. Since this term includes the effect of the armature demagnetising M.M.F., it is not a true reactance in the ordinary meaning of the word, but merely an equivalent reactance. If the synchronous reactance be denoted by r_x, then when it is combined with the resistance r_a of the armature the expression $\sqrt{r_a^2 + r_x^2}$ is obtained. This expression is termed the "synchronous impedance," and will be denoted by z_a, *i.e.* $z_a = \sqrt{r_a^2 + r_x^2}$. In general the resistance r_a will be small compared with r_x, so that for practical purposes the synchronous impedance may be taken as equivalent to the synchronous reactance.

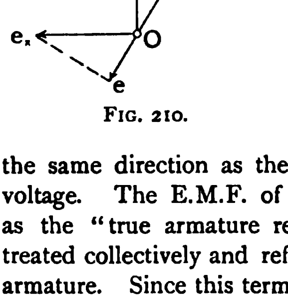


FIG. 210.

When the components r_a and r_x are known, then for any power factor $\cos\phi$ the terminal P.D. in terms of the armature current I is derived as follows. Referring to Figure 211, draw from O the vectors OR and OL at right angles to represent the quantities Ir_a and Ir_x respectively. Their vector sum OZ then represents the voltage consumed in the synchronous impedance of the armature. For an inductive load the terminal voltage will be in advance of the current, and is represented by OE, ϕ degrees in advance of the current vector OI. Since the magnitude of OE is equal to the vector difference of the normal induced voltage OE_i (obtained from the no-load saturation curve) and the E.M.F., OZ, consumed by the impedance, the point E_i is determined by constructing the triangle OEE_i, in which EE_i is drawn equal and parallel to OZ. An inspection of the diagram shows that

$$OE_i^2 = OE^2 + EE_i^2 + 2\ OE\ .\ EE_i \cos ZOE$$

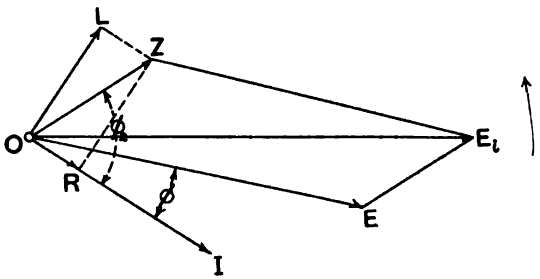


FIG. 211.

Hence the terminal voltage E is expressed by

$$OE^2 = OE_i^2 - EE_i^2 - 2\ OE\ .\ EE_i \cos ZOE$$

If the angle ROZ, which is constant and determined solely by the constants of the armature and not by the load, be denoted by ϕ_2, then the angle $ZOE = \phi_2 - \phi$, and

$$OE^2 + 2\ OE\ .\ EE_i \cos(\phi_2 - \phi) = OE_i^2 - EE_i^2$$

$$i.e.\ E^2 + 2\ EIz_a \cos(\phi_2 - \phi) = E_i^2 - I^2 z_a^2$$

Adding $Iz_a \cos(\phi_2 - \phi)$ to both sides of the equation

$$E + Iz_a \cos(\phi_2 - \phi) = \sqrt{E_{i2} + I^2 z_a^2 + I^2 z_a^2 \cos^2(\phi_2 - \phi)};$$

that is,

$$\begin{aligned} E &= \sqrt{E_i^2 - I^2 z_a^2 + I^2 z_a^2 \cos^2(\phi_2 - \phi)} - Iz_a \cos(\phi_2 - \phi) \\ &= \sqrt{E_i^2 + I^2 z_a^2\{\cos^2(\phi_2 - \phi) - 1\}} - Iz_a \cos(\phi_2 - \phi) \\ &= \sqrt{E_i^2 + I^2 z_a^2 \sin^2(\phi_2 - \phi)} - Iz_a \cos(\phi_2 - \phi) \\ &= \sqrt{E_i^2 + I^2 z_a^2 (\sin\phi_2\ .\ \cos\phi - \cos\phi_2 \sin\phi)^2} \\ &\qquad - Iz_a(\cos\phi_2 \cos\phi + \sin\phi_2 \sin\phi) \end{aligned}$$

Now, $\sin \phi_2 = \frac{r_x}{z_a}$ and $\cos \phi_2 = \frac{r_a}{z_a}$; hence

$$E = \sqrt{E_t^2 - I^2 (r_x \cos \phi - r_a \sin \phi)^2} - I (r_a \cos \phi + r_x \sin \phi) \quad \ldots (78)$$

In large alternators r_a is small compared with z_a, so that the terminal voltage E for any value of current is approximately expressed by

$$E = \sqrt{E_t^2 - (I r_x \cos \phi)^2} - I r_x \sin \phi$$
$$= \sqrt{E_t^2 - (I z_a \cos \phi^2)} - I z_a \sin \phi$$

Regulation Curves.—The regulation curve of an alternator shows for any particular power factor the variation of terminal voltage as the current output is increased from zero, the field excitation

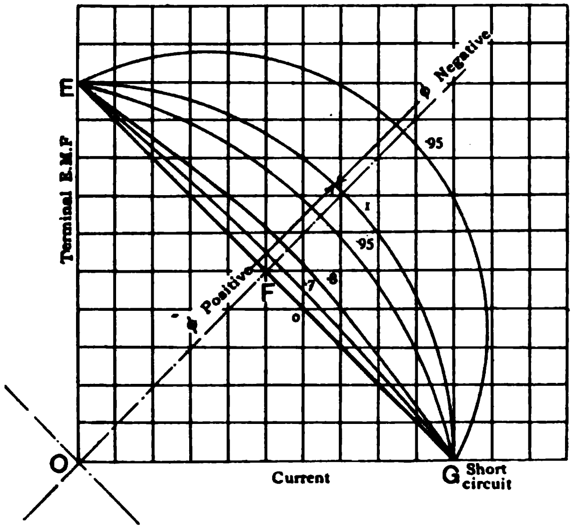


FIG. 212.—Load curves of an alternator.

remaining constant. If E_t denote the open circuit E.M.F. at normal excitation, then the terminal P.D. for various values of current I can be determined from equation 78. This equation represents a family of ellipses with the variable parameter ϕ, the centre of the ellipse being at the origin O (Figure 212). When $\phi = 90$ degrees,—*i.e.* when the power factor is zero

$$E = E_t - I r_x$$

and the regulation curve will be a straight line, as shown by EFG. The regulation curves for other power factors have also been plotted, from which it will be seen that the terminal P.D. falls off much more rapidly on inductive loads (ϕ being positive) than on non-inductive load ($\cos \phi = 1$), and may increase when the current leads with respect

to the terminal voltage. At low power-factors the curves become more nearly straight lines and lie closer together, thus showing that for inductive loads of low power-factors the regulation is almost equally bad, but that for power factors in the neighbourhood of unity the terminal P.D. is much more sensitive to changes in the power factor of the load. The reason for this is that the demagnetising action of the armature M.M.F. increases approximately in proportion to sin ϕ. The rate of increase of sin ϕ is rapid when ϕ is small, and decreases as sin ϕ approaches its maximum value of unity. Thus a given increase in the value of ϕ affects the regulation to a greater extent when the actual angle of lag is small.

The theoretical curves Figure 212 show the regulation from open- to short-circuit with constant excitation. In practice, however, only a small portion of these curves can be obtained from modern machines, as it is undesirable to short-circuit them when fully excited. In the early types of alternators this might be done without a very excessive short-circuit current being obtained, owing to the high value of the synchronous reactance; whereas in machines of modern design the full-load current is obtained on short-circuiting the armature with the field only excited to about 30 per cent. of the full-load value.

Experimental Determination of Short-circuit Characteristic and Synchronous Impedance.—When the effective armature resistance r_a and the synchronous reactance r_x is known, then the various points on the regulation curve for a load of known power factor cos ϕ can be calculated from equation 78. To obtain a value of the synchronous impedance, and from hence the synchronous reactance, for any field excitation, there must first be determined (1) the no-load saturation curve and (2) the short-circuit characteristic.

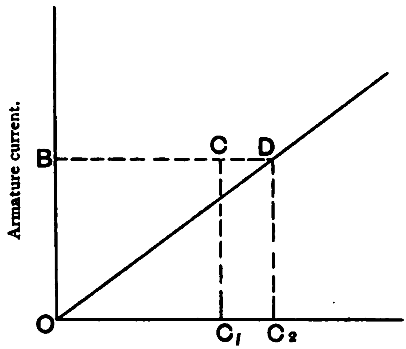


FIG. 213.—Short-circuit characteristic.

The short-circuit characteristic of an alternator is a curve connecting the field excitation with the corresponding values of current obtained on short-circuiting the armature through a non-inductive ammeter, the machine being driven at approximately constant speed. In the case of a polyphase alternator all the phases are short-circuited and an ammeter placed in each phase, the mean reading of the three ammeters being taken as the short-circuit current. The short-circuit characteristic (see Figure 213) will as a rule be a straight line for excitations up to about 70 per cent. of

the full-load value, and if there be no residual magnetism the curve will pass through zero. In order to get rid of residual magnetism as much as possible, the machine should be run short-circuited and with no field excitation for some time prior to making observations.

When a short-circuit current traverses the armature winding it produces a demagnetising effect, and an equivalent number of ampères must be sent through the field coils to compensate for it. These are, of course, proportional to the value of the short-circuit current, and in Figure 213 for an armature current = OB, the field current to compensate for the demagnetising action is represented by OC_1. If the resistance be considered negligible, then the voltage necessary to maintain the short-circuit current OB will be that which must be generated to overcome the armature self-induced E.M.F. From the open-circuit saturation curve the excitation (= CD) necessary to produce this voltage can be found, and when CD is added to the abscissa BC the point D will fall on the short-circuit curve.

Owing to the reduced permeability of the armature teeth at high inductions the inductance decreases with increasing excitation, and hence the voltage required for the current is less than proportional to the short-circuit current. This would mean that the curve would bend up away from the abscissa reference axis if the no-load saturation curve were a straight line; but the voltage increases less rapidly than excitation, so that the combined effect of saturation is to keep the short-circuit curve more nearly a straight line. Whether the curve finally bends up or down will depend upon the relative saturation of the teeth and other parts of the iron circuit.

In carrying out the short-circuit test it is not necessary to maintain the speed exactly constant; for since the impedance of the armature is nearly all due to the reactance r_x, it will be proportional to the frequency. Any change in the induced E.M.F. due to change of speed is compensated for by an equivalent increase or decrease in the reactance. The short-circuit current, when the machine is driven at approximately normal speed, thus remains constant for considerable speed variations.

The open-circuit and short-circuit curves for a 625-K.V.A., 5300-volt, 25 ~, 3-phase alternator having a full-load current of 68 ampères have been plotted in Figure 214. The synchronous impedance at any excitation is obtained by dividing the corresponding value of the open-circuit E.M.F. by the short-circuit current produced by the same excitation. The synchronous impedance obtained in this manner is shown by the curve so marked, from which it will be observed that it is not a constant value but decreases as the excitation is increased, due to the saturation of the magnetic circuit. In order to produce as nearly as may be the full-load excitation, and to obtain as high a degree of saturation in the field magnet system as possible, it

is important to take the short-circuit current as high as the heating of the winding permits. If it is found impossible to obtain points on

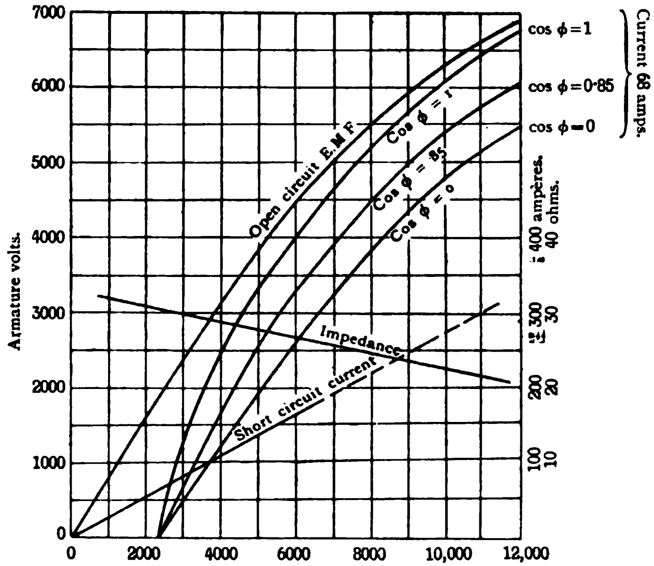


FIG. 214.—Regulation test.

the curve right up to the full-load excitation, these must be extrapolated, as shown by the dotted part of the short-circuit curve in Figure 214.

To predetermine the Regulation from the Short-circuit Characteristic

The regulation and load-saturation curves of an alternator may be determined by direct experiment if a suitable load and a sufficient amount of power is available to carry out the test. In the case of large alternators the power necessary for testing them may not be available until they are permanently erected and ready for service. Therefore it is necessary to have a method which enables the terminal voltage to be determined for any current and power factor. An account will here be given of the more important methods which have been proposed for predetermining the regulation of alternators.

Behn-Eschenburg's E.M.F. Method.—This method is based upon the fact that when the open-circuit saturation curve and the short-circuit characteristic are known the synchronous impedance can be computed in the manner described above. The synchronous impedance for various excitations having been determined, the full-load

saturation curve at zero power factor—*i.e.* $\phi = 90$, may be obtained by multiplying the value of the synchronous impedance at various excitations by the full-load current and subtracting the resultant voltage from the corresponding volts on the open-circuit characteristic. In Figure 214 the full-load characteristic for $\cos\phi = 0$ has been determined in this manner, the data necessary to plot the curves having been calculated as set forth in Table XIX.

TABLE XIX.—CALCULATION OF REGULATION OF THREE-PHASE ALTERNATOR

625 K.V.A. 5300 volts. 25 ∿ 150 R.P.M.

Open-circuit saturation curve } see Figure 214.
Short-circuit characteristic }
Full-load current = 68 ampères.

Field ampère-turns.	Impedance.				Terminal Volts for $I_a = 68$.		
	Short-circuit Current I_o	Open-circuit Volts E_t	Impedance $Z = \frac{E_t}{I_o}$	Voltage Drop due to Impedance $e_z = I_a z_a$	$\cos\phi = 0$ $E_t - e_z$	$\cos\phi = 0.85$ Equation 78.	$\cos\phi = 1$ Equation 78.
3000	80	2400	30.0	2040	360	500	1300
5000	135	3850	28.5	1950	1900	2640	3340
7000	190	5000	26.4	1800	3200	3810	4670
9000	250	6000	24.0	1620	4380	5000	5760
11000	300	6650	22.2	1500	5150	5700	6450

To determine the load curve for any other power factor between $\cos\phi = 0$ and $\cos\phi = 1$, the synchronous impedance z_a must be resolved into its two components. The effective resistance r_a can be readily measured experimentally, in which case the synchronous reactance r_x is given by the equation

$$r_x = \sqrt{z_a^2 - r_a^2}$$

When the components r_x and r_a have been thus determined, then for any power factor $\cos\phi$, the terminal volts may be calculated from equation 78. In Figure 214 there have been drawn the full-load saturation curves for power factors of unity and 0.85.

The objection to this method is that the value of the synchronous reactance, obtained from the short-circuit and open-circuit characteristic comes out too large. This is due to the fact that in determining the short-circuit curve, the machine is working with a much under excited field, or with a very excessive armature current, both of which circumstances would give rise to a greater armature reaction than would occur under normal conditions. Further, since the short-circuit

current is practically a wattless lagging current, its maximum value will occur when the armature coils are in such a position relative to the poles that the demagnetising ampère-turns are most effective. (see Figure 199). This causes the armature reaction to be still further exaggerated. Alternators working under normal conditions of load are therefore found to give better regulation than would be predicted from the experimental value of the synchronous impedance.

Rothert's Ampère-turn Method.—By the previous method the regulation is determined by the vectorial summation of voltages: a second method, which consists in combining vectorially M.M.Fs. or ampère-turns, will now be examined. This method was originally proposed by Rothert. It has undoubtedly the advantage of investigating more closely the existing conditions in an alternator, and has been used to a considerable extent by designers.

As in the E.M.F. method, the open- and short-circuit characteristics of the machine are supposed to be known, and it is then necessary to consider the magnetic flux which exists when the alternator is short-circuited with its full-load armature current flowing. The "real" E.M.F. induced in the armature under these conditions is simply that required to overcome the ohmic drop, and may be represented by a vector of direction OR (Figure 215), the flux inducing this voltage being represented by a vector OY in quadrature with it. Further, the short-circuit current produces the flux of leakage inductance, which may therefore be represented by OX in phase with OR. The resultant OZ of the fluxes OX and OY gives the flux which exists in the armature under these conditions. This actually existing flux is due to the resultant of the M.M.Fs. acting, namely, the field ampère-turns and the armature ampère-turns. The following diagram of ampère-turns can therefore be constructed:—

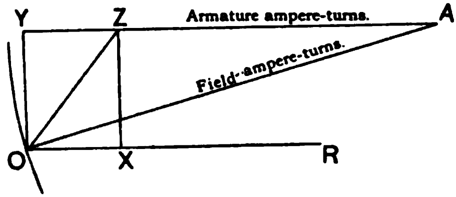


FIG. 215.

From the open-circuit characteristic there is found the ampère-turns to produce the flux OZ (the speed and number of armature turns being known); and, therefore, by altering the scale of the diagram the vector OZ may be taken to represent these ampère-turns. From Z is then drawn the vector ZA parallel to OX and equal in magnitude to the full-load ampère-turns of the armature. The closing side OA of the triangle represents the field ampère-turns when full-load short-circuit current is flowing. Conversely, if the leakage flux is not known in the first instance, AZ representing the armature ampère-turns may be drawn, and a circle, with radius AO equal to the corresponding number

of field ampère-turns, described about A as centre. The right-angled triangle OYA may then be constructed, the length OY equal to the ampère-turns for the resistance drop being derived from the no-load saturation curve. As a rule, the resistance drop will be very small, so that the vector OY is small and OA will be approximately equal to YA. Hence on short-circuit the sum of the armature ampère-turns and those corresponding to the leakage inductance are very nearly equal to the field ampère-turns. The armature ampère-turns may thus be obtained from the short-circuit characteristic.

When an alternator is supplying current to an external circuit at a definite terminal P.D., a similar combination of M.M.Fs. may be made by considering the field ampère-turns to be employed—

(1) In driving the current through the armature itself, and

(2) In maintaining the given terminal pressure.

Of course the first is known from the short-circuit test, and the second from the no-load saturation curve of the machine. The vector diagram will then be as in Figure 216, OE is the phase of the normal induced E.M.F., ON, in quadrature with OE, the ampère-turns obtained from the saturation curve, for the flux required to induce it. OC, lagging by the angle ϕ (where $\cos\phi$ is the power factor of the circuit), represents the armature ampère-turns corresponding to full-load current and as found from the short-circuit characteristic. NC, the closing side of the triangle, gives the field ampère-turns required to maintain the terminal P.D. Conversely, to find P.D. for any excitation and load, a line in the direction OE is drawn, and differing in phase from this by the angle ϕ, a second line representing the ampère-turns as found from the short-circuit test. Then with C as centre, and a radius equal to the field ampère-turns, an arc is described cutting OH in N. ON then gives the ampère-turns available for maintaining the terminal pressure, which latter is then found by referring to the open-circuit characteristic.

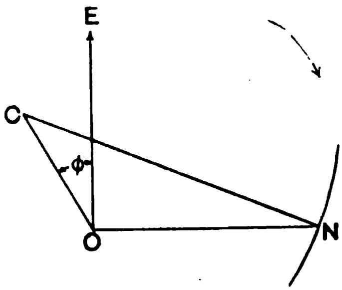


FIG. 216.

It should be noted that this vector diagram is not strictly accurate, as it assumes that the ampère-turns consumed by the armature itself are in phase with the armature current, or in other words, that the vector OA (Figure 215) lies along AY. Since the ampère-turns OY required to overcome the ohmic resistance of the armature are very small, the angle OAY is not more than 10 degrees in modern machines, and hence the neglect of the armature resistance in this method may be corrected for by making the angle EOC (Figure 216)

exceed ϕ by 10 degrees; or, as an alternative, the E.M.F. may be considered as necessarily to be increased by an amount equal to the component of the resistance drop in phase with it. That is, ON may be taken as the ampère-turns required to induce

$$E + Ir_a \cos\phi \text{ volts}$$

When the sides ON and NC of the triangle are known, the regulation, at full-load and power factor $\cos\phi$, may now be computed from the no-load saturation curve. Referring to the saturation curve of Figure 217, let OC represent the excitation corresponding to a normal induced E.M.F., OE. To compensate for the voltage drop EE_1 ($= I r_a \cos\phi$) due to the resistance of the winding, the excitation must be increased to OC_1. Marked off along the horizontal axis is a length OC_2, which represents the ampère-turns at full-load, the value of OC_2 being obtained graphically as in Figure 216. Since OE_2 is the open-circuit E.M.F. corresponding to the excitation OC_2, the rise in terminal P.D. between full-load and no-load is represented by the length EE_2. Further, if from a point F on the saturation curve corresponding to normal no-load excitation, a line be drawn parallel to the horizontal axis to meet C_2A in G, then G will be a point on the full-load saturation curve for the particular power factor for which AT is calculated.

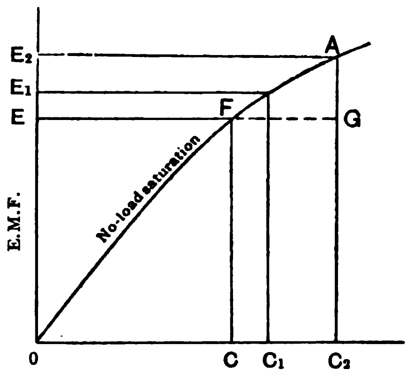


FIG. 217.

This method, like the previous one, is only capable of giving approximate results. The actual flux through the armature is more than that corresponding to ON in virtue of the secondary leakage that must be balanced by the main field, and this leakage has only been taken into account by grouping it with the armature reaction proper. Further, the leakage from the field magnets is that corresponding not to ON but to the total number of ampère-turns CN, and with highly saturated poles the leakage from the magnet will be greater than has been assumed. The field ampère-turns AT_F at full-load will, when calculated by this method, be less than that actually required, although by the selection of suitable correction coefficients the diagram has been widely applied for deducing the regulation of alternators. The correction factors, which take into account, for instance, the increased field and armature leakage at full-load, may be obtained as the result of tests of different types of machines.

Without the use of the arbitrary corrections, the simple vector diagram gives values of the terminal voltage considerably in excess of those that are obtained by a direct test. The discrepancy between the predicted regulation and the actual is more marked with strongly saturated poles than is the case when normal excitation is on the "knee" of the saturation curve. Rothert's method has therefore the opposite effect to that of the open- and short-circuit curve method of Behn-Eschenburg, which, as has already been explained, tends to predict worse regulation than the machine possesses.

Separation of Armature Reaction into Two Components.—As already stated neither of the two previous methods of determining the voltage regulation gives accurate results. Behn-Eschenburg's method is unsatisfactory in that the entire armature reaction is treated as producing an electromotive force of self-induction; whereas Rothert's method is at fault because all the armature reaction is repre-

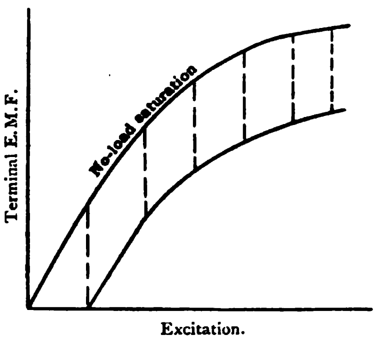


FIG. 218.

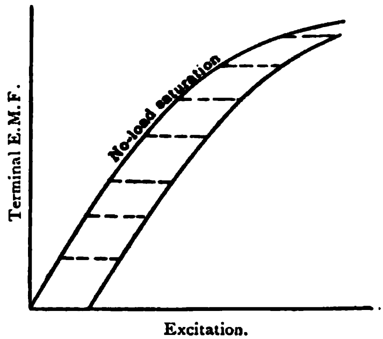


FIG. 219.

sented as having a demagnetising effect. Referring to Figure 218, with the former method the load curve is obtained by shifting the open-circuit characteristic downwards through a distance AB to allow for a constant voltage drop, whereas with the ampère-turn method the no-load curve is shifted a constant distance to the right (Figure 219) to allow for a constant demagnetisation. For an accurate predetermination of the terminal voltage under different loads, especially if the magnetic circuit be strongly saturated, it is necessary to separate the effect of the armature demagnetising ampère-turns from that due to leakage reactance.

Wattless Current Method.—In this method of distinguishing between the two components it is necessary, in addition to the open-circuit characteristic, to have a load saturation curve for approximately zero power factor. This second curve may be obtained by connecting the alternator to one or more choking coils or to an under-excited synchronous motor, so that the current taken by the latter is practically

wattless. The field excitation of the alternator is reduced step by step, and, with the current circulating between the two machines maintained at the full load value I, the corresponding terminal voltages are observed. In Figure 220 the lower curve BD gives the load characteristic for zero power factor, the top curve OA being the characteristic on open-circuit. The point B where the wattless current curve cuts the abscissa reference axis can always be obtained from the short-circuit test.

With an armature current lagging approximately 90 degrees behind the induced E.M.F., the ampère-turns of the armature are nearly all demagnetising; and if T_a denote the armature turns per pole, I_a the armature current, T_f the field turns per pole, then for any value of field current $OC = I_f$ the current which must be sent through the field winding in order to counterbalance the armature demagnetising magnetomotive force $= \frac{I_a T_a}{T_f} = I_h$. If from any point G on the wattless current curve BD there be drawn a horizontal line $GH = I_h$, then the ordinate JK passing through the point H would represent the E.M.F. induced in the armature winding by a magnetic flux corresponding to $(I_f T_f - I_a T_a)$ ampère-turns. The voltage e_x due to leakage reactance is represented by the ordinate HK, and is given by the difference between the induced E.M.F., JK and the terminal voltage JH ($=$CG). e_x having been determined, the self-induction of the armature can be calculated from the equation

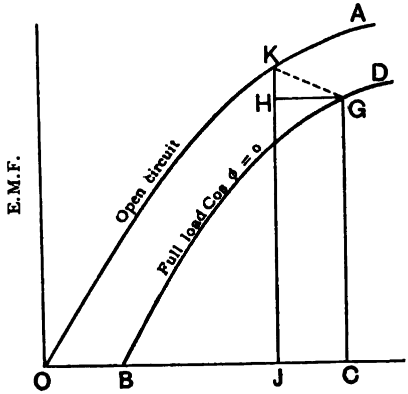


FIG. 220.

$$L_a = \frac{e_x}{2\pi \sim I_a}$$

Since the inductance has not a constant value but varies with the internal phase angle, it follows that this method of analysis can only give the inductance approximately. When the power factor is zero, the amplitude value of the current occurs when the sides of an armature coil are nearly midway between the poles. Consequently the flux of self induction is smaller than would be the case if the maximum value of the current were reached when the conductors are under the pole face. The difference in the inductance in these two positions may be quite considerable.

Kapp's Method.—A great drawback to the above method is that either a synchronous motor or choking coils capable of taking approximately full-load current at from three-quarters to full voltage must be

provided. With the object of avoiding the necessity of providing such special apparatus, and eliminating the inaccuracy of the graphical construction, Professor Kapp * has devised the following test for analysing armature reaction in star-connected 3-phase alternators. The principle of the test is to run the machine on short-circuit and to so regulate the excitation that the current in one particular phase remains the same whilst the number of phases included in the circuit is varied. By varying the current in the other phases the demagnetising M.M.F. of the armature is also varied, whilst at the same time the reactance voltage of the "particular phase" in which the current is maintained constant does not vary. Since the open-circuit E.M.F. is, neglecting the small correction due to armature resistance, the sum of the reactance voltage and the drop produced by the armature demagnetising M.M.F., the two effects can be separated by taking readings under two different conditions.

In Figure 221 the current in phase I is kept the same whilst it is varied in the other two. The neutral point O is connected by switches S_1, S_2, and S_3 with the 3-phase terminals, an ammeter A being inserted in phase I. The machine is run at its rated speed, the switch S_1 closed, and the excitation adjusted so that about full-load current flows through phase I. The excitation is noted, and the corresponding E.M.F. taken from the no-load characteristic. The switches S_2 and S_3 are then closed. This causes short-circuit currents to flow in phases II and III as well as in phase I, and the demagnetising action of the armature on the field is thereby increased so that the ammeter A will now indicate a smaller current than previously. To bring back the current to its initial value it is necessary to increase the excitation and observe the corresponding E.M.F. from the open-circuit curve. These two E.M.F. readings and one current reading are all that are required to separate the two components; but to eliminate errors of observation it is necessary to take several readings for different currents—in fact, to plot the short-circuit characteristics for one phase and for three phases in action. Let AT_1 and AT_3 be the field ampère-turns corresponding to a particular current I_a, when one and when three phases respectively are in action. Also let $AT_{DM} = CI_a$ denote the demagnetising ampère-turns of the armature, C being a constant, and let AT_x represent that excitation which will produce an E.M.F. e_x just sufficient to counterbalance the reactance voltage. Since I_a is the same in every phase the two tests give the following equations:—

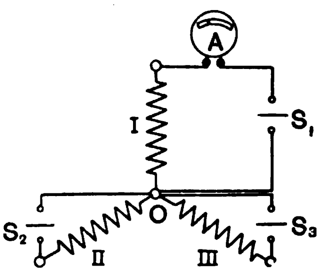


FIG. 221.

* *Inst. of Elect. Engineers* (1909), vol. xlii. p. 703.

$$AT_1 = AT_s + AT_{DM}$$
$$AT_3 = AT_s + 3AT_{DM}$$
$$\text{Hence, } \frac{AT_3 - AT_1}{2} = AT_{DM} = KI_a$$
$$\text{and } AT_x = \frac{3\,AT_1 - AT_3}{2}$$

From the no-load saturation curve there is obtained the value of e_x corresponding to AT_x, and hence the reactance per phase is expressed by

$$2\pi \sim L = \frac{e_x}{I_a}$$

Measurement of Leakage Reactance with Rotor Present. —To obtain what is known as the stationary reactance of an alternator, full-load current at the proper frequency is sent through the armature winding and the field magnets, which are fixed in a definite position, excited to their normal value. The P.D. is then measured at the armature terminals and the reactance calculated from the equation

$$I_a = E/\sqrt{r_a^2 + r_x^2}$$

The voltage E besides including a component to overcome the true armature impedance Z_a has a second component which balances the electromotive force set up by the cross magnetic flux of the armature. When the inductance of an armature is measured with the rotor in position, the value obtained will thus include the inductance due to the cross-magnetic flux Φ_{CM}. Now the cross flux varies for different positions of the poles relative to the coil sides, being a maximum when the poles are directly under the the coil sides (Figure 198) and a minimum when the coil sides are midway between poles (Figure 199). By this method of measurement, the position of maximum inductance will generally occur when the coil sides are approximately under the pole centres, the smallest value occurring when the coil sides are midway between poles. If the inductance be measured with the armature in various positions relative to the poles, a wavy curve such as is shown in Figure 222 will be obtained, the curve repeating itself twice in each alternator period. The exact position where the crest value occurs will depend upon the saturation of the armature teeth, the length of the pole face arc, the material of the pole shoes (whether laminated or solid), and the shape of the slots.

For a given position of the coils relative to the poles, the inductance has not a constant value but varies with the phase angle. If the power factor be zero the maximum or crest value of the current curve occurs when the coil sides are nearly midway between the poles. Consequently, the self-induced flux is smaller than would be the case if the maximum value of the current were reached when the coil sides are opposite the

middle of the poles, that is, when the power factor is unity. When the machine is at work with some such power factor as $\cos\phi = 0.8$, the crest of the current wave will occur at about the moment the coil sides are near the edge of the poles, and consequently the average inductance will lie somewhere between these extreme limits. When the values of

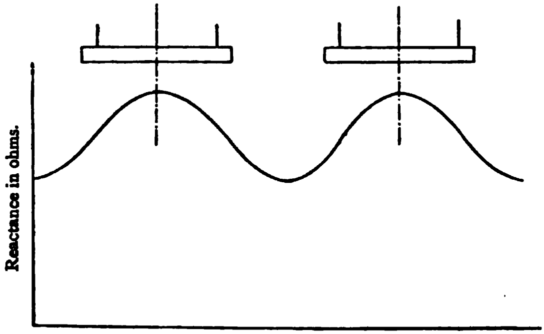


FIG. 222.

r_x have been determined for different pole positions, the average value is generally taken as the self-induction of the winding when the alternator is normally loaded.

Measurement of Leakage Reactance with Rotor Absent. —To measure that component of the armature reaction which is due solely to the armature leakage flux, the magnet wheel should be removed so as to include none of the true cross lines due to the distorting component of the armature M.M.F. When the rotor is removed from the stator and an alternating current of the proper frequency sent through the windings of the latter, the flux interlinked with the stator winding can be divided into four parts—

1. The flux inside the slots.
2. The flux across the tooth heads.
3. The flux associated with the end-connections.
4. The flux passing out of the teeth of one pole, through the bore into the two adjacent poles (*i.e.* along the same path as main flux when the rotor is present).

In the case of alternators, which have, as a rule, comparatively large air-gaps, the fluxes 1, 2, and 3 will not be materially different when the rotor is present from that when it is removed; but the part 4 only exists as a leakage flux when the rotor is absent. If for any value of armature current I_a, and terminal voltage E, the leakage inductance be calculated from the equation

$$\frac{E}{I_a} = \sqrt{r_a^2 + 4\pi^2 \sim^2 L_1^2}$$

then values obtained for L_1 will be in excess of the true value that is required for the determination of the fall of potential. The reason for this is that all the four components are included in the measured value of the leakage when the latter is taken with the rotor removed. Hence, if such a measurement is to be used for ascertaining the leakage flux under actual conditions, it is necessary to be able to find the part 4—which may reach 40 per cent. of the total leakage flux—and deduct it from this measured value. The remainder can then be taken, with very fair approximation, as equal to the actual leakage flux when the rotor is present.

Assuming that the stator winding gives a sinusoidal curve of M.M.F., M. Schenkel has shown * mathematically that the flux which passes across the stator bore from pole to pole when the rotor is removed is expressed by

$$\Phi = 2 \cdot \frac{4\pi}{10} \cdot AC_1 L_g \text{ cgs. lines}$$

where AC_1 = sum of all the ampère conductors inside a half pole pitch.

L_g = gross length of armature core in centimetres.

For the case of a 3-phase winding in which there are q slots per pole per phase, and C_1 conductors per slot, the mean value of the flux in the bore will be

$$\Phi = 2 \cdot \frac{4\pi}{10} \cdot 0.912\, q \cdot C_1 I_a L_g \times 1.5$$
$$= 3.24\, q \cdot C_1 \cdot I_a L_g$$

where I_a = armature current. Since this flux rotates with a speed $R = \frac{60 \sim}{p}$ it will induce in each phase of the stator winding an E.M.F.

$$E_B = 4.24 \sim pqC_1\, \Phi \times 10^{-8} \text{ volts}$$
$$= 13.75 \sim pL_g(qC_1)^2 I_a 10^{-8} \text{ volts}$$

For a star-connected winding

$$E_B = \sqrt{3} \cdot 13.75 \sim p \cdot L_g(qC_1)^2 I_a 10^{-8} \text{ volts}$$

This is the E.M.F. which must be deducted from the voltage (as measured) required to send the current I_a through the stator winding when the rotor is removed. The remainder will then represent with fairly close approximation the leakage voltage corresponding to the current I_a.

Compounding of Alternators

As has been previously shown, the terminal voltage of an alternator, having constant field excitation, will diminish to a greater or less extent with increasing load, so that to obtain approximately constant terminal pressure at all normal loads the exciting current must be regulated

* *Electrotechnik und Maschinenbau* (Feb. 28, 1909), No. 27, pp. 201–208.

either by hand or by some form of automatic gear, such as the Tirrill regulator. These methods of regulation are in some cases not the most desirable, as a certain time must elapse between the alteration in the terminal voltage and the adjustment of the excitation. An alternator to regulate well, when subjected to rapidly fluctuating loads, must respond instantaneously to the change of load, and to obtain this it is necessary to provide a compensating field winding analogous to the series winding of a compounded direct current dynamo. In addition to the constant excitation provided by the ordinary direct current winding on the poles, there must therefore be another source of excitation which varies not only with the magnitude, but also with the power factor, of the load. Since the early days of alternator construction many attempts have been made, with more or less success, at a solution of this problem, but only a limited number have been commercially successful. According to the principles involved the more important methods of compounding alternators are—

1. By operating upon the exciter with an alternating current so as to increase the voltage of excitation.

2. By maintaining a constant field excitation and utilising the armature M.M.F. to raise the induced voltage.

3. By supplying a rectified current to the main, or a second, field winding.

(1) Operating upon the Exciter with an Alternating Current. —Figure 223 illustrates for a 3-phase revolving field type alternator a

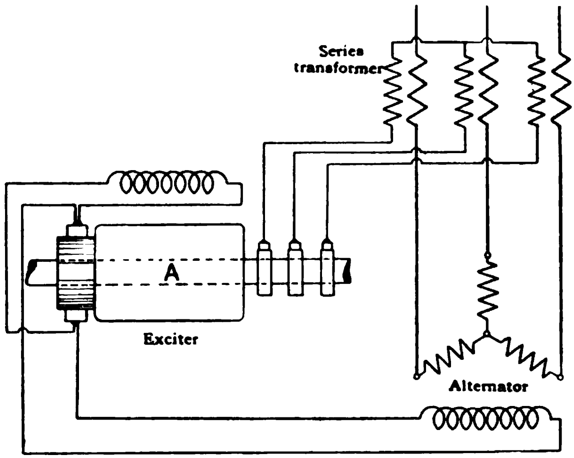


FIG. 223.—Rice's compensated exciter.

compounding arrangement, devised by E. W. Rice,* which has been adopted by the American General Electric Company. The device,

* *Electrical World*, April 27, 1907.

sometimes referred to as a "compensated exciter," utilises the effect which armature reaction produces upon the resultant field strength of the exciter. Mounted on the same shaft as the alternator field system is the armature A of a direct current exciter, the field system of which is provided with the same number of poles as the revolving field poles of the alternator. The armature of the exciter is, at the end remote from the commutator, connected to three slip rings in the manner of a 3-phase converter, the field of the exciter being shunt excited from the direct current or commutator side. The main current of the alternator is led through a 3-phase series transformer, the secondaries of which are connected to the exciter armature through the slip rings. A 3-phase current proportional to the load and in phase with the alternator current is thus led into the armature of the exciter, and in consequence of the synchronous rotation of the exciter armature, produces a magnetic effect which is stationary in space and therefore also in respect to the fixed field poles of the exciter. The effect of this reaction depends upon the magnitude and phase of the current producing it, and the latter, as is the case with armature reaction in synchronous motors, strengthens the field if the current is lagging, weakens it if leading, and merely produces a cross magnetising effect if in phase with the normal induced E.M.F. Hence, if the field winding of the alternator be connected across the brushes of the exciter, its excitation will automatically increase as the load in the main circuit increases, and also as the main load becomes more inductive, conditions which are essential for maintaining a constant terminal voltage. The extent of the "compounding action" depends upon the relative position of the synchronous field of the exciter armature and the field poles, and in order that this may be adjusted, if desired, the exciter field system can be fitted into a circular seating cast with the bed plate, and rotated through an angle in space by suitable gearing. This allows the amount of compounding for a load of any given power factor to be adjusted. To get over the difficulty of a very large exciter with as many poles as the alternator, the former may be driven from the main shaft through spur gearing and by the adoption of a suitable gear ratio it will be possible to use a 4-pole exciter.

The compounding arrangement now adopted by Parsons & Co. for their turbo-alternators is another example coming under this heading, and is shown diagrammatically in Figure 224. The magnetic circuit of the exciter, generally of the 2-pole type, is constructed so as to present to the flux generated in the field poles an alternative path F of laminated iron which is in parallel with the main path through the armature. This leakage path carries a winding which is supplied with current from the secondary of a current transformer. The primary of the latter is excited by the main current of the alternator. The alternating current in the leakage winding will therefore vary with the current output from

the alternator, and as the load increases the M.M.F. throttles the lines of force, thereby increasing the reluctance of the leakage paths. More flux will go through the armature, thus raising the voltage of the exciter and compensating for the tendency of the main voltage to drop. In

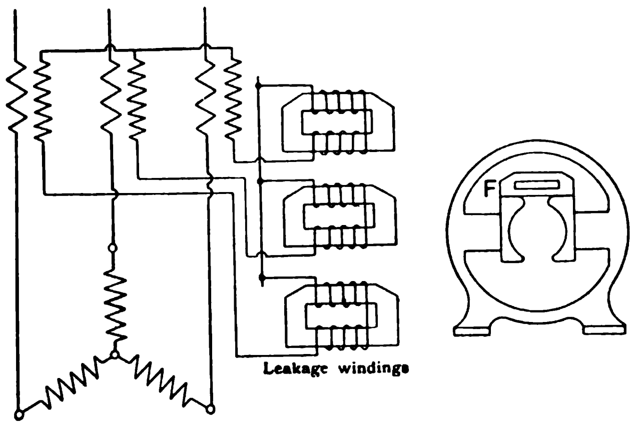


FIG. 224.

a 3-phase alternator there is generally one leakage path in each phase, as indicated in the figure. The leakage paths can be adjusted for any desired amount of compounding or for any power factor. When adjusted the compounding will however only be correct for one particular power factor.

(2) **Utilising the Armature Magnetomotive Force.**—Figure 225 illustrates an alternator field system designed so as to utilise armature reaction for strengthening the field on loads of moderately high power factors. This method is due to Miles Walker, and has been worked commercially by the British Westinghouse Company. Each pole of the alternator consists of two parts, one of which is saturated while the other is unsaturated. The saturated part carries the ordinary field winding, and there may or may not be another magnetising coil wound round the whole pole. This

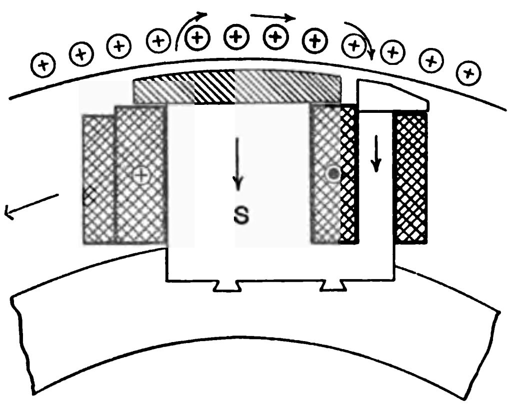


FIG. 225.—Walker's compensated winding.

secondary winding may be necessary for varying the normal voltage of the machine. In Figure 225, the broad pole is very highly saturated in the region marked by the shaded lines, whilst the narrow pole on the right is unsaturated, and at no-load carries no flux. If the field system rotates in an anti-clockwise direction then, supposing the main pole under consideration to be of S polarity, the direction of the E.M.F. induced in the conductors directly above the saturated part of the pole will be away from the observer, as indicated by the crossed circles. Any current in the armature winding which is in phase, or nearly in phase, with this electromotive force will set up a cross M.M.F. which tends to demagnetise the saturated part of the pole and magnetise the unsaturated part as shown by the arrows. The flux in the saturated part cannot be altered to any appreciable extent, whilst the unsaturated part becomes highly magnetised by the cross-magnetising component of the armature M.M.F., thus increasing the E.M.F. induced in the armature winding. When the armature current lags behind the induced voltage, the wattless component $= I_a \sin \phi_1$ will tend to demagnetise the unsaturated pole, while the power component $= I_a \cos \phi_1$ tends to magnetise it, so that should the internal phase angle ϕ_1 exceed a certain limit the M.M.F. due to the current $I_a \sin \phi_1$ will be greater than that due to $I_a \cos \phi_1$, with the result that the auxiliary pole becomes magnetised in a negative direction. The voltage drop will then be greater than if no auxiliary pole were present. This method of compounding can only be effective where the power factor is not less than 0.85. With very large lagging currents decompounding will result, so that this method is quite unsuited for generators subjected to heavy loads at low power factors. In the majority of generating stations the power factor at heavy loads is above 0.85, and at that power factor the magnetising effect is stronger than the demagnetising, so that when the load comes on the voltage rises. Owing to the field winding being embedded in slots, this method of compensation readily adapts itself to turbo-alternators of the cylindrical field type, and the Westinghouse Company have constructed several units up to 4000-K.V.A. which embody these principles.

(3) **Rectifying the Exciting Current.**—The rectification of an alternating current into a direct current has, up to the present, proved the most successful method by which the compounding of alternators can be affected. The generator may then be made either self-exciting, or, if given an initial excitation from a source of direct current, can be compounded in a manner analogous to that adopted in direct current machines. When the latter practice is adopted, the field poles must be wound with two circuits, one of which is separately excited whilst the other is self-excited by the main current. Referring to Figure 226, which shows the principle as applied to a single-phase 4-pole alternator, the rectifying commutator C mounted on the alternator shaft is con-

structed with as many segments as there are poles, alternate segments being in electrical contact with each other. The compound winding D—or the whole field winding in the case of a self-excited machine—is connected between two adjacent segments, and by means of brushes B_1 B_2 a single-phase current is led into the synchronously rotating commutator. The circuit between the two brushes is completed through the winding D, and since the connections between the latter and the brushes are reversed at each reversal of the alternating current, the compounding coils will be traversed by a unidirected but pulsating current. By having brushes sufficiently broad to bridge over the insulation between two segments, the field winding will be short-circuited just before the reversal of the current takes place, and the pulsations in the rectified current will thereby be considerably damped by the comparatively large inductance of the field coils. In order to obtain sparkless rectification, the connections between the brushes and the windings require to be reversed at the instant of zero current, so that the brushes must be very accurately set. Since the commutator rotates in synchronism with the field system, it is evident that the position of zero current will alter with the power factor of the load, and unless the brush position be adjusted for each change of power factor serious sparking will result. Owing to the difficulty of obtaining sparkless working, especially where large currents are concerned, this arrangement has now become almost absolete.

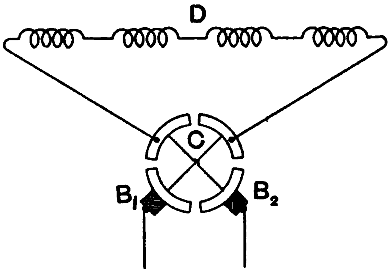


FIG. 226.

Though unsuited for single-phase currents, the above principles have been very successfully applied by A. Heyland to the self-excitation and compounding of 3-phase alternators. By employing more phases and dividing the field winding into several parallel sections, the pulsations of the exciting current can be considerably diminished, while the tendency to spark is almost eliminated by using a commutator with several segments per pole per phase. Figure 227 illustrates Heyland's arrangement as applied to a 6-pole 3-phase self-exciting alternator. The field poles, of the same construction as for an ordinary alternator, are wound with a four-circuit winding, the ends of each circuit terminating in a commutator segment as shown. In order to facilitate the damping out of the current pulsations, the four parallel circuits of the magnet winding should have the greatest possible mutual induction, and for this reason four circuits pass round each pole.

The commutator, which is fixed to the shaft of the generator and connected through brushes to the source of 3-phase current, has

six segments per pole, of which only four are in contact with the field winding. Between each group of four active segments are two dummies, which have no connection with the windings except through the brushes. The current carrying segments, which are spaced from each other by a double pole pitch, are always at the same potential, and may on that account be interconnected at the commutator by internal cross-connections as shown in the diagram. It is then possible to work with one brush per phase. The four sections of the field winding are electrically connected with each other at several points, these connections serving to equalise the currents and voltages which arise in the rectification of the current.

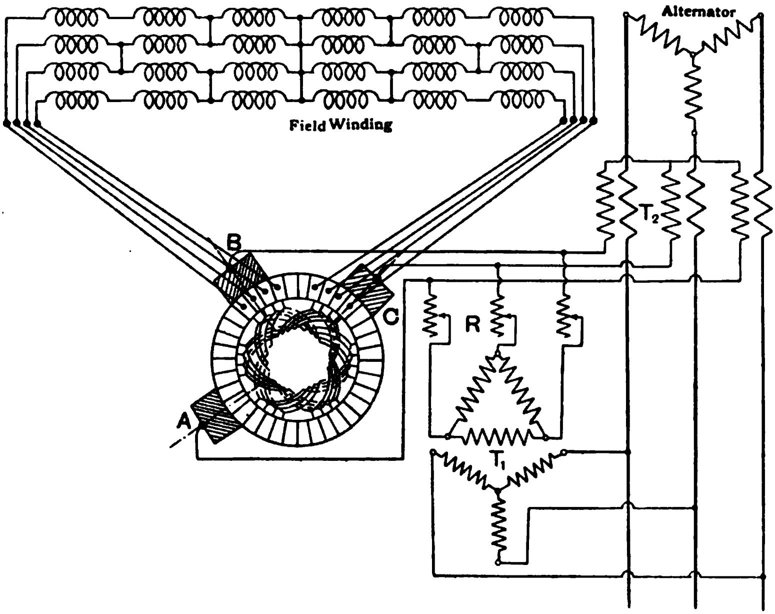


FIG. 227.

On the commutator are three brushes A, B, and C, which just cover three segments and are displaced relative to each other by 240 electrical degrees, or for the case considered by $240/3 = 80$ degrees in space. Since the commutator rotates synchronously with the voltage applied to the brushes, each individual segment has a definite mean voltage. Assuming a sine curve of applied E.M.F., then if these mean voltages for a *2-pole arrangement* be plotted as a function of the commutator circumference, an almost sinusoidal wave of E.M.F. is obtained (see Figure 228), which remains stationary relative to the commutator but rotates relative to the poles. Between segments 2 and 8 there is a mean potential difference $= E + E^1$, and this tends to send current through

that section of the field winding connected across these segments. The three brushes must, in order not to produce short-circuits among themselves, be separated by at least the width of a segment, so that for the instant shown only the circuits *a* and *c* are in direct connection with the brushes. The coils *b* and *d* are not without current, for because of the cross-connections between the various circuits of the windings these coils are in electrical connection with the same segments as *a* and *c*, and will therefore have currents flowing in their middle parts. When the commutator becomes displaced by one segment width from that shown in Figure 228, coils *b* and *d* come into direct, and *a* and *c* into indirect, connection with the current carrying segments, the outer parts of the latter circuits being without current. It will thus be seen that the current in the field coils is still of a pulsating nature, but owing to the large mutual inductance of the parallel circuits, the pulsations are so strongly damped that the resultant mean field current will affect the excitation in almost the same way as a direct current. The tendency to spark is obviated by the fact that there are always several paths through the field winding open to the current. The position of the brushes is determined, so that when the machine is running unloaded and normally excited, the middle point of the four segments connected with the field winding coincides with the amplitude value of the potential curve (see Figure 228). This will occur at that brush position for which the no-load voltage attains its maximum.

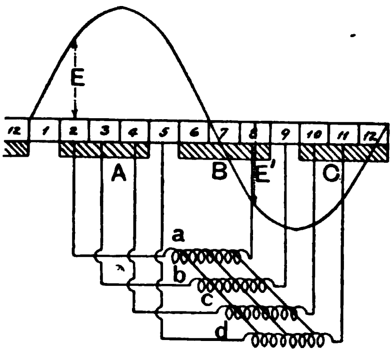


FIG. 228.

The 3-phase current supplied to the commutator is, with Heyland's arrangement, obtained partly from a potential transformer T_1 (Figure 227) connected in parallel with the alternator and partly from a current transformer T_2 which is in series with the mains. The potential transformer T_1 supplies a nearly constant exciting current whose phase is fixed by the voltage of the generator, whilst the current transformer T_2 supplies the compounding current proportional to and in phase with the armature current. In order to use the same set of brushes for the compounding and for the exciting current, the secondary of T_1 must be mesh-connected (see p. 138), so as to cause the exciting current to be a quarter of a period behind the compounding current supplied by T_2. The currents from T_1 and T_2 combine, as shown in the vector diagram of Figure 229, where OI_1 represents the current delivered by T_1 and OI_2 that delivered by T_2.

The left-hand diagram represents the condition at non-inductive load, in which case I_1 and I_2 are in quadrature, and the total current supplied to the rectifier is the resultant OI′. The line OP marks the position of OI_2 at non-inductive load. For an inductive load of power factor cos ϕ, the current vector OI_2 will make an angle ϕ with OP, so that the resultant current will be increased to a value represented by OI″. The compounding action by this method therefore increases with the power factor as well as with the load. In order to regulate the amount of compounding or over-compounding of a machine, adjustable resistances R are inserted in series with the transformer T_1.

Heyland has proposed a modification of the above method which can be employed for compounding machines of standard design, the direct coupled exciter in this case being operated upon with the rectified current. The exciter, in addition to the ordinary shunt winding, is provided with an interconnected multiple circuit winding on its field poles. This is supplied with the rectified current through a series transformer and a commutator in the same way as the main generator field would be supplied if the compound winding were on the main poles.

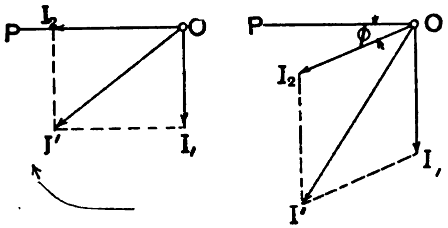


FIG. 229.

SUDDEN SHORT-CIRCUITING OF ALTERNATORS

If the armature of an alternator of ordinary regulating qualities be short-circuited while at rest and then run up to normal speed and fully excited, the armature current will not rise to more than 2.5 or 3 times its full-load value. This is because the current lags about 90 degrees behind the pole centre and so sets up a M.M.F. opposing the main flux. If, however, an alternator be suddenly short-circuited at the armature terminals while running at normal speed and fully excited, the current may rise momentarily to from 10 to 25 times its normal full-load value. This will set up enormous mechanical forces between the windings of the various phases, which in the case of turbo-alternators may cause serious damage to the windings, unless the end-connections are rigidly clamped in the manner mentioned in Chapter VI. The first rush of current is propelled by the full E.M.F. of the alternator, for during the first $\frac{1}{100}$ or $\frac{1}{50}$ part of a second there is no time for the field to become demagnetised. The rate at which the current rises is determined by the self-

induction of the armature winding, and also to a certain extent by the distributed capacity between the winding and the frame. Owing to the importance of this question in connection with large turbo-alternators, a brief study will be made of the essential factors which affect the difference between a *gradual short-circuit* and *a sudden short-circuit.*

When an alternator is gradually short-circuited with such an excitation that the armature current is equal to that at full-load, the field ampère-turns will have a certain value denoted by AT. Under such conditions a small flux Φ_a passes into the armature, which is just sufficient to compensate for the voltage consumed in armature impedance. In the pole and yoke there will be the flux Φ_a together with the leakage flux Φ_l between adjacent poles and due to AT ampère-turns. On short-circuiting the alternator with full-load excitation, $= AT_o$ ampère-turns, the armature current will be approximately $\frac{AT_o}{AT}$ times greater than the full-load current, and since the flux passing from pole to armature will have increased in approximately the same ratio,

$$\Phi_{ao} = \Phi_a \cdot \frac{AT_o}{AT}$$

Let Φ denote the flux per pole entering the armature under normal full-load conditions, then on sudden short-circuit, the flux through the armature, as well as that in the pole, has to change by the amount $\Phi - \Phi_{ao}$. It is this change of flux which is the cause of many of the phenomena to be observed on the occasion of sudden short-circuit. During the time this change is taking place, a larger flux is entering the armature from the field poles than after the end of the change. A higher E.M.F. is therefore induced, and consequently the short-circuit current will exceed the normal.

At the first moment of short-circuit, the E.M.F. is $\frac{\Phi}{\Phi_{ao}}$ times larger than at the normal, so that if the current could rise instantaneously with the voltage, the short-circuit current would be $\frac{\Phi}{\Phi_{ao}}$ times in excess of the normal short-circuit current, I_o. Owing to the self-induction of the stator winding, the current rise will lag almost a fourth of a period behind the E.M.F., so that in reality the flux will have decreased by a certain amount before the current attains its maximum value. In consequence of this the maximum value will be slightly smaller than $\frac{\Phi}{\Phi_{ao}} \cdot I_o$. The rate of increase of current is at

first but little affected by the change of magnetic flux through the main magnetic circuit, because as the current in the armature rises, there are eddy currents induced in the solid parts of the poles which maintain the flux through the armature almost at its full value. It is only as these eddy currents die down that the armature M.M.F. begins to demagnetise the field poles.

Referring to Figure 230, let A represent one phase of a 4-pole alternator, and suppose that this phase is short-circuited at the instant when the poles are in the position shown. As the current rises the increasing M.M.F. of phase A increases the magnetic flux in all such paths as PP. As soon as this flux begins to increase eddy currents are induced in the pole face which flow in the opposite direction to the current in the conductors of phase A, the return path of the eddy currents being along the sides of the poles and back along the face of the S-poles. This current opposes the increase of flux along the paths PP, so that the flux cannot increase at a greater rate than is just sufficient to generate the eddy currents against the opposition of the resistance and the self-induction of their path. The curve in Figure 231 shows the general way in which the current may be expected to rise, the zero line of current depending upon the instant at which short-circuit occurs. By the time that the S pole comes under the conductors A, the eddy currents will have begun to decrease, so that pole S will not be so strongly magnetised as the pole N was. Though the current may diminish under the influence of the S pole it does not always cross the zero line, but rises again under the influence of the next N pole, so falling and rising and describing a wavy line keeps its mean values above the zero line for several alternations.

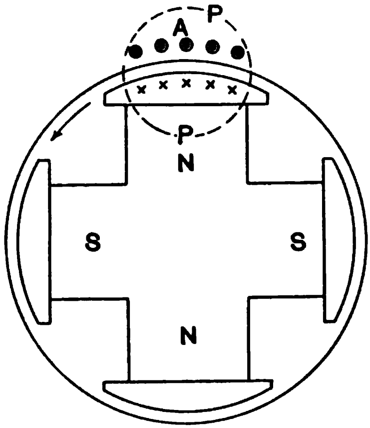


FIG. 230.

The instantaneous values of the current in the armature of a turbo-alternator at the time of short-circuit have been investigated experimentally by Miles Walker,* and Figure 232 shows a record of an oscillogram taken on a 5500-K.V.A. alternator with salient poles. The waves V show the voltage before short-circuit, which in this case has a R.M.S. value of 3900 and an amplitude value of 5700 volts. At the instant of short-circuit the voltage at the stator

* *Journ. of the Inst. of Elect. Engineers* (1910), vol. xliv. p. 295.

terminals fell to zero, and the current curve C sprang into existence. These curves clearly show the general nature of the current on short-circuit: it rises to such a high value during the first half of a period that the pole which passes during the next half period is hardly sufficient to bring it to zero. The current then rises and falls in waves of gradually diminishing amplitude, until, after a lapse of several seconds, it assumes the value it would have had if the short-circuit had been made before the field was excited.

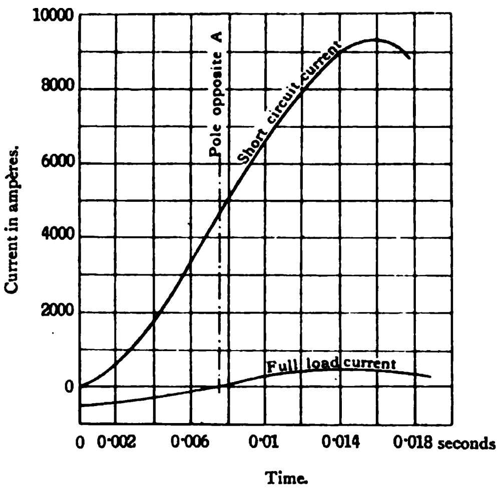


FIG. 231.—Magnitude and phase of short-circuit current as compared with full-load current lagging 90° for a 5500-K.V.A. 33 ~ alternator.

Note.—Zero line of short-circuit current depends on the instant at which short-circuit occurs, *e.g.* for the instant 0·004 draw zeroline through 1800 ampères.

Another important phenomenon attending the sudden short-circuiting of an alternator is the momentary rise in the exciting current to a value 4 or 5 times in excess of that at normal full-load, thus tending to further increase the initial value of the short-circuit current in the armature. Owing to the change in flux passing through the main magnetic circuit, an E.M.F. proportional to the rate of change of the flux $\frac{d\phi}{dt}$ is induced in the field winding, and acts in the same direction as the exciter voltage. During the interval this change is in progress the exciting current will therefore be greater than the normal value.

Let L denote the self-induction, R the ohmic resistance, T the number of turns, e the voltage of excitation, and Φ the normal flux for each pole of the field circuit, then if i denote the instantaneous value of field current and ϕ the instantaneous flux

$$e - T\frac{d\phi}{dt} \cdot 10^{-8} = Ri + L\frac{di}{dt}$$

Further, let i' denote the increase of field current above its normal value, then

$$-T \cdot \frac{d\phi}{dt}\ 10^{-8} = L\frac{di}{dt} + Ri'$$

Since R will always be small compared with L, the term Ri' may be neglected, in which case

$$-T \cdot \frac{d\phi}{dt}\ 10^{-8} = L\frac{di}{dt}$$

$$\text{and } i = \frac{-T}{L}(\phi - \Phi)\ 10^{-8} = \frac{T}{L} \cdot (\Phi - \phi)\ 10^{-8}$$

$\Phi - \phi$ is the change of flux at any given instant and $\frac{T}{L}(\Phi - \phi)\ 10^{-8}$ the increase of the field current at the same instant.

In general, the greater part of the self-induction L will be due to the leakage flux Φ_l between adjacent poles.

Hence, if i_1 denote the field current at the instant previous to short-circuit

$$L \cong \frac{T \cdot \Phi_l}{i_1} \cdot 10^{-8}$$

$$\text{and } i = i_1 \cdot \frac{\Phi - \phi}{\Phi_l}$$

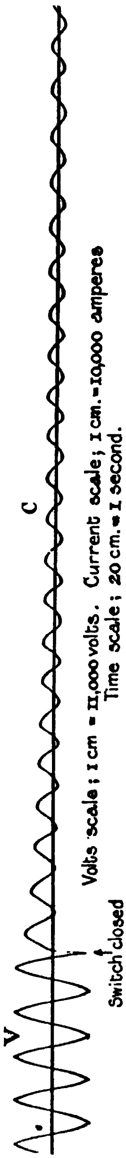
When, for instance, the flux has decreased to 0.2 of its normal value

$$\Phi - \phi = 0.8\ \Phi$$

and at that instant

$$i = i_1 \cdot \frac{0.8\ \Phi}{\Phi_l}$$

For a turbo-alternator Φ_l will be about 16 per cent. of Φ, so that substituting in the equation for i, there is obtained the result that when the



Volts scale; 1 cm = 11,000 volts. Current scale; 1 cm. = 10,000 amperes.
Time scale; 20 cm. = 1 second.
V = voltage measured at terminals of the generator. To get the right phase of voltage in the leg of the star reverse the polarity and subtract 30°

FIG. 232.

flux has fallen to 0.2 of its normal value the rise of field current above normal is

$$i = i_o \frac{0.8\ \Phi}{0.16\ \Phi} = 5\ i_o$$

In order to protect a generator against the effects of a sudden short-circuit, Hobart has proposed to put a quick-acting maximum cut-out in the field circuit. This cut-out could be conveniently arranged so that a resistance is inserted in the field circuit as soon as the field current increases by more than 30 to 50 per cent.

CHAPTER X

ALTERNATORS:—LOSSES—EFFICIENCY AND HEATING

The losses occurring in an alternating current generator may be considered as made up of five parts.

Iron Losses—

(1) Hysteresis in armature core and teeth.

(2) Eddy currents in armature core, armature teeth, pole shoes, etc.

Copper Losses—

(3) Armature I^2R.

(4) Field excitation.

Mechanical Losses—

(5) Bearing friction, windage, and vibration.

Iron Losses

Hysteresis.—When iron is subjected to an alternating magnetic field, it was shown on page 103 that the loss due to hysteresis can be expressed approximately by the equation

$$W_h = \eta \sim B_m^{1.6}\ V\ 10^{-4} \text{ watts}$$

where V = Volume in decimetres^3

B_m = Maximum induction.

η = Hysteresis constant which for armature iron has a value ranging from 0.0025 to 0.003.

The alteration in magnetism which occurs when iron is subjected to an alternating magnetic field differs somewhat from the change that takes place in the armature core of an alternator or direct current dynamo. Consequently, one might expect the relation between W_h and B_m to be different also. The nature of the hysteresis loss in the armature core of an alternator will be investigated by aid of Figure 233. When the field system rotates relative to the armature core, the molecules of iron forming the latter tend to take up such a position that their magnetic axis will always lie in a direction parallel to the magnetic flux. Considering a particular molecule which is situated in a plane near to the air-gap periphery, then during the movement of

the poles through a double pole pitch the magnetic axis of the molecule will take up successively the positions shown at a_1, a_2, a_3, and a_4, Hence, as the armature core moves relative to the poles, each molecule of iron in the body of the core rotates through 360 degrees in each period. The change in magnetism which occurs in the core of an

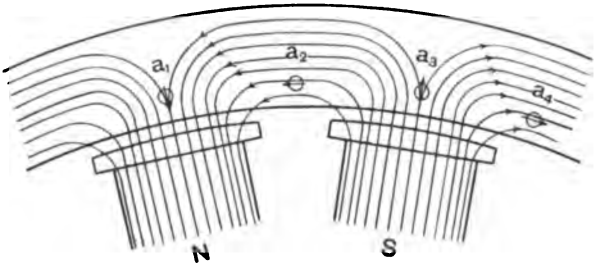


FIG. 233.—Rotating magnetic field in armature core.

armature is therefore termed a rotating magnetic field as distinguished from an alternating magnetic field, which is produced by a periodic reversal of the magnetic field. If the alternator has p pairs of poles, then during one revolution of the field system each molecule of iron will make p periodic movements, *i.e.* the number of magnetic cycles per second is the same as the machine frequency.

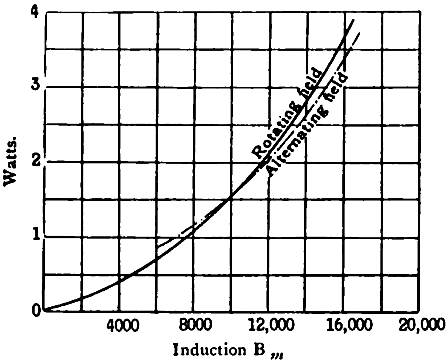


FIG. 234.—Hysteresis loss in alternating current field and rotating field.

Rotating field ——

Alternating field - — - —

For inductions below $B_m \cong 13000$ several observers have found that the rotating field causes a somewhat greater hysteresis loss than the corresponding alternating field. For higher inductions the reverse is the case, and when the reversals are produced by rotation the energy loss reaches a maximum for values of B_m between 17000 and 18000. For still higher inductions the loss rapidly decreases to an almost negligible amount at $B_m = 23000$. The curves in Figure 234 have been experimentally obtained by A. Dina from tests made to determine the hysteresis loss in armature iron when subjected to (1) an alternating and (2) a rotating magnetic field. For inductions between $B_m = 6000$ and $B_m = 16000$ it will be seen that the energy loss produced by a rotating field is approximately the same as that due to an

alternating field. In consequence of this it is permissible to use Steinmetz's formula for the determination of the hysteresis loss in armature cores.

Hysteresis Loss in Armature Core.—In a toothed armature it is necessary to distinguish between the energy loss occurring in the armature core proper and that occurring in the teeth. If the magnetic flux in the core were to distribute itself uniformly over the cross-section, then the energy loss would be expressed by

$$W_{hc} = \eta \sim B_a^{1.6} V_c \times 10^{-4} \text{ watts}$$

$$\text{where} \quad B_a = \frac{0.5 \times \text{Flux in armature per pole}}{\text{cross-sectional area of core}}$$

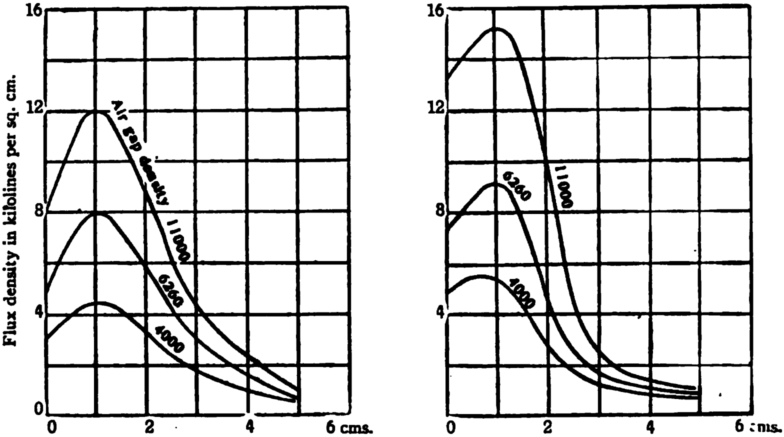
$$V_c = \text{Volume of iron in decimetres}^3$$

The magnetic flux is not, however, distributed uniformly over the cross-section of the core, but tends to take such paths that the total reluctance of the combined paths is a minimum. If, on the perimeter corresponding to the roots of the teeth, any two points, similarly situated with respect to adjacent poles, be considered, then the lines of force passing from the point under an N pole to the corresponding point under an adjacent S pole will tend to take the path conforming to the chord which joins the two points together. Should this path be rigidly followed, the flux will tend to be concentrated along a narrow path situated a short distance above the tooth roots. The resultant high induction would, however, considerably lower the permeability of the iron just below the teeth, consequently a longer path of higher permeability would then offer less reluctance. The lines of force, therefore, spread down into the core, but their distribution across any section never becomes completely uniform. The extent to which the flux distribution departs from a uniform value is dependent upon the ratio of pole arc to pole pitch, and also upon the shaping of the pole shoes.

Professor W. M. Thornton* has made a most exhaustive series of investigations regarding the distribution of flux over a cross-section of the armature core for both smooth and toothed armatures. The original tests were carried out on a smooth core armature placed inside a 4-pole field system. There were three different sets of poles, giving three different values of the ratio of pole arc to pole pitch, namely, 0.48, 0.60, and 0.72. In Figure 235 the distribution of the flux density B_a for two different values of pole arc $\div$ pole pitch is, for different air-gap inductions, plotted as a function of the radial depth of core up to 5 cms., the inductions being those along a radius passing through the centre of the interpolar space. These curves show that for a given air-gap density the deviation of the flux distribution from a uniform

* *Electrician* (August 1904), vol. liii. p. 959 (Reprint of British Association Paper).

value is more pronounced the greater the ratio pole arc ÷ pole pitch. In the second paper mentioned below,* Thornton puts forward the



Depth below surface.

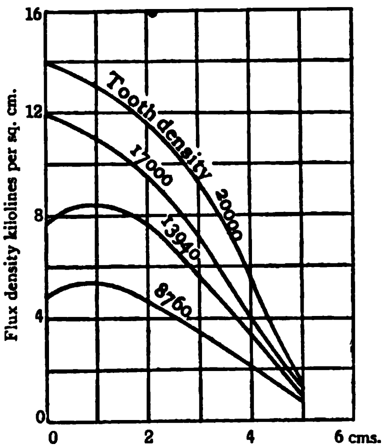
$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0{\cdot}6.$

Depth below surface.

$\frac{\text{Pole arc}}{\text{Pole pitch}} = 0{\cdot}72.$

FIG. 235.—Variation of flux density in smooth core armature.

results of similar investigations made on slotted armatures. Figure 236 gives the relation of the core induction as a function of the depth below the teeth, the curves being plotted for various tooth inductions. It is evident from these curves that with high tooth densities the maximum value of B_a occurs directly under the slots, whilst at low tooth inductions the maximum core induction is in similar position as that in a smooth core armature. The explanation of this is that at the higher inductions the flux is already distributed to some extent through the slot, and as soon as a greater cross-section is offered it bends off rapidly. Figures 237 and 238 show the distribution of the flux in the armature core and slots over one pole pitch, the armature in this case being the rotating member. From these figures it will be evident that the inner part of the armature iron carries very



Depth from bottom of slot.

FIG. 236.—Variation of flux density in core of toothed armature.

* *Journ. of the Inst. of Elect. Engineers* (1906), vol. xxxvii. p. 125.

little flux. A decrease in the radial depth of core will therefore increase the core induction only to a small extent.

The complete equation for the hysteresis loss in an armature core is

$$W_{hc} = K_3 \eta \sim B_a^{1.6} V_c 10^{-4} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

where the coefficient K_3 is introduced to allow for the increase in loss

FIG. 237.

due to the non-uniform distribution of the flux. The value of K_3, as determined experimentally for machines of various designs, ranges from about 1.05 at no-load to 1.15 at full-load.

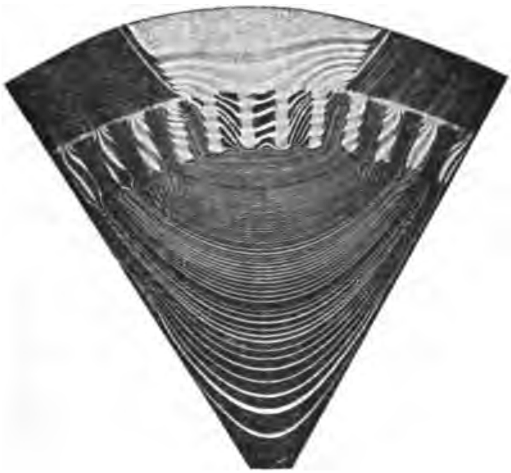


FIG. 238.

Hysteresis Loss in Armature Teeth.—In order to simplify the investigations, the formula for the hysteresis loss in the teeth will be derived on the assumption that (1) all the lines of force entering each tooth at the top pass down the tooth unchanged, and (2) the energy

loss obeys the law W is proportional to $B^{1.6}$. The latter assumption is approximately correct, since the flux density at minimum section seldom exceeds 17000 lines per cm.^{2}

If for the open slots of Figure 239 L_n denotes the nett length of iron in the armature core, and B_x the flux density across an element of width t, depth dh, and volume V_x cms.^{3}, the hysteresis loss in the element is

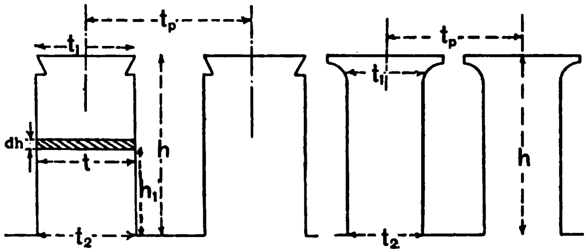
$$10^{-7} . \eta . \sim . B_x^{1.6} . V_x = 10^{-7} . \eta . \sim . B_x^{1.6} . t . L_n . dh$$

The teeth in Figure 239 are shown with parallel sides, but in the actual alternator the top would be narrower than the root. Let the flux density at the root of the tooth be denoted by B_{min}, then if the effect of the dovetails for the wedges be neglected,

$$B_x^{1.6} = B^{1.6}_{min} . \left(\frac{t_2}{t}\right)^{1.6}$$

and the hysteresis loss per element becomes

$$10^{-7} . \eta . \sim . B_{min}^{1.6} . t_2^{1.6} . L_n . t^{-0.6} . dh$$



Open slots. Closed slots.

FIG. 239.

Since the element under consideration is distant h_1 cm. from root of tooth, the width of the element is

$$t = t_2 - \frac{(t_2 - t_1)h_1}{h}$$

where h denotes height of slot. Substituting this value in the above equation, the hysteresis loss per tooth

$$= 10^{-7} . \eta . \sim . B^{1.6}_{min} . t_2^{1.6} . L_n \int_{h_1=0}^{h_1=h} \left\{ t_2 - \frac{(t_2 - t_1)h_1}{h} \right\}^{-0.6} dh$$

Now $\int \left\{ t_2 - \frac{(t_2 - t_1)h}{h} \right\}^{-0.6} dh = \frac{2.5h}{t_2 - t_1} \left\{ t_2 - \frac{(t_2 - t_1)h_1}{h} \right\}^{0.4}$

and between the limits h and o

$$= 2.5 \frac{h}{t_2 - t_1} \left\{ t_2^{0.4} - t_1^{0.4} \right\} = 2.5 \frac{h . t_2^{0.4}}{t_2 - t_1} \left\{ 1 - \left(\frac{t_1}{t_2}\right)^{0.4} \right\}$$

Since the volume of one tooth $= V_t = \frac{t_2 + t_1}{2} . h . L_n$, the depth of a tooth $= h = \frac{2 V_t}{L_n(t_2 + t_1)}$. When these values are inserted in the above equation the hysteresis loss per tooth

$$= 10^{-7} . \eta . \sim . B^{1.6}_{min} . t_2^2 . V_t . 5 . \frac{1 - \left(\frac{t_1}{t_2}\right)^{0.4}}{t_2^2 - t_1^2}$$

Hence, if V_t denote the total volume of the teeth in decimetres[3], the energy loss due to hysteresis is expressed by the equation

$$W_{ht} = K_4 . \eta . \sim . B^{1.6}_{min} . V_t . 10^{-4} \text{ watts} \quad . \quad . \quad (80)$$

$$\text{where} \quad K_4 = 5 . \frac{1 - \left(\frac{t_1}{t_2}\right)^{0.4}}{1 - \left(\frac{t_1}{t_2}\right)^{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

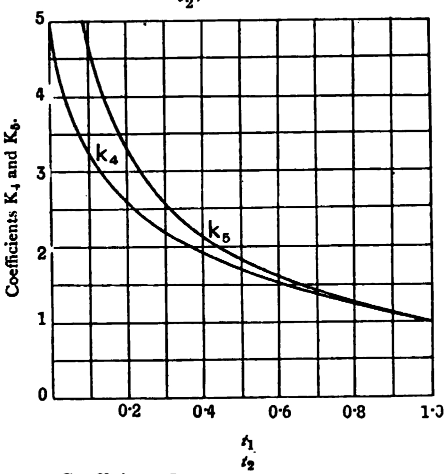


FIG. 240.—Coefficients K_4 and K_5 for calculating iron loss in teeth.

The values of the coefficient K_4, which takes into account the variation of induction down the teeth, have been plotted in Figure 240 as a function of $\frac{t_1}{t_2}$. In the case of semi-closed slots the teeth projections will have very little influence upon the hysteresis loss. Hence, the latter can also be calculated from equation 80, where the letters t_1 and t_2 will have the significance indicated in Figure 239.

Eddy Current Loss in Armature Core.—Besides the hysteresis loss in the core there is also a loss accompanying the eddy currents which circulate in the core plates. So long as the flux in the core changes it will be impossible to prevent the induction of E.M.Fs., but the resultant eddy currents can be considerably reduced by building the core of sufficiently thin laminations. With frequencies ranging from 25 to 60

cycles per second the standard thickness of stamping is 0.5 mm. For higher frequencies it is better to use thinner plates down to 0.3 mm.

On p. 107 it is shown that when iron is subjected to an alternating magnetic field the eddy current loss per cubic centimetre in very thin plates is

$$W_e = \frac{1.65 . \sim^2 . B_m^2 . t^2 \times 10^{-16}}{\rho} \text{ watts}$$

where B_m = Maximum flux density in the iron.
t = Thickness of plate in centimetres.
ρ = Specific resistance of iron.

For ordinary iron laminations at 15° C. $\rho = 1.2 \times 10^{-5}$, so that

$$W_e = 1.4 \sim^2 . B^2 . t^2 . 10^{-11} \text{ watts}$$
$$= \epsilon \sim^2 . B^2 . t^2 . 10^{-11} \text{ watts}$$

The value of the constant ϵ will depend upon (1) the composition of the iron; (2) the extent to which the induction in a cross-section of the core deviates from a uniform value; and (3) the form factor of the induced E.M.F. Now, in deriving the above equation, it was assumed that the induced E.M.F. varied sinusoidally and had a form factor = 1.11. With a pulsating magnetic field the wave form of the E.M.F. induced in the core plates will have a pronounced peak, in consequence of which the constant ϵ will have a higher value than that for iron plates subjected to a rotating magnetic field. The value ϵ for armature laminations could quite easily be determined mathematically, but unfortunately it would be of no practical utility. The discrepancy between the theoretical value of ϵ and the actual value as found experimentally ranges from 100 to 200 per cent.

This is due to the fact that the value derived mathematically does not take into account the increased loss due to—

(1) Eddy currents induced in solid metal parts, *e.g.* core flanges, bolts, spider arms, etc.

(2) Neighbouring plates making contact with each other.

The eddy currents in the end-flanges of the armature core are caused by the fringing magnetic field which passes from the flanges of the pole pieces into the armature core, as shown in Figure 183. In order to eliminate this loss as much as possible the armature core is sometimes made to project on either side about 1 cm. beyond the edge of the pole. The greater portion of the flux leaving the pole flanges will then enter the armature by the tops of the teeth. During the process of milling or filing the slots, the edges of the laminations are liable to become burred, thus destroying the insulation between them, and forming more or less continuous conductors lying parallel to the armature winding. The burring of the laminations therefore increases the strength of the eddy currents, and consequently the energy required for their maintenance.

Owing to the uncertain factors that require to be introduced it will now be evident that any attempt to derive a theoretical value for the constant ϵ is entirely out of the question. The constant ϵ must be determined experimentally for machines of various designs, and since it is dependent upon the amount of machining and style of construction, its value for machines of the same output will vary considerably. If V_c and V_n expressed in decimetres³, denote the volume of the core and teeth respectively, then the eddy current loss for the core

$$W_{ec} = \epsilon \, . \sim^2 . \, B_c^2 \, . \, t^2 \, . \, V_c \, . \, 10^{-11} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

and for the teeth

$$W_{et} = K_5 \, . \, \epsilon \, . \sim^2 . \, B^2_{min} \, . \, t^2 \, . \, V_t \, . \, 10^{-11} \text{ watts} \quad . \quad . \quad . \quad . \quad (83)$$

where the constant ϵ has an average value $\cong 6$. The coefficient K_5, derived in the same manner as the coefficient K_4 of equation 80, is calculated from the expression

$$K_5 = \frac{4.6}{1 - \left(\frac{t_1}{t_2}\right)^2} \, . \log\left(\frac{t_1}{t_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

In Figure 240 K_5 has also been plotted as a function of $\left(\frac{t_1}{t_2}\right)$.

Empirical Formula of Core Losses.—Owing to the uncertain factors which require to be introduced, it will be evident that any attempt to estimate the eddy current losses from the above formula must necessarily be very unreliable, so that empirical estimations of the eddy current losses, together with the hysteresis losses, are generally resorted to in practice. It is most convenient to treat these losses collectively and refer to them as the core losses or iron losses of the machine.

In practice the core losses are usually predetermined from the results of tests on previous machines of similar construction, and the results obtained in this manner have proved to be much more reliable than those derived from theoretical formula. The author has found the following formula to give results very consistent with those afterwards measured experimentally—

$$W = K_g \, . \sim . \, \beta \, . \, c \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

where W = Sum of hysteresis and eddy current losses.
K_g = Kilogrammes of iron.
β = A coefficient the value of which depends upon $\sim$
c = A coefficient which depends upon the flux density.

The values of the coefficients β and c can be obtained from the curves in Figure 241, which have been plotted from the test data relating to a large number of alternators. In estimating the iron losses from the

above formula the losses in the teeth and the core must of course be calculated separately.

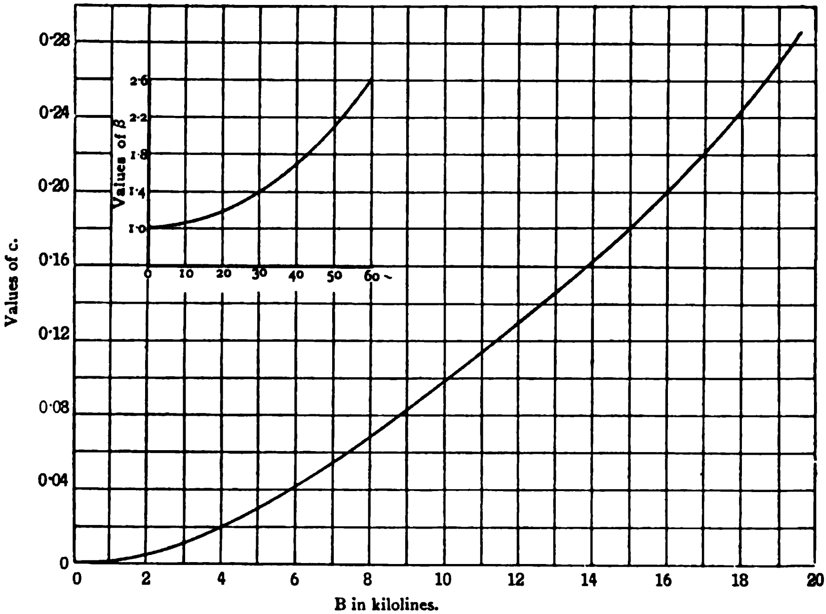


FIG. 241.—Coefficients for calculating core losses in armatures.

Eddy Current Losses in Pole Shoes.—In Chapter VII. it was shown that in a slotted armature the flux density at any point under the pole face will vary according to the position of the armature teeth relative to that point; consequently, as the field system rotates the flux

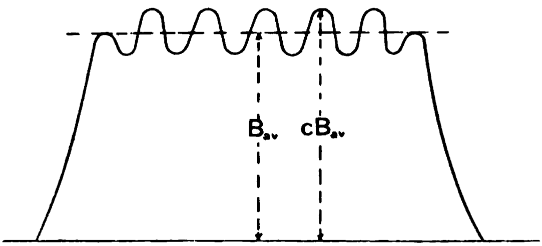


FIG. 242.—Flux distribution along air-gap of slotted armature (uniform air-gap depth).

in the pole shoe will pulsate. The magnitude of these pulsations is mainly dependent upon the value of the mean flux density B_{av} over the entire pole face and on the ratio of slot opening to air-gap. If cB_{av} denotes the maximum flux density opposite a tooth, then the curve of flux distribution, shown in Figure 242, is obtained by adding to the

curve of average induction another curve of amplitude $(c-1)\,B_{av}$. The E.M.Fs.—of frequency $\sim_1 = \frac{n_t\,R}{60}$, where n_t denotes the number of armature teeth—set up by the flux pulsations produce corresponding eddy currents which extend to a depth beyond which the induction is constant. These eddy currents, in turn, tend to damp out the fluctuations in the field—*i.e.* they set up a screening effect,—and in consequence of this they are confined chiefly to the outer surface of the pole shoe. The loss corresponding to these eddy currents is dependent upon (1) the amplitude of the field pulsation; (2) the frequency of the currents; and (3) the specific resistance ρ and permeability μ of the pole shoe material.

Owing to the complicated nature of the screening effect of the eddy currents an exact mathematical investigation of the losses is exceedingly difficult. Under the assumption that the flux pulsates according to the sine law, the following theoretical formula for the approximate calculation of the eddy current loss in solid pole shoes has been derived by Potier and Rüdenberg *

$$W_{ep} = 0.4\left\{\frac{(c-1)B_{av}}{1000}\right\}^2 v \cdot \sqrt{\frac{10 \cdot t_p \cdot v}{\mu\rho_1}} \cdot A_p \text{ watts} \quad . \quad . \quad . \quad (86)$$

where v = Peripheral speed of armature in metres per second.
t_p = Slot pitch in centimetres.
A_p = Pole face area in decimetres².
ρ_1 = Specific resistance of pole shoe iron in ohms per mm. cube.
μ = Permeability of the iron.

For cast iron, cast steel, and laminated iron ρ_1 has the values 10^{-3}, 2×10^{-4} and 1.5×10^{-4} respectively. The value of μ can be taken as that corresponding to the mean value of flux density in the pole shoe, which is approximately equal to B_{av}.

The above formula was deduced on the assumption that the flux pulsations are sinusoidal. Curves of flux distribution obtained by an exploring coil, however, indicate that the ripples are decidedly peaked, thus leading to greater E.M.F. amplitudes, and consequently greater losses than that calculated from the above theoretical formula.

In Professor Arnold's laboratory at Karlsruhe two independent series of investigations have recently been made to determine an empirical law for the eddy current loss in pole shoes. In these researches the measurements were made as follows:—The air-gap was made so large that no losses due to eddy currents occurred in the pole shoe, and the power thus required to drive the machine was accurately measured by means of a spring dynamometer. Then with the same flux density and a decreased air-gap the increase of power supplied was

* *Electrotechnische Zeitschrift* (1905), vol. xxvi. p. 181, and *Industrie Électrique* (1905), p. 35.

taken as equal to the losses in the pole shoe due to the pulsations of the flux, caused by the armature teeth. Thus, by keeping the gap constant and using successively solid and laminated pole shoes, the increase of the losses would be given by the difference of the readings, whilst the absolute losses in each case could be found by the above method.

The first series of investigations were made by Dexhumir, and as a result of his research the following formula was deduced *:—

$$W_{ep} = C\{(c-1)B_{av}t_p\}^{2.2}\left\{\frac{n_t R}{60}\right\}^{1.7} . A_p \times 10^{-11} \text{ watts.} \quad . \quad (87)$$

where the coefficient C has the following average values:—

For laminations 0.55 mm. thick $C = 0.8$
„ „ 0.20 „ „ $C = 0.35$
„ solid cast steel $C = 2.3$

The factor c, the ratio of maximum to average flux density, has the same value as the air-gap correction coefficient given on page 238.

According to the more recent experiments of Wall and Smith,† the equation to the eddy current loss in the pole shoes would appear to be as follows:—

$$W_{ep} = C\left(\frac{s_o}{\delta}\right)^{3.5} B_{av}^{2.1} . \left(\frac{n_t R}{60}\right)^{1.5} . A_p . 10^{-11} \text{ watts} \quad . \quad . \quad . \quad (88)$$

where δ = Radial depth of air-gap.
s_o = Slot opening.

The value of the constant C depends upon the grade of iron employed, and for wrought-iron pole pieces = 0.046. The above equation shows that the factor which has the greatest influence on the pole shoe loss is the ratio of $\frac{\text{slot opening}}{\text{depth of air-gap}}$ —*i.e.* $\frac{s_o}{\delta}$. Hence, in order to keep down the eddy current loss in the pole shoes, the slot opening should be made as small as possible consistent with a reasonable slot inductance. As a result of their investigations, Wall and Smith found that with solid pole shoes the loss is quite negligible for ordinary inductions provided the ratio $\frac{\text{slot-opening}}{\text{air-gap}}$ does not exceed 2.0. As alternators are nearly always constructed with laminated pole shoes, it follows that the limit value of $\frac{s_o}{\delta}$ at which the losses become appreciable is increased above that for solid pole shoes. Besides that due to eddy currents, there will be a small loss in the pole shoes due to hysteresis. This, however, will be negligibly small in comparison with the loss due to eddy currents.

* Arnold, *Die Gleichstrommaschin*, vol. i. 2nd ed. p. 648.
† *Journ. of the Inst. of Elect. Engineers* (1908), vol. xl. p. 577.

Copper Losses

Armature I^2R Loss.—The copper loss in the windings of an alternator having m-phases is expressed by the equation

$$W_{ca} = I_a^2 \cdot r_a \cdot m \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad (89)$$

where I_a denotes the armature current and r_a the effective resistance per phase of the winding. If l_a denotes the mean length per turn in centimetres, a_a the cross-sectional area of conductor in sq. cms., and T the number of turns in series per phase, then the effective resistance per phase for a mean rise in temperature of T° is given by equation 11, page 102, namely—

$$r_a = \frac{1.7 \times 10^{-6}\, k l_a \mathrm{T}\ (1 + 0.004\ \mathrm{T}^\circ)}{a_a}$$

The factor k allows for the increase in resistance resulting from eddy currents induced in the body of the conductors by the flux which enters the core through the slots, and also by the rotating flux set up by the harmonics in the armature M.M.F. wave. The values to be assigned to k must of course be determined from test data, and will be approximately as follows:—

	Single-Phase	Polyphase
25 ~ to 40 ~	1.5 – 1.7	1.3 – 1.5
40 ~ to 60 ~	1.9 – 2.2	1.5 – 2.0

The increased values in a single-phase winding as compared with a polyphase winding is due to the greater pulsations of armature flux in the former machines.

Excitation Loss.—Let l_f denote the mean length per turn for the field coil in cms., a_f the area of cross-section of wire in cms.², T_c the number of turns per coil, and p the number of pairs of poles, then, for a temperature T° above 15° C., the resistance of the field winding is

$$r_f = \frac{1.7 \times 10^{-6} \cdot l_f \cdot \mathrm{T}_c \cdot 2p\,(1 + 0.004\ \mathrm{T}^\circ)}{a_f} \text{ ohms}$$

If the exciting current I_e is varied by regulating the exciter volts, the excitation losses are

$$W_{\alpha} = I_e^2\, r_f \text{ watts}$$

In large central stations where several alternators would be operated in parallel, the field windings are often excited from a constant pressure supply of E_e volts, and the field current adjusted by a rheostat in the field circuit. Under such conditions the excitation loss would be $W_{\alpha} = I_e E_e$ watts. This latter method of excitation involves a greater loss in the regulating resistance than would be the case with a resistance in the exciter circuit, and on this account is less frequently used. It has, however, the advantage that the changes in magnetic flux respond quicker to any change in the regulating resistance than is possible with a shunt wound exciter.

Mechanical Losses.—The mechanical losses are those due to bearing friction, windage, and vibration, and of these only the first permits of calculation. If K denotes the bearing pressure in kilogrammes, d the diameter of the bearing, and l the length, both of which are expressed in centimetres, then the specific bearing pressure is

$$P = \frac{K}{d \cdot l} \text{ kgs./cm.}^2$$

Further, if μ denotes the coefficient of friction, and $v = \frac{\pi d R}{6000}$ the peripheral speed of shaft in metres per second, then the power absorbed in bearing friction $= \mu \cdot K \cdot v$ kilogramme metres per second, and the friction loss in watts

$$W_{mb} = 9.81 \cdot \mu \cdot K \cdot v = 9.81 \mu \cdot P \cdot d \cdot l \cdot v$$

According to the results of Tower and Dettmar, the coefficient μ is expressed by the equation

$$\mu = \frac{c}{T^\circ} \cdot \frac{\sqrt{v}}{P}$$

where T° denotes the temperature of the bearing in °C, and c is a coefficient dependent on the quality of oil. Substituting this value for μ, the expression for the bearing friction loss becomes

$$W_{mb} = 9.81 \frac{c}{T^\circ} \cdot d \cdot l \cdot \sqrt{v^3} \text{ watts} \quad . \quad . \quad . \quad . \quad (90)$$

For the usual range of peripheral speeds, the temperature T° increases almost in direct proportion to the velocity, according to the equation

$$T^\circ \cong 8v$$

whilst the coefficient $c \cong 0.03$.

The loss caused by windage depends greatly upon the construction of the armature and field system, and ranges from 10 per cent. of the bearing friction in small machines running at 20 metres per second, to 100 per cent. in high speed machines direct coupled to steam turbines.

Efficiency.—In an electric generator the power output is delivered at the terminals of the machine, and is given by $W = m \cdot E \cdot I \cos \phi$, where m denotes the number of phases and E and I denote the E.M.F. and current per phase respectively. The power input is that supplied by the prime mover, and is equal to $W + w$, where w denotes the sum of all the losses. The equation for the efficiency is

$$\eta \text{ per cent.} = \frac{\text{Power output}}{\text{Power input}} \cdot 100 = \frac{W}{W + w} \, 100$$

The efficiency of an alternator will vary at different loads, and the form of the curve showing the variation of efficiency with output will depend upon the relative values of the various losses.

In alternators designed for a constant terminal voltage the losses vary with the load as follows:—

1. Hysteresis loss. $W_h = W_{hc} + W_{ht}$ increases slightly with the load

due to the increased flux through the armature necessary for the maintenance of a constant terminal voltage.

2. Eddy current loss. $W_e = W_{ec} + W_{et} + W_{ep}$. This also increases slightly with the load, for the same reason as the increase in hysteresis loss.

3. Armature copper loss. W_{ca} increases with the load in proportion to the square of the armature current.

4. Excitation loss. W_{ce} increases with the load at a rate proportional to the voltage drop.

For an increasing load of constant power factor the *increase* in the copper loss,—*i.e.* the armature I^2R and the excitation losses—can be represented by an equation of the second degree, thus

$$\text{Increase of loss with load} = C_1 I_a + C_2 I_a^2$$

where C_1 and C_2 are constants. Hence, if W_{co} denotes the copper

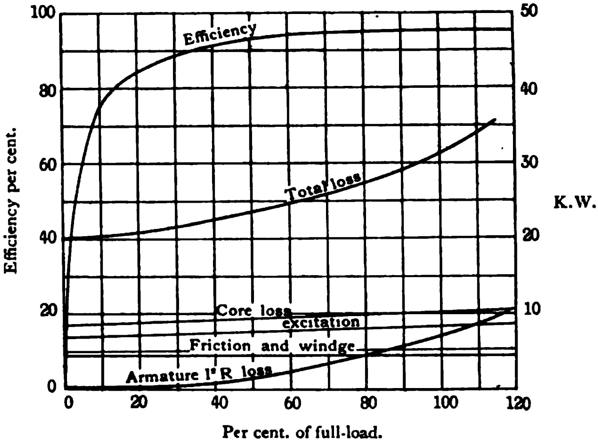


FIG. 243.—Efficiency and losses of 550-K.W. alternator.

loss at no-load, the copper loss corresponding to any armature current I_a is

$$W_c = W_{co} + C_1 I_a + C_2 I_a^2$$

5. Mechanical losses W_m vary only with the speed, and are therefore constant for all loads.

The sum of all the losses is

$$W_1 = W_h + W_e + W_c + W_m$$

The efficiency will be zero at no-load, and will remain low for small outputs, as the constant losses are then large in comparison with the power output. The efficiency increases with the output in the manner shown in Figure 243, and attains a maximum at that output for which the no-load losses are equal to the losses proportional to the square of the armature current. Should the output be further

increased the efficiency will diminish. That maximum efficiency corresponds to equality of $I_a^2 r_a$ and no-load losses may be proved as follows:—Since the constant or no-load losses and the variable losses can respectively be denoted by

$$\text{constant losses} = W_h + W_e + W_m + W_{co} = W_o$$
$$\text{variable losses} = C_1 I_a + C_2 I_a^2$$

the efficiency corresponding to any armature current I_a is

$$\eta = \frac{W}{W + W_o + C_1 I_a + C_2 I_a^2}$$

Now, $W = m\ EI_a \cos\phi = C_o EI_a$, where $C_o =$ a constant; hence, substituting this value in the equation for efficiency and dividing top and bottom by $C_o I_a$

$$\eta = \frac{E}{E + \frac{W_o}{C_o I_a} + \frac{C_1}{C_o} + \frac{C_2}{C_o} \cdot I_a} = \frac{E}{U}$$

Since E is a constant, the efficiency will be a maximum, when the denominator of this equation is a minimum. Thence, differentiating the equation

$$U = E + \frac{W_o}{C_o I_a} + \frac{C_1}{C_o} + \frac{C_2}{C_o} \cdot I_a$$

with respect to I_a, and placing the differential coefficient $= 0$

$$\frac{dU}{dI_a} = \frac{C_2}{C_o} - \frac{W_o}{C_o I_a^2} = 0$$

The efficiency is therefore a maximum when $W_o = C_2 I_a^2 \cong m I_a^2 r_a$. That is, the maximum efficiency occurs at that load where the loss proportional to the square of the armature current equals the sum of the no-load losses.

The efficiency curve for a 550-K.V.A. 5000-volts 50 ~ alternator, revolving at 300 R.P.M., is shown in Figure 243. In the same figure there is also shown the variation of the component losses with the output, and attention is directed to the slight increase in the iron loss caused by distortion of the field and the increase in the magnetic flux necessary to compensate for the impedance drop. In alternators of normal design the relation between the various losses is generally such that the output corresponding to maximum efficiency is somewhat beyond the rated full-load value.

Testing of Alternators for Efficiency

In the preceding part of this chapter the methods of predetermining the losses and efficiency of alternating current generators have been discussed. There will now be considered a few of the more important tests to which complete machines are subjected for deter-

mining their actual losses and efficiency. In the case of large generators a suitable prime mover and load may not be available at the manufacturing works for working the machine at full-load and measuring the efficiency directly. The usual method of carrying out an efficiency test is therefore to measure the various losses individually and to compute the efficiency from the known output.

The efficiency of an alternator is expressed by the equation—

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power output} + \text{losses}} = \frac{W}{W + W_i + W_m + W_{ca} + W_{e}}$$

where W = Power output.

W_i = Iron losses, *i.e.* sum of hysteresis and eddy current losses.

W_m = Mechanical losses.

W_{ca} = Armature copper loss.

W_e = Excitation loss.

To measure the losses W_i, W_m, and W_{ca}, the alternator to be tested is coupled to a suitable direct current motor, the losses of which are accurately known. To obtain the best results, the output of the driving motor should be from 5 to 10 per cent. of the total output of the machine to be tested.

Iron Losses.—The alternator A to be tested is driven at normal speed, and the input to the direct current motor is measured for various flux densities in the core of A. The input to the motor, when A is unexcited, is consumed in the friction and windage losses of the two machines, and the core and I^2r losses in the driving motor; while the input when A is excited also includes the iron losses in its armature core which correspond to the partial excitation at which the input is measured. Thus, to determine the iron losses of A, the other losses for each input reading must be known.

In carrying out this test various precautions require to be observed. To avoid including the volts drop due to brush contact resistance at the commutator of the motor, the armature voltage should be read direct from the commutator segments instead of across the terminals. The condition is fulfilled by insulating one brush in each of the positive and negative sets from the brush spindle and connecting the voltmeter across them. By this means the actual voltage on the commutator is measured independently of any IR drops due to brush contact resistance.

The iron losses should be measured for a particular speed, so that the friction and windage losses of the set remain constant. The iron losses of the motor should preferably be kept constant, so that they can be eliminated in the final calculation. To meet this requirement both the speed and exciting current of the motor must be held constant. Thus it will be necessary to separately excite the field of

the motor and control the speed by the voltage supplied to the armature.

The diagram of connections would be as shown in Figure 244. The field winding of the alternators is connected to a source of E.M.F. through a regulating resistance R_1 and ammeter A_1. The field of the driving motor is similarly connected through R_2 and A_2. The armature of the motor is connected through a starting rheostat R to a

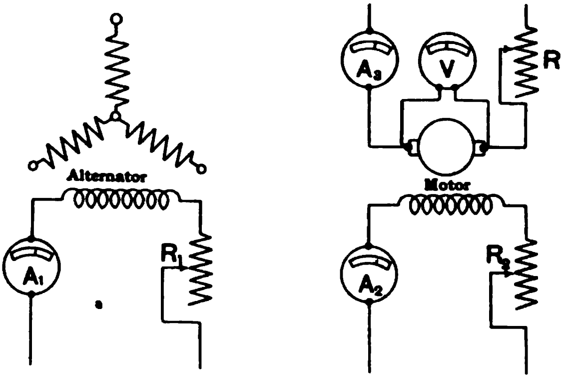


FIG. 244.—Diagram of connections for core loss tests.

dynamo whose voltage can be controlled. The ammeter A_3 and voltmeter V indicate respectively the current input to, and voltage across, the armature of the motor. During the test the current supplied to the motor and its voltage should be observed for various excitations between zero and about 25 per cent. above that corresponding to the normal voltage of the alternator. Knowing the resistance r of the motor armature, the core losses can be calculated and tabulated thus—

Driving Motor.					Alternator under Test.		
Input to Armature.			Armature I^2r.	$IE-I^2r$.	Core Loss in Watts.	Field Current.	Armature Volts from No-load Saturation Curve.
Current I.	Volts E.	Watts IE.					

$IE - I^2r$ gives for each observation the sum of the iron losses in the alternator, the constant losses in the iron of the motor, and the

friction and windage of the set. The reading taken with A unexcited is called the friction reading; thus if the value of $IE - I^2r$ for the friction reading be subtracted from the value of $IE - I^2r$ when the field is excited, the result in each case gives the iron losses of the alternator for that field excitation. The frequency of reversal of flux, as given by $\frac{Rp}{60}$, will remain constant. A curve can now be plotted with alternator volts as abscissæ and iron losses in watts as ordinates.

Mechanical Losses.—To measure the mechanical losses, *i.e.* the losses due to bearing friction, windage, and vibration, the unexcited alternator is driven at normal speed by the motor. The losses under consideration are then obtained by subtracting the copper and no-load losses of the motor from the total power supplied.

Armature Copper Loss.—When measuring the copper loss in the armature of an alternator, it should be remembered that the resistance as measured by means of a direct current is always less than the effective resistance as computed from alternating-current measurements, owing to the skin effect. Again, there is an additional loss due to eddy currents induced in conductors themselves. To obtain conditions which would be approximately the same as those occurring in actual working, the armature copper loss should be measured when an alternating current of the rated frequency flows in the armature windings, and with the field system driven at normal speed by the direct-current motor. With the field of the alternator unexcited the power taken by the armature of the motor is observed. The alternator armature winding is then short-circuited through an ammeter of low resistance, and the field excitation adjusted so as to give a value of armature current for which the corresponding copper loss has to be measured. The increase in power W_c given out by the motor is due to the armature copper loss and a certain amount of iron loss. But the excitation necessary to send a full-load current through the armature on short-circuit is negligible compared with that ordinarily employed at full pressure; hence the increase of power W_c can be taken as the armature copper loss corresponding to the current indicated by the ammeter in the armature circuit.

Excitation.—For the excitation loss the only measurement to be made is that of the resistance of the field windings, which is usually performed by sending a measured direct current through the winding and noting the fall of potential. The power required for excitation is then = current^2 × resistance.

Example.—The efficiency of a 550-K.V.A., 3-phase, 50 ~, 5000-volt, Y-connected alternator is worked out from the various test results as set forth below. The core loss curve is given in Figure 245.

	No-load.	Quarter-load.	Half-load.	Three-quarter-load.	Full-load.
Terminal voltage	5000	5000	5000	5000	5000
Induced voltage (estimated)	5000	5100	5200	5300	5400
Iron loss in watts (from test; see Fig. 245)	8800	9200	9600	10000	10600
Armature copper loss in watts (from test)	0	440	1700	3900	7000
Mechanical losses in watts (from test)	4500	4500	4500	4500	4500
Exciting current in ampères (estimated)	17.2	17.6	18	18.5	19
Resistance of field winding hot (from test).	24.0 ohms				
Excitation loss in watts	7000	7400	7800	8200	8700
Total loss in K.W.	20.3	21.5	23.6	26.6	30.8
Output in K.W. for $\cos\phi=0.8$.	0	137	275	415	550
Input in K.W. for $\cos\phi=0.8$	20.3	158.5	298.6	441.6	580.8
Efficiency for $\cos\phi=0.8$	0	87.0	93.0	94.5	95.0

In Figure 243, page 321, the losses and efficiency of the alternator under consideration have been plotted as a function of the output.

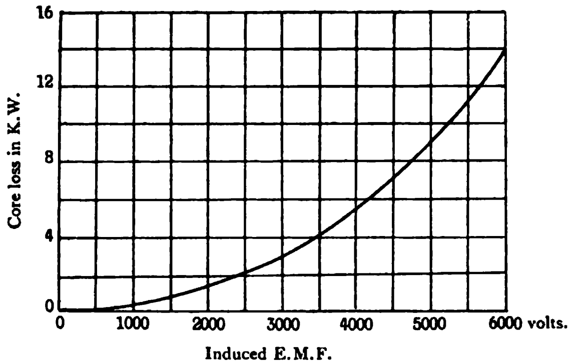


FIG. 245.—Core loss curve for 550-K.W. alternator.

Hopkinson's Test of Two Alternators.—When two similar alternators are available a convenient method of determining their efficiency is by the Hopkinson method, whereby one machine operates as a generator driving the other as a motor, the power lost in the two machines being supplied from an external source. The method of carrying out the test is as follows:—Referring to Figure 246, the rotors of the two alternators A_g and A_m to be tested are coupled rigidly together and driven at normal speed by a small direct-current motor M, which should be large enough to supply the power corresponding to the losses in the two machines. The motor M should be accurately tested for the losses at various outputs and speeds, so that the exact amount

of power transmitted by it to the alternators is known for any given value of current and E.M.F. supplied to the motor.

The armature windings of the two alternators are connected to oppose each other through an ammeter A and current coil of the wattmeter W, the voltmeter V indicating the pressure at the alternator terminals. The field windings of the alternators are each connected through a regulating resistance to a source of direct current, which is here supposed to be the same as that which supplies current to the motor M. When the alternators have been run up to normal speed the test requires that one alternator, say A_g, should act as a generator and the other A_m as a motor, the power supplied by A_g to A_m being measured on the wattmeter W. In order to set up a circulating

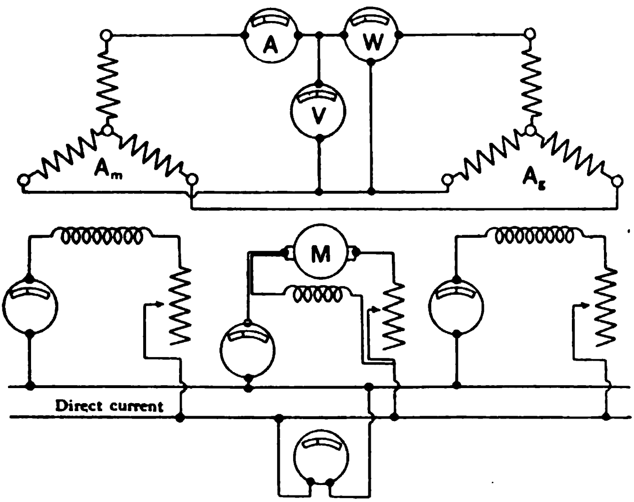


FIG. 246.—Diagram of connections for Hopkinson test.

current between the two machines it is not sufficient to have their excitations slightly different, but there must also be a phase difference between the internal E.M.F.'s of the two armatures. In order to obtain this phase difference the two machines should be coupled with about a 25 degrees difference in the position of their armature coils relative to the poles.

The alternators can be considered as two machines working in parallel, and Figure 247 gives the vector diagram for the test. OA is the direction of the vector of terminal voltage, which of course is alike for both machines. OE_1 is the vector of the E.M.F. of the generator, which has a certain lead ϕ with respect to OA. OE_2 is the corresponding vector of E.M.F. of the motor, which is almost opposite to the vector OE_1. The resultant E.M.F. acting through the armatures of both machines is represented by OE; this pressure sets up

the circulating current OI, which owing to the armature circuits being highly inductive lags behind the vector OE by an angle slightly less than 90 degrees. By choosing the proper values of the angle ϕ and by adjusting the two exciting currents there can be obtained any current at any power factor. In order to obtain $\cos\phi = 1$ (supposing the two alternators to be exactly alike) the exciting currents should be very nearly equal. By over-exciting the generator and under exciting the motor the vector OE_1 is increased and OE_2 diminished. The phase of the current I is therefore shifted so that it will be lagging behind the terminal voltage OA. From the vector diagram it will also be obvious that by over-exciting the motor and under-exciting the generator a leading current is obtained.

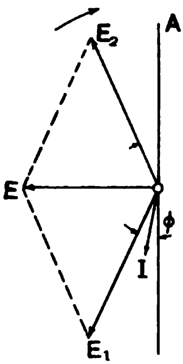


FIG. 247. — Vector diagram for Hopkinson test.

Let W denote power supplied by the generator to the motor as read on the wattmeter, w_e the excitation loss in the generator, and w_m the power supplied to the direct-current motor less the watts absorbed by the motor itself. Since the two machines are nearly equally loaded it may be assumed that the loss w_m is equally divided between the two machines. This is not strictly so, as for a lagging current the generator will have a larger excitation and core loss than the motor. The percentage efficiency of each machine is expressed by

$$\eta \text{ per cent.} = \frac{W}{W + \frac{w_m}{2} + w_e} \cdot 100$$

Retardation Method of Determining Iron and Friction Losses.—This method is especially applicable to alternators of the fly-wheel type, and though it involves a knowledge of the moment of inertia of the rotor it is frequently adopted in practice. It depends upon the determination of the retardation which takes place in the rotating member of a machine when the driving torque is removed. The alternator under test is run up to a speed somewhat above the normal by means of a motor and belt, or by its own exciter. The belt is then slipped off, or the current shut off from the exciter, as the case may be, and observations are taken of the rate at which the machine slows down, by reading a tachometer at small intervals of time. For each observation the corresponding angular velocity $\omega = \frac{2\pi R}{60}$ is computed, and a curve (see Figure 248) is now plotted connecting the angular velocity with time. If at any given moment corresponding to an angular velocity ω a tangent be drawn to the curve at a point corresponding to the moment in question, then this

tangent is a measure of the retardation as expressed by the time-rate of change of the angular velocity, or $\frac{d\omega}{dt}$.

Let M denote the moment of inertia of the rotor in C.G.S. units, *i.e.* in grammes of mass $\times$ cms.^{2}, then at any instant when the rotor is slowing down the rate of change of angular momentum is equal to the retarding torque T acting on it, T of course being in C.G.S. units. Thus

$$-T = -M \frac{d\omega}{dt} \text{ dyne-centimetres}$$

Since the product $T\omega$ is a measure of the rate at which the energy of the rotor is expended in overcoming the retardation,

$$T\omega = M\omega \frac{d\omega}{dt} \text{ ergs per second}$$

$$= M\omega \frac{d\omega}{dt} \times 10^{-7} \text{ watts}$$

But a kilogramme-(metre)$^2 = 10^7$ C.G.S. units. The moment of inertia M is therefore best expressed in kilogramme-(metres)^2, as the factor 10^{-7} cancels out and the expression becomes

$$W_o = M\omega \cdot \frac{d\omega}{dt} \text{ watts}$$

As the speed is generally measured in revolutions per minute, and plotted in relation to time in seconds, the previous equation can conveniently be expressed thus—

$$W_o = M \cdot \frac{2\pi R}{60} \cdot \frac{2\pi}{60} \cdot \frac{dR}{dt}$$

$$\cong M \cdot R \frac{dR}{dt} \times 10^{-2} \text{ watts} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91)$$

where M is in kilogramme-(metres)^2. For equation 91 the retardation curve should be plotted with speed in revolutions per minute as ordinates and time in seconds as abscissa.

The watts thus obtained are at any moment equal to the rate at which energy is dissipated in friction of bearings and brushes, windage, hysteresis, and eddy currents. Thus if the retardation curves be determined, first with the field unexcited and then with the field excited at various strengths, a complete analysis of the friction and iron losses can be made. When the field is excited it will be found that the retardation occurs more rapidly than when unexcited; this, of course, is due to the effect of hysteresis and eddy currents.

In order to make use of equation 91, the values of the moment of inertia M must first be known. In general the form of the rotor will be too complex to allow of it being calculated from the drawings of the machine, but it can be found experimentally or as follows. After

taking the retardation curves the alternator is driven at a speed of R revolutions per minute by a suitable motor the losses of which are known. The power W_o absorbed in overcoming the friction, windage, and iron losses of the alternator can thus be computed, and the moment of inertia calculated from equation 91, of which a value can be assigned to every factor except M. Another method of determining W_o is to run the alternator as an unloaded synchronous motor and measure the power absorbed. The losses are thus obtained for a particular excitation and speed R.

Example.—The retardation curves for an 800-K.V.A., 3-phase, 250-R.P.M. alternator are plotted in Figure 248, from observations

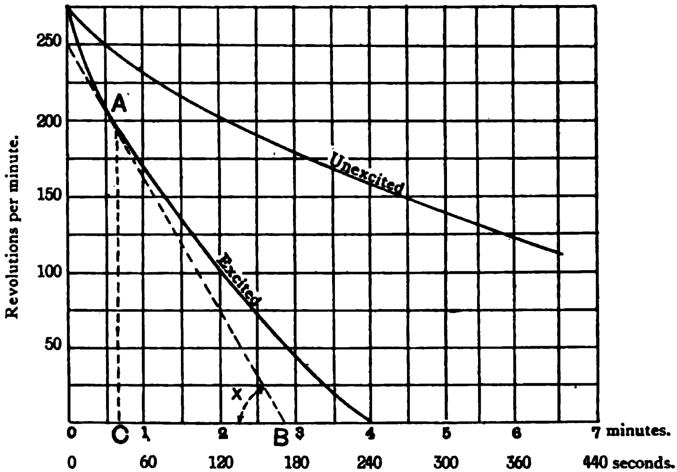


FIG. 248.—Retardation tests on 800-K.V.A. alternator.

taken every half-minute. The upper curve is for the field unexcited, while the lower curve is for the field excited so as to send the full-load flux through the armature. When the alternator was driven at 200 R.P.M. from a direct-current motor the losses of which were known, the friction, windage, and iron loss corresponding to full-load excitation was computed to be 10,400 watts. From this data the curves of Figure 249 are derived as follows:—

At the point A, in the lower curve of Figure 248, corresponding to 200 R.P.M., a tangent is drawn cutting the abscissa axis at B. The value of $\frac{dR}{dt}$ is

$$\frac{dR}{dt} = \tan x = \frac{AC}{BC} = \frac{200}{132} = 1.51$$

$$[CB = 2.2 \text{ minutes} = 132 \text{ seconds}]$$

$$\text{Now, } W_o = M \cdot R \cdot \frac{dR}{dt} \cdot 10^{-2}$$

$$i.e., \; 10{,}400 = M \cdot 200 \cdot 1.51 \times 10^{-2}$$

Hence, moment of inertia of rotor $= \Sigma mr^2 = 3500$ kilogramme-(metres)^2.
Referring to the lower curve

$$\text{at time} = 0 \qquad R = 270$$
$$\text{also at time} = 15 \text{ seconds} \qquad R = 240$$

$$\text{Hence for any average speed } R = \frac{270 + 240}{2} = 255 \text{ R.P.M.}$$

$$\frac{dR}{dt} = \frac{270 - 240}{15} = \frac{30}{15} = 2.0$$

The power absorbed by the friction, windage, and iron losses at a speed of 255 R.P.M. is therefore

$$W_o = \Sigma mr^2 . R . \frac{dR}{dt} . 10^{-2} = 3500 \times 255 \times 2.0 \times 10^{-2} = 17{,}800 \text{ watts}$$

From a series of such values the top curve of Figure 249 is plotted showing the variation of friction, windage, and iron losses with the speed. In a similar manner the curve of friction loss is computed from the top curve of Figure 248. By deduction of the friction and windage loss from the total the curve of iron loss is obtained.

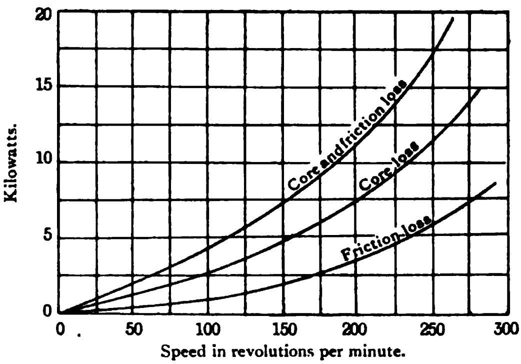


FIG. 249.—Losses of an 800-K.V.A. alternator as computed from retardation tests.

HEATING AND COOLING

The limit of output for alternating-current machines is determined either through regulation or temperature rise. In slow- and medium-speed alternators the regulation limit is generally reached long before the heating limit. In the case of high-speed turbo-machinery the conditions of limiting output are generally the reverse. The maximum temperature to which any part of an alternator may attain must be such that the insulation does not deteriorate, and that no appreciable increase occurs in the core loss due to ageing of the iron. To be within safe limits the usual practice in this country is to fix 60° C. as the maximum. As the temperature of a well-ventilated engine-room seldom exceeds 20° C., this permits of a temperature rise of 40° C.

Of all the calculations made in the design of an electric machine those relating to the probable temperature rise of the various parts are

without doubt the most uncertain. This is because the facilities for cooling, and consequently the temperature rise, are so much affected by details of construction. The succeeding formula can therefore only be expected to give approximate results.

Field-magnet Coils.—Though a magnet coil differs considerably from a homogeneous body in which the heat is radiated uniformly all over its surface, yet experimental results show that the curve of temperature rise does not diverge greatly from that of an exponential curve. The specific losses due to the I^2R loss are uniform all over the coil, but since the cooling facilities must be different for various parts, it follows that the temperature of all parts cannot attain to a uniform value. The highest temperature always occurs at the centre of gravity of the coil section, and the difference between the heating of the innermost parts and the surface will depend upon the thermal conductivity of the materials through which the heat must flow. When the coils are wound with cotton insulated wire the flow of heat from the central parts will be considerably retarded, so that a large temperature gradient would be expected. The field coils of alternators are, however, nearly always of bare copper strip wound on edge; the facilities for heat conditions are thereby considerably improved and a much more uniform temperature rise will result.

Owing to the temperature gradient between the innermost parts of a coil and the exposed outside surface, the temperature rise should always be obtained by measuring the ohmic resistance when hot and when cold, and computing the mean temperature rise from the equation

$$R_{t2} = R_{t1}\left\{1 + a(t_2 - t_1)\right\}$$

$$\textit{i.e.}\text{ mean temperature rise} = t_2 - t_1 = \frac{\dfrac{R_{t2}}{R_{t1}} - 1}{a}$$

where R_{t2} and R_{t1} are the resistance of the coils at the temperatures t_2 and t_1 respectively. For copper the temperature coefficient of resistance $a = 0.004$. In machines of normal design the mean temperature of the coils, as computed from the increase of resistance, varies from 40 to 60 per cent. in excess of the temperature of the outside layers as determined by a thermometer. The greater the radial depth of the winding the greater will be the departure of the heating from a uniform value. In order to prevent the maximum temperature exceeding a safe value the radial depth of the field coils is generally limited to about 4.0 cms.

If A_m denotes the cooling surface per coil in decimetres², W_m the excitation watts per coil, and K_m the heating coefficient, then when the field system is at rest the temperature rise would $= K_m \times \dfrac{W_m}{A_m}$. The cooling surface will here be taken as the external cylindrical surface only, the flanges and inside surface not being considered. The

heating coefficient will, however, be considerably influenced by the peripheral speed of the field coils, and test data show the additional cooling effect due to the rotation of the field system is equivalent to an increase in the radiating surface of $0.1\ v_m . A_m$ decimetres², where v_m denotes the peripheral speed of the rotor in metres per second. The equation for the temperature rise of the armature, in degrees centigrade, can therefore be expressed thus—

$$T_m = K_m . \frac{W_m}{A_m(1 + 0.1\ v_m)} \quad \ldots \ldots \ldots \quad (92)$$

If this equation gives the average temperature rise over the whole coil as computed from the increase of resistance, then the values to be assigned to the heating coefficient K_m will be approximately as follows:—

$K_m = 7$–9 for coils of copper strip wound on edge.

$K_m = 9$–10 for coils of d.c.c. wire wound in several layers to a depth not exceeding 4 cms.

In alternators with revolving armatures it would at first appear as if the cooling of the field coils would be considerably helped by the fanning action of the armature. Such is certainly the case when the armature is running light; but when the machine is loaded the heated air rising from the armature tends to retard the cooling, and on this account the rise in temperature of the field winding is practically unaffected by the peripheral speed of the armature. For stationary field coils the heating formula will therefore be

$$T_m = K_m . \frac{W_m}{A_m}$$

where K_m has the range of values given above for rotating field alternators.

Armature.—In the armature of any electrical machine the losses per unit volume are greatest in that zone which lies between the air-gap and the roots of the teeth. It is natural, therefore, to expect that the greatest temperature rise will occur at this part. In the end-connections the specific losses are smallest, and since the cooling through air ventilation is the most effective, the steady temperature to which the end-connections attain will be much lower than that for the active belt near the air-gap. Hence in considering the maximum temperature rise of the armature it is only necessary to take into account the watts lost in the copper within the slots W_c, *i.e.*,

$$W_c = \frac{L_g}{L_a} . W_{ca} = \frac{L_g}{L_a} . m . I_a^2 . r_a \text{ watts}$$

where L_g denotes the gross length of core between flanges. Now, if W_h and W_e denote the total hysteresis and eddy current losses respectively, then the armature losses coming into consideration are,

$$W_a = W_c + W_h + W_e$$

Owing to the complicated structure of the core it would be rather impracticable to attempt to estimate the total exposed surface from which radiation can take place; hence the usual practice is to introduce only the cooling surfaces indicated by the dark lines in Figure 250. If D and D_1 denote the external and internal core diameters, and n_d the number of ventilating ducts, then the cooling surface of the armature

$$= A_a = \frac{\pi}{4}(D^2 - D_1^2)(2 + n_d) + L_g \,.\, \pi(D + D_1)$$

Since the heating is proportional to the watts expended per unit area of cooling surface, the maximum temperature rise occurring at the armature surface, and as determined by the increase of resistance of the winding, is expressed by the equation

$$T_a \doteq K_a \times \text{watts per dcm}^2 = K_a \frac{W_a}{A_a}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (93)$$

The heating coefficient K_a gives the temperature rise in degrees centigrade per watt per square decimetre of cooling surface, and since this for any particular size of machine must necessarily depend upon the manner in which the machine is ventilated, the exact value of the coefficient should be determined experimentally for each type of construction. For standard types of construction with rotating field magnets the value of K_a ranges from 2.0 to 2.3; that is, in good machines a cooling surface of from 5 square centimetres to 6 square centimetres per watt lost should be provided. In using the above formula it is, of course, assumed that the rate of radiation from the armature is unaffected by the rotation of the field system. For the same reasons as mentioned in connection with stationary field coils, this condition is approximately fulfilled only when the machine is loaded. For a revolving armature the temperature rise will be

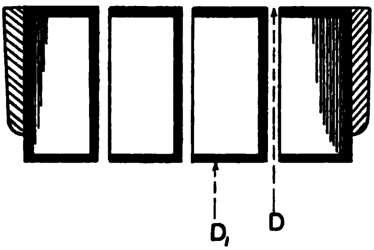


FIG. 250.—Cooling surface of armature core.

$$T_a = K_a \frac{W_a}{A_a(1 + 0.1\, v_m)}$$

where v_m is the peripheral speed of the armature in metres per second. The heating coefficient K_r has a value ranging from 2.5 to 3, according to the construction of the armature. The values of the heating coefficients K_m and K_a have been worked out for the designs tabulated on pages 400 to 417, and these should prove useful in selecting a value of the heating coefficient for any new design.

Heating and Ventilation of Turbo-alternators. — Under otherwise equal conditions the output of an electrical machine increases approximately in proportion to its volume or as the cube of the linear dimensions, whilst the radiating surface only increases as the square of the same. Hence there must be a limit of size where the surface is no longer sufficient to radiate the heat generated in the machine. This is the reason, for instance, why transformers above a certain output can only be built with artificial ventilation.

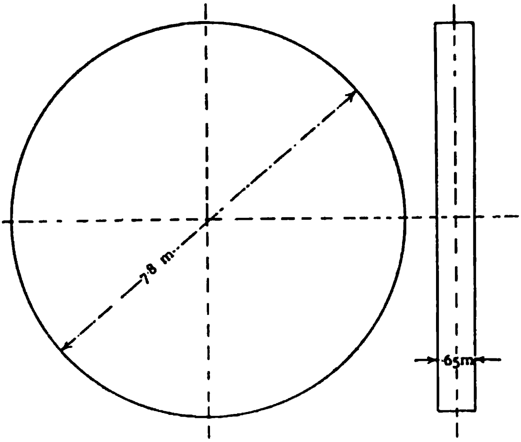


FIG. 251.—Over-all dimensions of 2000-K.V.A. slow-speed alternator.

In the case of turbo-generators the over-all dimensions are much smaller than for medium-speed machines of the same output, whilst the energy loss may be approximately the same in both types. The external surface of the high-speed machine will therefore be quite inadequate to dissipate the heat and keep within the permissible temperature rise. In order to make this quite clear, Figs. 251 and 252 give the over-all dimensions of a slow-speed alternator for 2000 K.V.A. and for a turbo-alternator for 6000 K.V.A. The slow-speed machine would go into a cylinder of 7.8 metres diameter and 0.65 metres length, which gives a volume of 31 cubic metres and a surface of 112 square metres. The total losses in this machine were about 100 K.W. The corresponding data for the turbo-alternator are 22.4 cubic metres and 45 square metres. However, the losses in this machine are about 250 K.W., or 2.5 times as great as in the other machine. For every 100 K.W. loss there is therefore only a volume of 9 cubic metres available as against 31 cubic metres in the slow-speed machine, and the available surface is only 18 square metres per

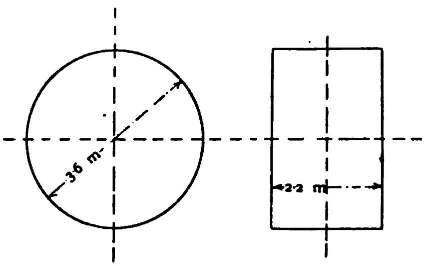


FIG. 252.—Over-all dimensions of 6000-K.V.A. turbo-alternator.

100 K.W. as against 112 square metres in the first machine. This concrete example shows how essential it is to provide special means for dissipating the heat, which is generated in a relatively small space. This is the more important, as the high-peripheral speed at which modern turbo-alternators run is liable to produce noise, and in order to reduce this as much as possible it is necessary to enclose the whole machine. The conditions of cooling by simple radiation and conduction to the surrounding air are therefore made still worse. In consequence of this the rotors of turbo-alternators must be designed so that, either by their own shape or by

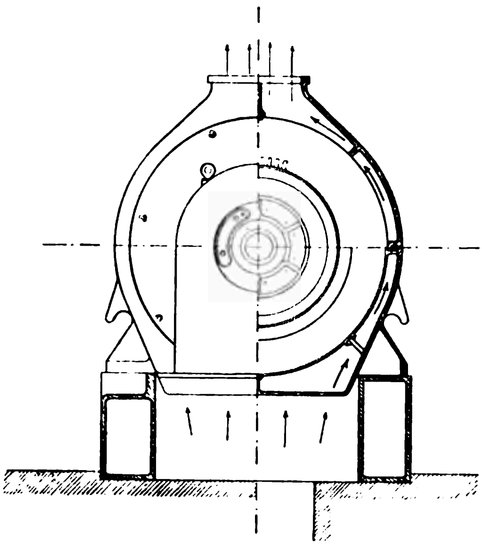


FIG. 253.

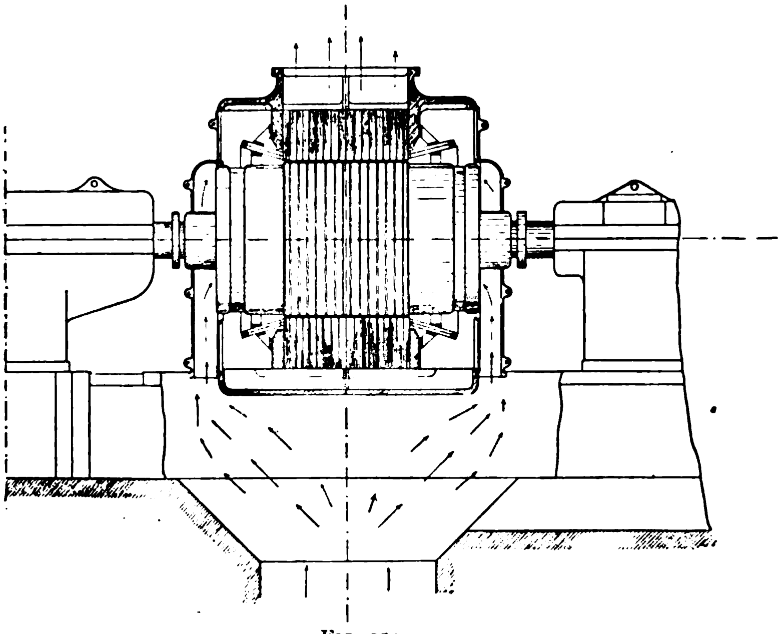


FIG. 254.

additional fans at their end they produce a strong draught through the machine.

For the ventilation of turbo-alternators the usual method adopted is to extend the stator housing at the end to form two chambers covering the hub (see Figs. 253 and 254). Into these chambers air is admitted from below. This air is caught by fans fastened on the rotor and forced into the shell of the housing, through the ducts between the laminations and out at the vent holes at the top.

In the case of very large units forced-draught ventilation is also employed, the usual plan being to mount a high-speed blower at one end of the bed-plate, direct coupled, the blower supplying air under a considerable head to the inside of the bed-plate, which is sealed with the exception of an opening to the frame of the enclosed generator, the hot air exhausting up the chimney.

Calculation of Air.—The quantity of air required for cooling is derived as follows:—Let t_1 denote temperature of air at inlet, and t_2 that at outlet, then since one cubic metre of air weighs 1.2×10^3 grammes at 15° C., the quantity of heat dissipated per cubic metre of air

$$= S(t_2 - t_1)\ 1.2 \times 10^3 \text{ calories}$$
$$= 4.2\ S(t_2 - t_1)\ 1.2 \times 10^3 = (t_2 - t_1) 1.2 \times 10^3 \text{ joules}$$

where S = specific heat of air = 0.238. Since the number of cubic metres required per kilo-joule $= \dfrac{1000}{(t_2 - t_1)\ 1.2 \times 10^3}$, the volume of air required per second per kilowatt dissipated

$$= \frac{1000}{(t_2 - t_1)\ 1.2 \times 10^3} = \frac{0.83}{(t_2 - t_1)} \text{ cubic metres}$$

Let T° Centigrade denote the permissible temperature rise above surrounding air, then the usual practice is to allow for a 0.5 T° rise in temperature of the cooling air between inlet and outlet; hence, if KW denotes the kilowatts to be dissipated, the quantity of air required is

$$V = \frac{0.83}{0.5\ T^\circ} \cdot KW = 1.66\ \frac{KW}{T^\circ} \text{ cubic metres per second}$$

$$100\ \frac{KW}{T^\circ} \text{ cubic metres per minute}$$

In deriving this equation the heat radiated from the case has not been taken into account, which allows for a certain factor of safety.

Heating Tests.—In order to determine the temperature rise of an alternator, the machine should be kept running under normal full-load conditions until the temperature approaches a steady value. Since the time during which an alternator of large output would have to be run for this purpose may range from ten to twenty hours, such a heat test would prove very expensive if the machine were actually

run fully loaded. Numerous methods have therefore been devised by means of which the heat test may be carried out with the expenditure of only a relatively small amount of power. When two similar machines of equal capacity are available they may be coupled together and loaded as for the Hopkinson efficiency test for D.C. machines. In the case of large alternators, however, there not only exists the inconvenience and expense of bringing them into mechanical connection, but it seldom happens that two similar large alternators are available for testing at the same time. This has rendered it necessary to divide a single large alternator electrically into two units,—namely, into a generator unit and a motor unit,—and to circulate power between these units. For this purpose various methods have been proposed, of which, however, only a few have been successful in practice. For any test to have commercial value it is obvious that the losses given by that test must be approximately the same as under actual conditions; that is to say, they must have the same magnitude and produce the same heating effect as the actual losses. When these conditions are fulfilled the values of the temperature rise given by the test will be correct. The only method which has been used to any great extent in testing modern alternators is that due to Behrend.

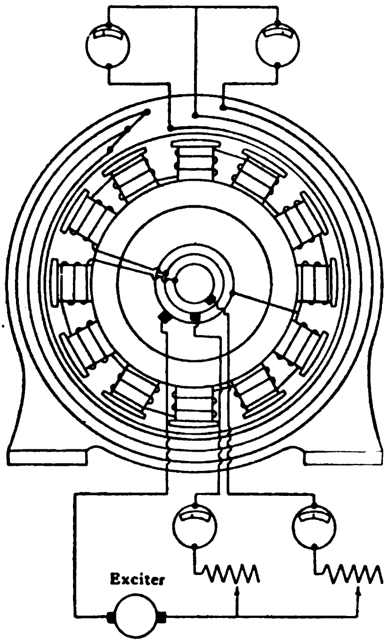
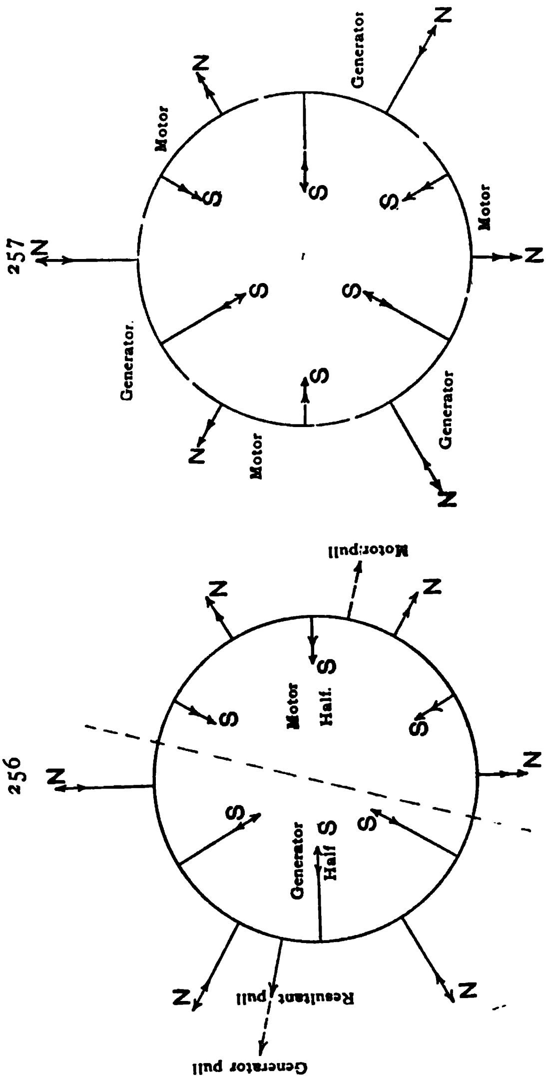


FIG. 255.—Scheme of connections for Behrend's heat test.

Behrend's Method.—This method is used to a very great extent in America, and, as will be seen from Figure 255, consists merely in splitting up the field into two halves. In the case of a revolving field alternator the mid point of the field winding is connected to the shaft, which then serves as a common terminal for both halves of the field. If full-load current be then passed through each branch no current whatever can flow in the armature, provided the system is symmetrical. By gradually reducing the excitation of one branch, the current in the other branch remaining constant, the current in the short-circuited armature coils can be raised to any desired value,—for instance, to full-load. The greatest objection to this method is that great unbalanced strains

are put on the machine, thus causing vibration, which in some cases would be positively dangerous. To explain this, Figure 256 has been drawn to illustrate diagrammatically the magnetic unbalancing of



FIGS. 256 and 257.—Diagram showing unbalanced pull in Behrend's test.

⟶ Flux in generator poles due to field ampère turns.
⟶ Flux in motor poles due to field ampère turns.
⟷ Resultant flux in generator poles.
⟶ Resultant flux in motor poles.

the 12-pole machine shown in Figure 255, which occurs when the same is tested by this method. The meaning of the various arrows is explained on the diagram. With a weak excitation on one half of the machine it is obvious that, with an open-circuit armature, the magnetic pull on the generator side will be much greater than that on the motor

side. On short-circuit the circulating current set up in the armature winding will be practically 90° out of phase with the E.M.F., and will lag in the generator and lead in the motor part of the winding. The generator field will thus be weakened whilst the field of the motor is strengthened. This effect will tend to equalise the field of the motor and generator poles, but does not remove all magnetic out-of-balance. To further diminish the pull on the generator side, the number of generator poles is sometimes made to exceed the number of motor poles, the flux per pole of the former being made weaker in proportion to the increased number. However, from the very nature of the test there must always be an unbalanced magnetic pull, for the current is due to the E.M.F. set up by the resultant flux—*i.e.* total generator flux *minus* total motor flux—and as soon as the motor pull equals the generator pull the flux in the motor portion equals that in the generator, and no current can flow owing to both E.M.F.'s being equal and opposite. In the case of slow and medium-speed alternators this resultant magnetic pull may not be so serious as to make the Behrend test impossible; but with high-speed machines this method becomes dangerous, and in some instances is absolutely prohibitive.

In order to overcome the above difficulties resulting from unbalanced pull the motor or weakly excited poles could be symmetrically interspersed amongst the generator or strongly excited poles, the field still being split up into two halves. For example, in a 12-pole machine the coils on poles 1, 2; 5, 6; 9 and 10 would form one group, the other group consisting of 3, 4; 7, 8; 11 and 12. As will be seen from Figure 257, the whole magnetic system is balanced with respect to the shaft, and mechanically this modified method is quite satisfactory.

Electrically both the above methods are open to objections, for the following reasons:—In the first place, since only about half of the field coils carry the full-load current, it is obvious that the proper heating will only occur on those poles, whilst the heating of the motor poles will be considerably below normal. What the effect of this on the other parts of the machine might be in the case of an open-type alternator it is difficult to estimate, although the influence on the cooling of the stator might be appreciable. With turbo-alternators which have generally to be totally enclosed this is a most important factor; for, with only a fraction of the field heating the internal temperature will be obviously lower than under actual full-load conditions.

As regards the stator, although with full-load armature current there is obtained the proper copper loss, it is very improbable that the temperature rise of the stator will be correct on account of the core losses not being the same as under actual full-load conditions. This is at once apparent from the fact that only the core loss under the generator poles will be that corresponding to normal conditions.

Another method of carrying out the heating test has recently been devised by S. P. Smith * in which all the field coils are in series and carry the full-load exciting current, in consequence of which there is obtained the full heating of the rotor. This method, however, is limited to machines with twelve or more poles.

* For full description see *Inst. of Elect. Engineers Journ.* (1908), vol. xlii. pp. 196-208.

CHAPTER XI

ALTERNATORS

Parallel Working

When a number of alternators, designed for the same frequency and equal terminal voltages, are required to supply energy to a given network, they must be connected in parallel. When paralleling direct-current generators it is only necessary to ensure that the incoming machine is at approximately the same voltage as that of the bus bars. The paralleling of an alternator is somewhat more complicated, for, in addition to equal E.M.F.'s, there must also be equal periodicities and equality of phase.

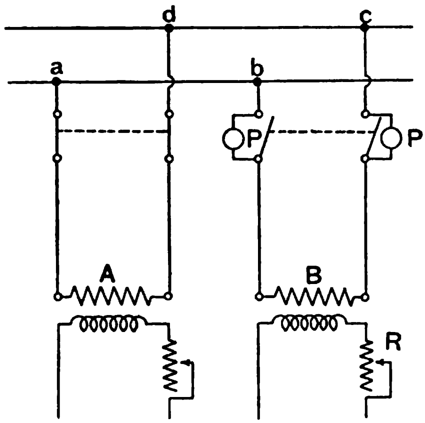


Fig. 258.—Synchronising of single-phase alternators.

To connect an alternator B in parallel with an alternator A, B must be raised to the same frequency and voltage as A, and in order to obtain synchronism of phase the incoming alternator should generate an E.M.F. which at any instant has the same magnitude and direction as the E.M.F. generated in the other alternators supplying current to the system. In order to observe this state, phase lamps P are joined up as shown in Figure 258. Suppose alternator A to be normally loaded, and that machine B is running but has not yet been excited. An alternating current will then pass from the working alternator through the circuit *a b* B *c d* A, thus causing the phase lamps to glow. If the field of the incoming alternator be now excited and the voltage adjusted by the resistance R so as to give the same voltage as the bus bars, an alternately opposing and assisting E.M.F. will then be set up in the lamp circuit. Assuming the frequency of alternator B to be approximately the same as

that of A, then as long as the machines are not in synchronism there will be a difference of potential across the contacts of the switch in each lead, and the lamps will show various states of incandescence. When the two machines are completely out of phase the voltage across the contacts of each switch will rise to about twice that of the bus bar pressure, thus causing the lamps to attain their maximum brilliancy. Ultimately when, by fine adjustment of the speed of the prime mover, machine B is brought into step with A, the E.M.F.'s of A and B acting on the lamp circuit will be directly opposed to each other, so that the lamps will be black. If the switch then be closed the two alternators will continue to run in parallel, the ratio in which the loads divide between the two machines being regulated by the governors of the respective prime movers. When once the machines are working properly, then, as will be afterwards explained, the mutual control which they exert electrically on one another tends to keep them in synchronism.

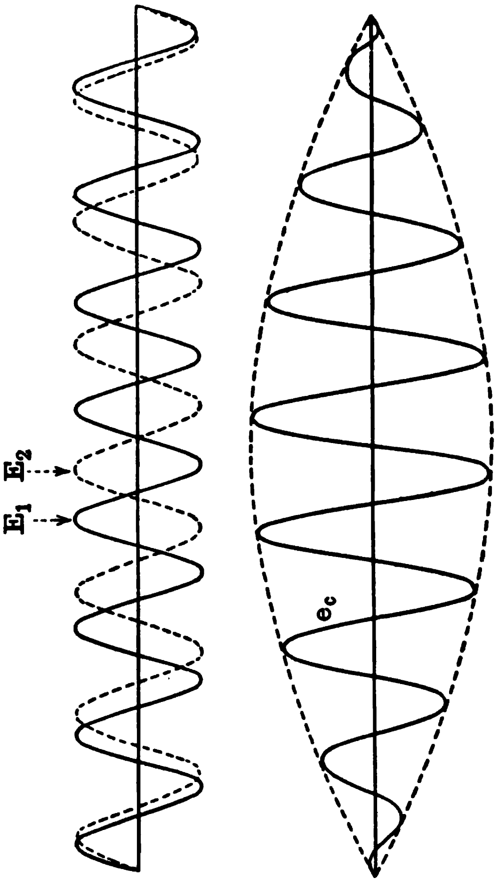


FIG. 259.

The curves of Figure 259 have been drawn to illustrate the nature of the pulsations in alternation of the phase lamps when two alternators, generating E.M.F.'s of the same amplitude E_{max} and wave form, are almost in synchronism. Starting from an instant of time when the instantaneous voltages induced in the respective machines are rising from zero to a positive maximum, let the full line curve E_1 represent the instantaneous values across the bus bars. This E.M.F. is expressed by the equation:

$$e_1 = E_{max} \sin pt$$

Suppose that the incoming generator gives an E.M.F. which differs in frequency from the bus bar E.M.F. by a very small amount δp, then the voltage of the alternator being synchronised is given by the equation

$$e_2 = E_{max} \sin (p + \delta p)t$$

and is represented by the curve E_2. If the algebraic sum of the two curves E_1 and E_2 be plotted as a function of time, then the periodic curve e_c is obtained which gives the difference of potential on the lamps, proportional to which is the current through them. Since the resultant E.M.F. can be expressed analytically, thus

$$e_c = E_1 + E_2 = E_{max} \sin pt + E_{max} \sin (p + \delta p)t$$
$$= E_{max} \cos \frac{\delta p \cdot t}{2} \cdot \sin \left(p + \frac{\delta p}{2}\right) t$$

it will be seen that the lamps are traversed by an alternating current the frequency of which is equal to the mean frequency of the two component E.M.F.'s. The amplitude values of the lamp current vary periodically, thus causing alternate intervals of brightness and darkness. If a curve plotted which passes through the amplitude values of the train of curves e_c it will represent a single sine function given by the expression

$$2\, E_{max} \cos \frac{\delta p}{2} \cdot t$$

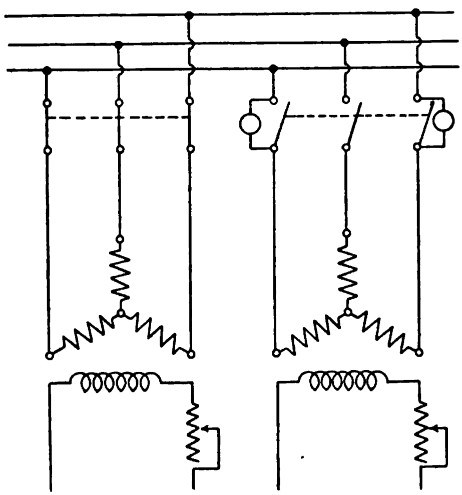


FIG. 260.—Synchronising of 3-phase alternators.

The dotted curve drawn through the amplitude values also shows that one "beat" or pulsation of the light from complete darkness to maximum brightness and back to darkness occupies an interval of time corresponding to that in which the number of complete periods passed through by the voltages E_1 and E_2 differ by 1. For example, if the frequency of E_1 and E_2 be 25 and 24.5 respectively, then at the end of two seconds the lamps will have passed through one pulsation. By fine adjustment of the speed of the prime mover each pulsation of light should be extended to from 7 to 10 seconds before the main switch is closed.

The above method of synchronising alternators has been discussed with reference to single-phase machines, but in the case of polyphase alternators the same procedure would be adopted. The corresponding

connections between the machines and the bus bars for a 3-phase system is shown in Figure 260, the synchronising lamps being placed across the two outside blades of the main switch. When a new polyphase machine is first connected to the bus bars it is necessary to verify the connections of each phase by using two sets of lamps as above. If the connections are correct both sets of lamps will, at any moment, give the same illumination.

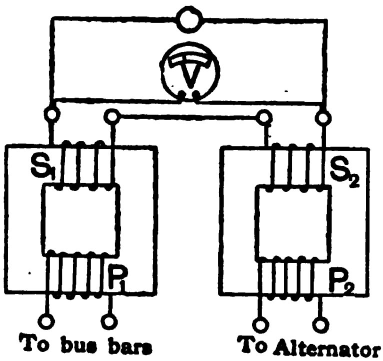


FIG. 261.

Synchronisers. — The insertion of phase lamps across the contacts of the main switch is the simplest form of synchroniser, but it is only suitable for low-voltage machines. The other forms of phase indicators in general use are, however, based upon somewhat similar principles. In order to make use of synchronising lamps for high-voltage machines they must be inserted in a circuit formed by connecting the secondaries S_1 and S_2 of two small transformers in series; see Figure 261. The primary P_1 of one transformer is connected across the bus bars, while that of the other P_2 is across the terminals of the incoming machine. The

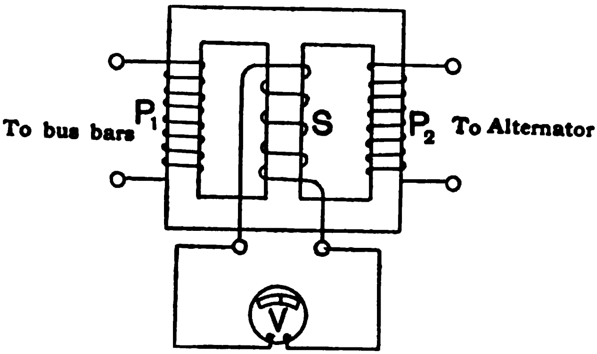


FIG. 262.

connections of the secondaries are generally so arranged that when the incoming machine is in synchronism with the machine already running the transformers assist each other in lighting the lamp. Another method is to wire the secondaries in opposition, in which case synchronism will occur when the lamps are black. The former arrangement is the best, as the moment of maximum brightness is more definitely indicated than the middle of an interval of total darkness.

In order to enable the switchboard attendant to ascertain more accurately the correct moment of closing the switch, a synchronising voltmeter V is frequently provided in addition to the lamps, the latter forming a convenient visual signal for the engine driver. When lamps are used the maximum voltage per lamp should be somewhat less than their rated voltage. Instead of using two separate transformers, a more usual practice is to employ a single one with three cores arranged side by side. The primaries P_1 and P_2 (Figure 262) are wound on the outside cores, and a single winding S on the central core is connected to the synchronising lamps or voltmeter. The primary coils are so wound that when their respective voltages are in phase

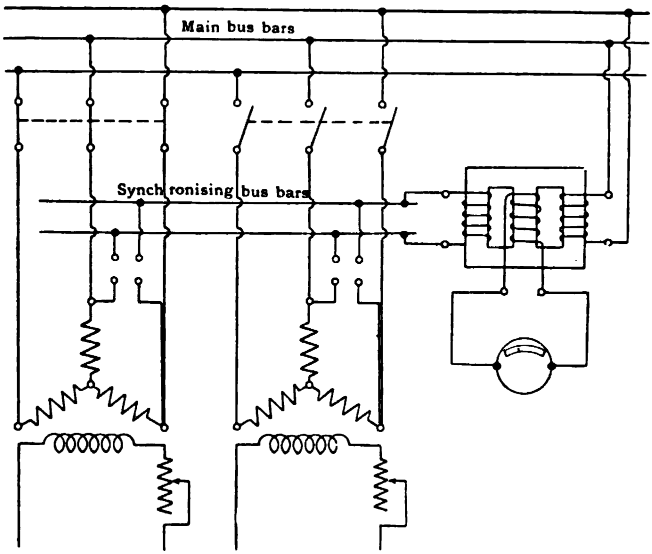


FIG. 263.

they each tend to send a magnetic flux through the central core in the same direction. The electro motive force induced in the coil S will therefore be a maximum, and the lamp in series with it will be brightest, when the voltage induced in the incoming generator is in synchronism with that of the bus bars.

In a large generating station it is quite unnecessary to provide a synchronising set for each machine, as the same synchroniser could be used for several machines in succession as required, the primaries of the transformers being switched off as soon as the main switch is closed. In order to enable the same synchronising set to be used for all machines, it is necessary to provide synchronising bus bars, the scheme of connections for moderate voltage machines being as shown in Figure 263.

The forms of synchronisers so far described, although indicating any difference of frequency or phase, are incapable of showing whether the incoming generator is running above or below synchronism, for the light will fluctuate in the same way for speeds 5 per cent. above or below the synchronous speed. The time occupied in paralleling an alternator may therefore be somewhat prolonged. Hence the more modern types of synchronisers are provided with additional visual signals, which indicate to the engine-driver when the speed is too high or too low.

A rotary synchroniser having these refinements has been independently invented by Everett-Edgecumbe in this country and Paul Lincoln in the United States. This instrument is in principle a small alternating-current motor, the rotor of which rotates in one or other direction according as the incoming generator is running too fast or too slow. Referring to Figure 264, the rotor, consisting of a laminated core running in ball bearings and carrying a pointer, is wound with an ordinary 2-phase winding, the coils A,A being connected in series with one another and with a non-inductive resistance or lamps R, whilst the coils B, B are in series with a choking coil C. The two circuits are connected in parallel across the terminals T_1 and T_2, current being led into the rotor windings through the slip rings S. The rotor terminals are connected to those of the incoming generator, and by means of the choking coil a lag of nearly 85 degrees is produced between the two circuits, and an almost uniform rotating field is the result. The stator consists of a 4-pole laminated core, which is also provided with a 2-phase winding, the 2-phase current being produced in a manner similar to that of the rotor. The stator terminals are connected up to the bus bars, and the internal connections are such that the direction of rotation of the stator and rotor fields are the same. When the incoming machine is in step with the other machines connected to the bus bars, the two fields will be rotating at the same speed, and consequently the rotor will stand still. If, on the other hand, the alternator to be paralleled is above or below the synchronous speed, the rotor will have to revolve against or with the

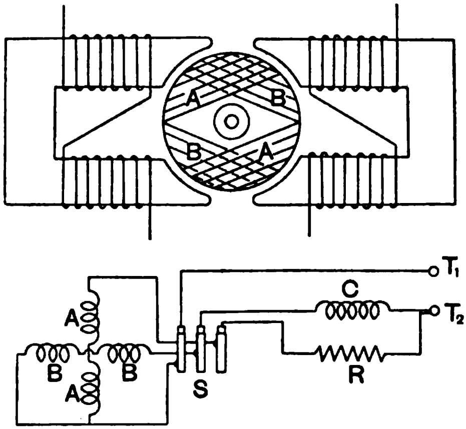


Fig. 264.

flux respectively in order that the two fields may keep in step. Thus, for example, a clockwise rotation of the rotor will show that the incoming machine is above or below synchronism according as the rotation of the field is counter-clockwise or clockwise. The speed of the rotation is also an indication of the difference between the two frequencies, one complete revolution representing a difference of two cycles per second. These synchronisers can be fitted with an arrangement of two lamps, coloured respectively red and green, which are automatically exposed to view according to the direction of rotation of the rotor. The engine-driver can thus see at a considerable distance whether the prime mover is going too fast or too slow. When this synchroniser is used on a 3-phase circuit, its stator

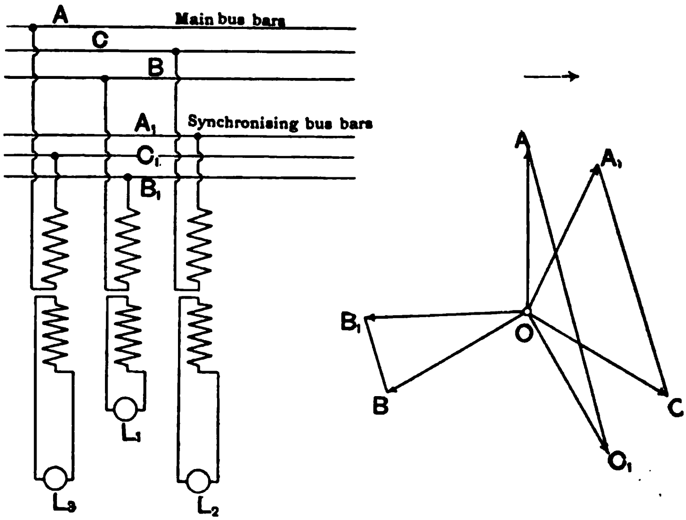


FIG. 265.—Three-phase synchroniser.

terminals are connected across two of the bus bars, and its rotor terminals to the corresponding terminals of the generator. Of course for high-tension generators the synchroniser must be connected up to the bus bars and the machine through step down transformers.

In a 3-phase system it is possible to observe from the varying degrees of brightness of three lamps whether the incoming alternator is running too fast or too slow. The three lamps L_1, L_2, and L_3 are set up in triangular fashion and connected across the secondaries of a 3-phase transformer, whose primaries are connected between the main and synchronising bus bars, as shown in Figure 265. Suppose the E.M.F. of the bus bars to be represented by the vectors OA, OB, and OC, then if the incoming alternator be running at synchronous speed but not quite in phase with the bus bar pressure, its E.M.F.

vectors would be represented by OA_1, OB_1, and OC_1. The voltage across the lamps L_1, L_2, and L_3 will then be proportional to the vectors BB_1, A_1C, and AC_1, so that L_3 will be in a state of higher incandescence than L_2, whilst L_1 will be quite dim. When equality of phase is attained the latter will be completely black, and L_3 and L_2 will be equally bright. If the incoming generator be running above synchronism the vector system A_1, B_1, C_1 will revolve faster than the system A, B, C, and the lamps will experience a cyclic change of terminal pressure causing the lamps to glow in the succession L_1, L_2, and L_3. When the machine runs too fast the system A_1, B_1, C_1 will move more slowly than A, B, C, and the lamps will now glow in the order L_3, L_2, and L_1. Hence the order in which the lamps brighten will be an indication of the relative speed of the incoming alternator. This form of synchroniser, due to Siemens and Halske, is largely used in connection with 3-phase plants.

Effect of Engine Governor on Distribution of Load.—In a steam-engine the function of the governor is to regulate the amount of steam admitted to the cylinders so that, for considerable fluctuations in load, the speed is maintained within certain narrow limits. This regulation is generally effected by throttling the steam and thereby varying the pressure in the cylinders. At the lower limit of speed the centrifugal force of the governor balls will not be sufficient to move them out against the controlling springs, so that there is no throttling and the steam pressure is a maximum. At a somewhat higher speed the balls move out to a position where the greater centrifugal force is again balanced by the extended springs. As the position of the governor balls alter they are made to operate some mechanism which adjusts the position of the throttle valve and thereby reduces the mean pressure in the cylinders. Should the speed go on increasing, the balls will ultimately move out to their fullest extent, in which case the supply of steam is entirely cut off. For each position of the governor there is therefore a definite speed, and if the mean speed of the engine, as controlled by the governor, be plotted as a function of the load on the alternator, a curve such as ω_m, Figure 266, will be obtained. The difference between the speeds corresponding to no-load and full-load, *i.e.* for minimum and maximum steam pressures, is called the "drop"

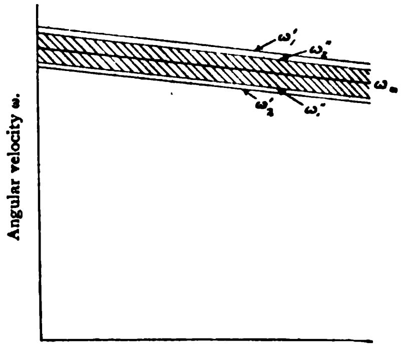


FIG. 266.

or "regulation" of the governor, and this varies according to the type of governor from 3 to 6 per cent. of the no-load speed.

When several alternators are worked on the same bus bars, the amount of power supplied to the network by each machine depends almost entirely on the angle of advance θ_s (see Figure 276) of its induced voltage E_i with respect to the terminal or bus bar voltage E. The greater the vector difference between E_i and E, the greater will be the voltage available for overcoming the armature impedance, and hence the greater will be the current that the armature can supply. The magnitude of the angle θ_s is dependent only on the load of the alternator, and if the speed of the engine alters with the load, it can be regulated by the governor.

During each revolution of a reciprocating engine the angular velocity varies, as shown in Figure 272, between two limits ω_1 and ω_2, and if $\omega_m = \frac{\omega_1 + \omega_2}{2}$ denotes the mean angular velocity, the extent of this speed variation can be expressed thus—

$$\text{Cyclic irregularity} = \sigma = \frac{\omega_1 - \omega_2}{\omega_m}$$

With a very sensible governor these fluctuations in the angular velocity of the prime mover would set up periodic oscillations of the governor and so cause hunting. To avoid such a tendency the governor should be damped sufficiently so that it does not respond to these regular fluctuations in the engine speed.

Owing to the friction of the mechanism, valves, etc., the governor of any steam-engine will always have a certain amount of lag,—*i.e.* there must be a certain change in speed before the throttle valve operates. If ω'_1, and ω'_2 denote the greatest and least angular velocity which is possible without the governor acting, then the expression

$$\epsilon = \frac{\omega'_1 - \omega'_2}{\omega_m}$$

may be termed the coefficient of insensibility. In Figure 266 there have also been plotted the values of ω'_1 and ω'_2 as a function of the load. Now since the angular speed during one revolution can vary within the limits $\pm \frac{1}{2} \sigma \omega_m$ without the governor being affected, this amount must be subtracted from ω'_1 and added to ω'_2 so as to give the curves ω''_1 and ω''_2 which represent respectively the greatest mean velocity and the smallest mean velocity which the engine can have without the governor acting. With a given load on the machine the mean angular velocity of the set can therefore vary between narrow limits, that is, the governor possesses a certain amount of stability shown by the shaded portions below and above the mean curve of Figure 266. With steam or water turbines the cyclic irregularity would be zero, so that the curves ω'' and ω' would coincide.

When several identical and equally excited alternators are connected to the same bus bars and are driven by identical engines, the load will not necessarily be equally shared between all the machines. For if there be drawn in Figure 267 a horizontal line corresponding to the mean angular velocity of all the machines, it will, for a considerable distance, be inside the shaded part, and thus at this speed the machines can be loaded quite unequally. This difference of loading will be greater the flatter the speed curves are—*i.e.* the smaller the drop of the governor. If the governors are of isochronous or astatic type, the speed will remain constant independent of the position of the governor balls. The curves of Figure 267 will therefore be horizontal, and the load between the various machines will remain as adjusted. Such governors are therefore quite unsuited for engines driving alternators in parallel.

With non-isochronous governors the load will be the more uniformly distributed between the various machines, the less the difference between the two curves ω''_1, and ω''_2—that is, the more the value of σ and ϵ approach each other. The governor must, however, be damped sufficiently so that ϵ is never less than σ, for otherwise the governor would operate every revolution, and this, as already mentioned, would cause hunting. Further, with a governor of given stability the steeper the speed curves the shorter will be the length of AB, and the more uniformly will the load distribute itself between the various machines. On this account a large governor drop is an advantage in so far as parallel running is concerned, but a large drop also involves a large alteration in the frequency of the current between no-load and full-load. Hence the regulation of the governor should not be chosen greater than is actually necessary for good parallel working. Where the frequency must be the same for all loads, it is necessary to alter the sensitiveness of the governor as the load varies, and this may be performed either by the hand or automatically. For lighting plant hand regulation is quite sufficient, but for power plant, where the load undergoes rapid fluctuations, automatic regulation, which usually takes the form of an electric relay controlled from the switch-board, is essential.

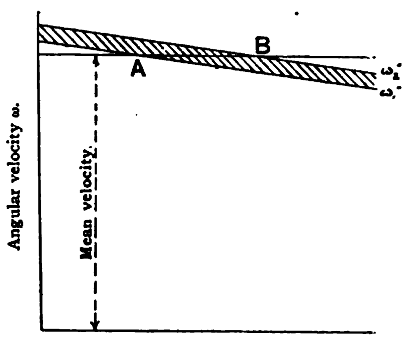


FIG. 267.

In Figure 268 the mean angular velocity ω_m has been plotted as a function of the load on the alternator from three identical engines, the governors of which had been adjusted to different positions in

each case. The speed curves, though running almost parallel to each other, will owing to the different settings of the governors not coincide, so that for a given angular speed the mean loads W_1, W_2, and W_3 will be different. Hence it is possible to alter the loading of any machine by simply adjusting the setting of the governor. If the position of the governor on any machine be altered in such a direction as to cause the engine to accelerate, the angle θ_r, and hence the load W on the alternator, will increase. To shift the load from one machine to another the governor of one machine is shifted in one direction as far as the governor of the other machine is shifted in the opposite direction, and when the load has been entirely removed the machine can then be disconnected from the bus bars without disturbing the working of the other sets.

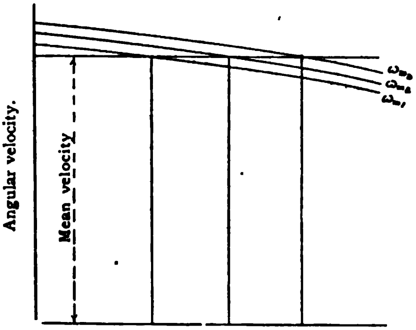


FIG. 268.

Effect of Varying the Field Excitation.—In order to examine how the parallel operation of alternators is influenced by varying the field current of one set, it will be supposed that the alternator A to be considered has been duly synchronised, and that the supply of steam, or gas, to the prime mover is just sufficient to overcome the friction and iron losses of the set, no energy being given out to the network. Let E_1 denote the bus bar voltage, and suppose that, at the instant of paralleling, the open-circuit or induced voltage of the considered alternator was also equal to E_1. If, after paralleling, the field excitation of A be now increased, its induced E.M.F. will rise to a value E_2, producing a difference $E_2 - E_1$ between the voltage of the bus bars and the machine. This would appear to mean that the alternator A should take a share of the load, the power developed by the engine being correspondingly increased. But the engine cannot develop any additional power, as the throttle valve just admits sufficient steam to cope with the no-load losses of the set and keep the alternator running at synchronous speed. What actually happens is that the voltage $E_2 - E_1$ acting through the windings of machine A and the windings of other machines connected to the bus bars sets up an alternating current I_c between machine A and the bus bars. Since the ohmic resistance of the armature windings is small in comparison with the reactance, the current I_c will lag approximately 90 degrees with regard to $E_2 - E_1$. This wattless current will produce a direct demagnetising effect on the main field of alternator A, thus altering the vector sum of the induced

E.M.F. and the reactance voltage, until the terminal voltage becomes equal to that of the bus bars. The result of an increase in excitation is therefore not an increase in the induced voltage, but a correction current I_c circulating between the bus bars and the alternator under consideration. The phase relationship between the current I_c and the various E.M.Fs. is shown in Figure 269, where the lettering E_1, E_2, $E_2 - E_1$ has the same meaning as above.

Should the excitation of the considered alternator be reduced to a lower value than that corresponding to equal terminal and bus bar voltages, the current I_c will be in the opposite phase and will lead with respect to E_2, and therefore increase the magnetism of A, bringing the terminal voltage up to the same value as that of the bus bars. The above remarks show that it is still possible to operate an alternator in parallel with others, though unequally excited, but in order to reduce to a minimum these circulating currents,—together with the resultant copper loss,—the various machines working in parallel should be nearly similarly excited. Hence, whenever an alternator is connected in parallel with others, its excitation should be adjusted so that the main ammeters indicate approximately zero. More steam can then be admitted, thus allowing the alternator to pick up the load. The loading and unloading of a direct current dynamo which is working in parallel with others depends both on the supply to the prime mover and on the field excitation. With alternators, on the other hand, the loading and unloading depends entirely upon the supply to the prime mover, the variation of field current only producing a wattless armature current.

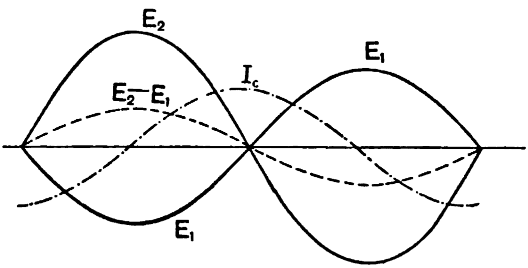


FIG. 269.

Phase Displacement and Synchronising Current.—If the steam admitted to the prime mover of a synchronised alternator is increased, the speed of the alternator will momentarily increase above the synchronous speed, and thus cause the rotating field system to lead with respect to the phase of the other alternators connected to the bus bars. In other words, the field system will be ahead of the synchronous position. The reverse will happen if the admission of steam is decreased. Suppose that, at any instant of time, θ, denotes the angle expressed in electrical degrees by which the field system is ahead of the synchronous position, then for this instant the phase relation between the alternator and bus bar pressures would be as in

Figure 270. The vectors OE_1 and OE_2' of equal magnitude represent respectively the phase relation between the bus bar voltage and the induced voltage of the considered machine when the rotor of the latter is revolving at a perfectly uniform speed corresponding to exact synchronism. On increasing the supply of steam any momentary acceleration of the rotor will cause the field system to overshoot the synchronous position by an angle θ_s, with the result that the vector OE_2 will now be θ_s degrees ahead of its former position. The voltages OE_1 and OE_2 will now have a resultant OE_r, which, since the armature resistance is small compared with the reactance, sets up the current OI_s approximately in quadrature with OE_r. Now, as OI_s

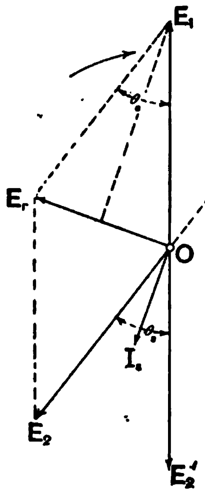


FIG. 270.

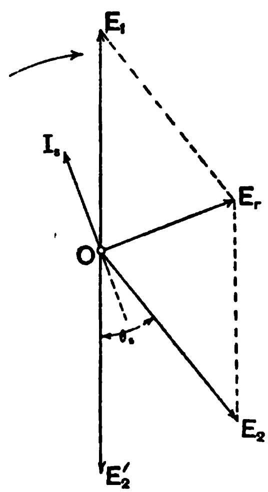


FIG. 271.

makes an angle of nearly $\pm \frac{\theta_s}{2}$ to each of the voltage vectors OE_1 and OE_2, the current I_s is nearly a true watt current. If Z_a denotes the armature impedance of the considered machine, then $I_s = \frac{OE_r}{Z_a}$. Since the component of OI_s, in phase with OE_2, acts in the same direction, the current I_s is generated with respect to E_2 and so tends to retard the alternator speed. Next, suppose that the field system, due to a decrease in the steam supply, falls slightly behind the synchronous speed. The vector OE_2 will now lag behind OE'_2 (see Figure 271), thus causing the vector OE_r of resultant E.M.F. to fall to the right-hand side of the ordinate reference axis. Since the current I_s set up by the resultant E.M.F. is in the opposite direction to OE_2, it is a motor current in relation to the alternator, and hence accelerates it. The circulating current I_s has therefore, for both positive and negative values

of θ_s, the effect of decreasing the divergence between the relative rotor positions of the considered alternator and the others connected to the bus bars, and besides doing actual work in heating its own circuit, keeps the machine in synchronism by maintaining the correct phase position. I_s is therefore known as the "synchronising current," the magnitude of which varies periodically with the phase angle θ_s.

In the case considered above it has been assumed that the machines represented by the vector OE_1 have an infinitely greater power than the considered alternator. A second case must, however, be taken into consideration; namely, where an alternator A_1 is working in parallel with another alternator A_2 of approximately the same output without any external load. Any phase displacement of A_2 relative to A_1 will now give rise to a synchronising current, $I_s = \frac{E_r}{2Z_a}$, where Z_a denotes the impedance of each machine. This current will affect machine A_2 in the same manner as described above, but in addition, A_1 will be oppositely affected. Thus if the phase displacement of A_2 be positive, the synchronising current is a generator current in relation to A_2 but a motor current with regard to A_1. The latter machine is thus accelerated owing to the increased input, whilst A_2 is retarded.

Free Oscillations.—When, owing to an increase in the steam supply to the prime mover, an alternator accelerates, the mass of its rotating system will obtain an increased momentum, and this, as already stated, causes the field system to lead the synchronising position by a small angle θ_s, and overshoot the new mean position. The resulting synchronising current exerts a torque which pulls the rotating system back. On its backward course the rotor gains sufficient momentum in the opposite direction to cause it to pass the correct mean position, whereupon the direction of the torque reverses, thus causing the rotor to again accelerate and pass the mean position. Oscillations about the mean position—or phase swinging, as it is sometimes called—are thus set up, and are accompanied by current surges between the alternator under consideration and the other machines connected to the bus bars. These free oscillations, when once started, would, if no damping forces were present, continue indefinitely. The oscillation of power between the bus bars and the alternator is, however, opposed by electrical and mechanical damping, which cause the oscillations to be of a gradually decreasing amplitude, until finally the alternator settles down to a steady state of running. Free oscillations similar to the above are also set up when an alternator is connected to the bus bars slightly out of phase, but in general they will prove quite harmless, so long as they are not large enough to throw the alternator out of step in the first instance, and provided no other oscillations interfere with them.

Synchronising Power.—The synchronising current which, as

already explained, is almost in quadrature with the resultant voltage E_r producing it, is expressed by

$$I_s = \frac{E_r}{Z_a}$$

where Z_a denotes the apparent impedance of the machine. The latter will be given by the quotient of the E.M.F. on open-circuit, and the short-circuit current for the same excitation and its value will be assumed to remain approximately constant independent of the excitation and load. If the output of the alternator under consideration be small compared with that of all the other machines connected to the bus bars, then the impedance of the other sets will be almost negligible in comparison. Now, suppose that when the machines are running in parallel the excitation be taken off all the alternators except the one under consideration. This will mean a dead short-circuit, and the short-circuit current would approximate to that which would flow if the terminals of the excited alternator were shorted by a heavy cable. If I_o denote the short-circuit current for the excitation that on the open-circuit generates the voltage $E = OE_2$ (Figure 270), then

$$Z_a = \frac{E}{I_o}$$

When this value of Z_a is substituted in the above equation, the following expression is obtained for the synchronising current—

$$I_s = I_o \, . \, \frac{E_r}{E}$$

From Figure 270 there is obtained the relation

$$E_r = OE_r \simeq 2 \, OE_2 \, . \, \sin \frac{\theta_s}{2} = 2 \, E \sin \frac{\theta_s}{2}$$

Hence,

$$\frac{E_r}{E} = 2 \sin \frac{\theta_s}{2}$$

$$\text{and } I_s \simeq I_o \, . \, 2 \sin \frac{\theta_s}{2}$$

When two alternators of the same size, and therefore having approximately the same impedance, are running in parallel, the conditions on short-circuit are quite different. The removal of the excitation from one machine would mean a short-circuit with single E.M.F. and double impedance, and therefore the current I_o would just be one half of that in the former case. In order to get the full short-circuit current through both machines it would be necessary to parallel the two machines fully excited and completely out of phase. Then the double E.M.F. shorted on the double impedance would generate the normal short-circuit current.

In the case of an alternator working in parallel with others of infinitely greater power, the synchronising power per phase corresponding to the current I_s is, from Figure 270, expressed by

$$W_s = EI_s \cos I_sOE_2 \cong EI_s \cos \frac{\theta_s}{2}$$

$$= EI_o \,.\, 2 \sin \frac{\theta_s}{2} \,.\, \cos \frac{\theta_s}{2} = EI_o \sin \theta_s \;*$$

Since all the phases of a polyphase generator have an equal effect, the synchronising power of an *m*-phase machine is given by the equation

$$W_s = mEI_o \sin \theta_s$$

As θ_s is small, $\sin \theta_s \simeq \theta_s$, and therefore

$$W_s = mEI_o\theta_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (95)$$

Oscillations due to Synchronising Torque.—If $\omega_m = \frac{2\pi R}{60}$ denote the mean value of the angular velocity of the rotor in radians per second, where R is the revolutions per minute, and M_s the synchronising torque in kilogramme-metres, then the synchronising power is also expressed by

$$W_s = M_s \times \frac{2\pi R}{60} \text{ kilogramme-metres per second}$$

$$= M_s \times \frac{2\pi R}{60} \times 9.81 \cong M_sR \text{ watts.}$$

When this value is substituted for W_s in equation 95, there is obtained the following expression for synchronising torque

$$M_s = \frac{m \,.\, E \,.\, I_o}{R} \,.\, \theta_s \text{ kilogramme-metres,}$$

Now for a given machine and full-load excitation, the factor $\frac{mEI_o}{R}$ is approximately constant, and denotes the synchronising torque f_s per radian of electrical displacement, *i.e.*

$$M_s = f_s \,.\, \theta_s$$

If θ denote the angle of mechanical displacement corresponding to the phase displacement θ_s, then $\theta = \theta_s p$ and

$$M_s = f_s p \,.\, \theta = c\theta$$

where p = number of pairs of poles,

and c = a constant $= \frac{m \,.\, E \,.\, I_o p}{R}$

Suppose that the load on the generator is slightly increased, then the rotor will lead the synchronous position, and the torque opposing the rotation of the rotor, which normally is M_n, will be $M_n + M_s = M_n + c\theta$. If now the additional load be removed, there will be a balance of

* See Rosenberg, "Parallel Operation of Alternators," *Inst. E.E. Journal*, 1909, vol. xlii. p. 524.

torque of amount $c\theta$ available for retarding the rotor of the generator, which accordingly will tend to return to the true synchronous position. After a time it will arrive in this position to which corresponds the torque M_n, but will then be moving with a velocity less than that corresponding to steady running. In consequence of the difference of velocity the generator will not exert the steady motion torque M_n, but there will be a balance of torque tending to accelerate it or retard it, according as its speed is less or greater than synchronous speed. For a small difference of angular velocity, the unbalanced torque will be proportional to the difference in velocity and $=k \cdot \frac{d\theta}{dt}$, where k is a coefficient denoting the moment per unit of angular velocity. Let Σmr^2 denote the moment of inertia of the rotor, then, since the moment of inertia of a rotating body multiplied by its angular acceleration equals the moment of the forces about the axis of the body, the general equation of the motion of the rotor is therefore approximately expressed by

$$\Sigma mr^2 \frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + c\theta = 0.$$

The solution of this differential equation for small values of $\frac{k}{\Sigma mr^2}$ is

$$\theta = \theta_0 \epsilon^{-\frac{k}{2}t} \cdot \sin(T_0 t + \alpha)$$

where $\frac{2\pi}{T_0}$ is the period of the oscillation, $T_0 \cong \sqrt{\frac{c}{\Sigma mr^2}}$ and θ_0 and α are constants depending on the initial conditions.

The importance or otherwise of the free oscillations depends almost entirely upon the sign of the coefficient k, that is, upon the way in which they are damped. If k is positive, the oscillations are of a continually diminishing amplitude and are finally damped out. On the other hand, if k be negative, even though very small, the amplitude of the oscillations will continually increase according to an exponential law until the machine falls out of step. In actual machines, however, there are a number of causes which give rise to real viscous forces, and generally overcome this tendency to instability. Such are the local currents induced in the substance of the armature conductors, and (generally but not always) the effect of currents induced by the oscillations in the pole pieces, the field coils, and in any special coils—*i.e.* damping coils—put there for the purpose. The eddy currents induced in the iron and the copper of an alternator are generally of such magnitude as to assign a positive value to k. The free oscillations are therefore quickly damped and stability of working will be assured.

Periodic Time of Free Oscillations.—The free oscillations of an alternator may be compared with those of a metal ring which is connected by means of a clock spring to a fixed centre. If the ring

be displaced by a momentarily applied tangential force out of its position of equilibrium, the controlling force tends to bring it back to its central position. The ring is thus set vibrating, and if the magnitude of the controlling force varies in direct proportion to the displacement, the motion of any point P on the ring will be a simple harmonic motion with regard to time. Let Σmr^2 denote the moment of inertia of the ring, then if f_m be the restoring moment which would act if the ring were rotated from its position of equilibrium through unit angle, the time of a complete oscillation is expressed by

$$T = 2\pi\sqrt{\frac{\Sigma mr^2}{f_m}}$$

In the case of an alternator, the controlling moment of the spring is replaced by the synchronising torque, and the ring by the rotating field system. Hence, if Σmr^2 denote the moment of inertia of all the rotating parts (crank, disc, etc., included), and f the synchronising torque per radian of mechanical displacement which acts at the instant the alternator deviates from the synchronous position, then the time of a free oscillation is

$$T_1 = 2\pi\sqrt{\frac{\Sigma mr^2}{f}}$$

It is, however, usual to express the periodic time in terms of the electrical displacement θ_s; hence, since $f = f_s \times p = \frac{M_s}{\theta_s} \cdot p$,

$$T_1 = 2\pi\sqrt{\frac{\Sigma mr^2}{p \cdot \frac{M_s}{\theta_s}}} = 2\pi\sqrt{\frac{\Sigma mr^2}{\frac{60 \cdot p}{2\pi R} \cdot \frac{W_s}{\theta_s}}}$$

where $\frac{W_s}{w_m} = \frac{W_s \cdot 60}{2\pi R}$, has been substituted for M_s. Now $\frac{W_s}{\theta_s} = m\mathrm{EI}_o$ and $p = \frac{60 \sim}{R}$; hence, substituting these values in the above equation, the expression for the periodic time of a free oscillation becomes

$$T_1 = \cdot 042\ R\sqrt{\frac{\Sigma mr^2}{m\mathrm{EI}_o \sim}} = \cdot 042\ R\sqrt{\frac{\Sigma mr^2}{m\mathrm{EI}\,\frac{I_o}{I} \cdot \sim}}$$

$$= \cdot 042\ R\sqrt{\frac{\Sigma mr^2}{\mathrm{K.V.A} \sim \frac{I_o}{I}}} \quad \cdots\cdots \quad (96)$$

where I is the normal full-load current per phase, and K.V.A. is the output of the alternator in kilovolt-ampères. The periodic time of a free oscillation therefore varies as the square root of the moment of inertia, and inversely as the square root of the ratio of short-circuit current to normal full-load current.

In modern alternators the time of a free oscillation will be long in comparison with the periodic time of the normal current supplied by the alternator. Where free oscillations are the only ones, no trouble will be experienced in the parallel working of alternating current generators. This is proved by the fact that turbo-alternators, owing to the uniform driving torque of the turbines, have given practically no trouble when operated in parallel. When alternators are driven by reciprocating engines the case is vastly different.

Forced Oscillations.—The free oscillations which have just been discussed are those executed by the alternator when disturbed by a temporary change of conditions, *e.g.* when switched into parallel with other machines before exact synchronism is attained, or when the load on the machine is suddenly altered. As already stated, these oscillations in themselves are not likely to cause instability of working, but should their periodic time be the same as, or approximate to, other oscillations impressed upon the alternator by the prime mover, steady motion will, unless the free oscillations are otherwise damped, be impossible, and the phenomenon of hunting occurs.

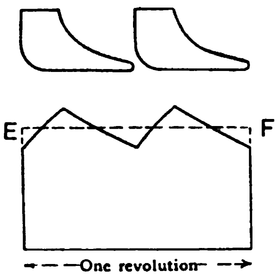


FIG. 272.—Angular velocity curve for single-cylinder double-acting engine.

Consider a single-crank double-acting steam-engine; then on each inlet of steam there is produced a knock upon the piston which will be transmitted to the field system so as to accelerate it. This occurs twice for every revolution, and the velocity curve of the engine will therefore be a series of waves (see Figure 272), with two peaks for each revolution. A two-crank engine with cranks at 90 degrees will show four velocity peaks per revolution, the peaks, however, being less pronounced. The "coefficient of speed variation" or "cyclic irregularity" as expressed by

$$\sigma = \frac{\text{maximum velocity} - \text{minimum velocity}}{\text{average velocity}} = \frac{\omega_1 - \omega_2}{\omega_m}$$

is smaller, the less the difference between the maximum and minimum velocities, and varies inversely as the moment of inertia of the rotating system.

The variations of speed produced by the irregular turning moment of a reciprocating engine may be considered as periodic oscillations which are forced upon the field system. The time in seconds of an oscillation for a single-crank double-acting engine is $T_2 = \frac{30}{R}$; for a 2-crank engine with cranks at 90 degrees, $T_2 = \frac{15}{R}$; and for 3-crank engines

with cranks at 120 degrees, $T_2 = \frac{10}{R}$, where R denotes revolutions per minute. If for any alternator the time of a forced oscillation is equal, or nearly equal to the time of a free oscillation, "resonance" will result, and the extent of the phase swinging will gradually increase and cause the alternator to fall out of step.

The relative crank positions of two sets supplying energy to the same network also influence the nature of the oscillations. Considering two similar sets, in Figure 274, there will evidently be a greater turning moment than in Figure 273; hence, if two machines are paralleled when the crank of one is in either of the positions shown in Figure 273, while that of the other engine is in either of the positions represented by Figure 274, the different turning moments will produce a phase

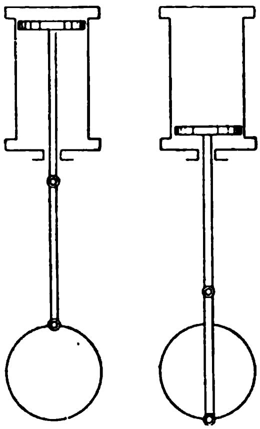


FIG. 273.

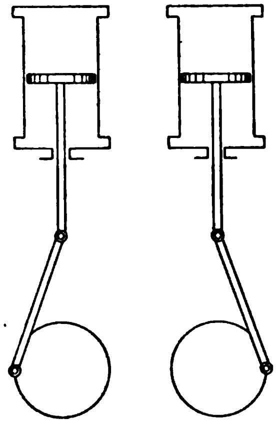


FIG. 274.

displacement between the two alternators, and phase swinging will result. Better conditions of working would obviously be obtained if the machines could be paralleled at an instant when electrical and mechanical synchronism coincided. This, however, makes the process of synchronising somewhat more complicated, as, besides the phase lamps or meters, a couple of bells must be observed, the bell circuits being momentarily closed for a particular position of the cranks. Owing to the complications involved, synchronising of the crank positions is only resorted to in the case of very slow-speed machines which are liable to give trouble through hunting.

In order to obtain an expression for the phase displacement resulting from unequal turning moment of the prime mover, it will be assumed that the load is constant. The curve showing the deviation of crank effort from the mean value should be expressed by a Fourier series of sine terms, but analysis of actual results shows that the error

caused by substituting a sine curve for of the irregular curve introduces only a small discrepancy, comparable with the assumption that the current curve of an alternator is a sine wave. Of course, in certain types of gas-engines this assumption would be far removed from actual facts, and in such cases the actual crank effort curve should be resolved into its various harmonics, and those of most importance investigated separately.

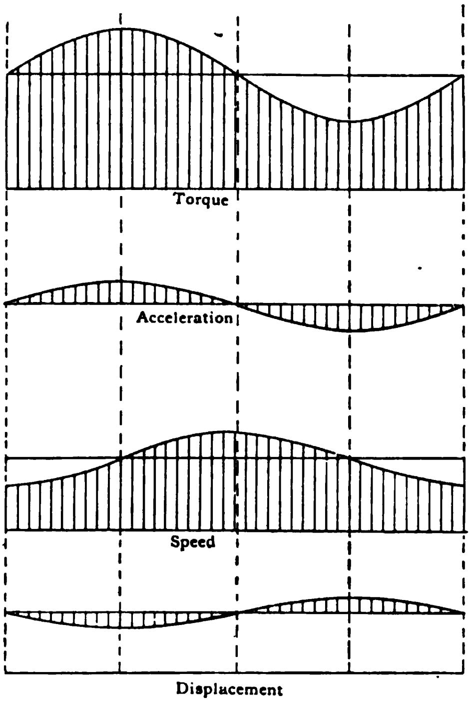


FIG. 275.

The equivalent curve of Figure 275, showing the variation of driving torque with time, can be treated as made up of a constant torque M_c and a superimposed oscillating torque of amplitude m_{max}. Assuming the latter to vary sinusoidally, the torque at any instance of time t

$$= M_c + m_{max} \sin \frac{2\pi t}{T_2}$$

If Σmr^2 denote the moment of inertia of the rotating system, then the oscillating torque causes an oscillating acceleration expressed by

$$\gamma = \frac{m_{max}}{\Sigma mr^2} \sin \frac{2\pi t}{T_2} = \gamma_{max} \sin \frac{2\pi t}{T_2}$$

where $\frac{m_{max}}{\Sigma mr^2}$ denotes the amplitude value of the acceleration curve.

The oscillating torque will, of course, cause an oscillation of the magnet wheel about the synchronous position, but the angular displacement will be by no means in phase with the oscillating torque. As the acceleration varies according to a sine wave, the angular speed increases so long as the acceleration is positive (see Figure 275), and when the acceleration goes through the zero value, the speed will be a maximum. When the acceleration is negative, the speed decreases, and the minimum value of the speed will be attained when the acceleration goes again through zero. This means that the oscillations of speed will lag by just a quarter of a period behind the oscillations of torque.

Let ω denote the instantaneous value of the angular velocity, and ω_m the constant velocity upon which the oscillating velocity is superimposed, then

$$\omega = \text{Average velocity} + \text{variable velocity}$$

$$= \omega_m + \int \gamma \, . \, dt = \omega_m + \gamma_{max} \int \sin \frac{2\pi t}{T_2} \, . \, dt.$$

$$= \omega_m - \gamma_{max} \frac{T_2}{2\pi} \, . \cos \frac{2\pi t}{T_2}$$

$$= \omega_m + \gamma_{max} \frac{T_2}{2\pi} \sin \left(\frac{2\pi t}{T_2} - \frac{\pi}{2} \right)$$

$$= \omega_m + \omega_{max} \sin \left(\frac{2\pi t}{T_2} - \frac{\pi}{2} \right)$$

where the amplitude value of the variable velocity $= \omega_{max} = \gamma_{max} \dfrac{T_2}{2\pi}$.
Now the co-efficient of speed variation is given by

$$\sigma = \frac{\omega_1 - \omega_2}{\omega_m} = \frac{(\omega_m + \omega_{max}) - (\omega_m - \omega_{max})}{\omega_m} = \frac{2\omega_{max}}{\omega_m}$$

$$i.e. \ \omega_{max} = \frac{\sigma \, \omega_m}{2}$$

So long as the oscillating speed is positive, the displacement will increase and attain its maximum value when the oscillating velocity passes through zero. Now, the amplitude of the displacement is the integral of velocity and time—*i.e.* displacement $s = \int \omega' \, . \, dt$, where $\omega' =$ variable velocity at any instant, so that if the maximum displacement $S_{max} = \omega_{max} \dfrac{T_2}{2\pi}$ the equation for the displacement becomes

$$S = \int \omega' \, . \, dt = \omega_{max} \int \sin \left(\frac{2\pi t}{T_2} - \frac{\pi}{2} \right) . \, dt$$

$$= \omega_{max} \, . \, \frac{T_2}{2\pi} \, . \sin \left(\frac{2\pi t}{T_2} - \pi \right) = S_{max} \sin \left(\frac{2\pi t}{T_2} - \pi \right)$$

The oscillating displacement when superimposed upon the average position will lag 90 degrees behind the oscillating speed, and 180 degrees behind the oscillating torque or acceleration (see Figure 275). If no other variable forces are acting upon the rotating system, then the displacement is exactly inverse to the primary oscillating torque. At the instant when the accelerating torque is greatest, the backward displacement will be a maximum, and the machine at this instant supplies less than its normal power to the bus bars.

Since $\omega_{max} = \dfrac{\sigma \, \omega_m}{2}$ the maximum displacement can be expressed thus

$$S_{max} = \omega_{max} \frac{T_2}{2\pi} = \frac{\sigma \, \omega_m}{2} \, . \, \frac{T_2}{2\pi}$$

Now, T_2 is the time of one cycle; hence $\omega_m T_2$ is the angular distance traversed per cycle. Further, if n_c denotes the number of cycles per revolution

$$\omega_m \cdot T_2 = \frac{2\pi}{n_c} \text{ and}$$

$$S_{max} = \frac{\sigma}{4\pi} \cdot \frac{2\pi}{n_c} = \frac{\sigma}{2n_c}$$

If p denotes the number of pairs of poles, then the displacement expressed in electrical degrees is

$$\theta_e = \frac{\sigma}{2n_c} \cdot \frac{360}{2\pi} \cdot p. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

From this equation it will be observed that the maximum phase displacement of the rotor produced by the uneven turning moment of the prime mover, is directly proportional to the coefficient of speed variation and inversely proportional to the number of cycles per revolution.

Size of Fly-wheel

For the steady running of alternators in parallel the coefficient of speed variation σ must be kept within certain defined limits by giving sufficient fly-wheel effect to the engine, otherwise the phase displacement θ_e may exceed the value corresponding to stable working. In order to determine the size of fly-wheel required, there must first be obtained from the combined indicator diagrams of the several cylinders, a crank effort curve. If a mean effort line EF be drawn then the largest of the areas lying above or below the mean effort line is called the fluctuation of energy $\delta\mathcal{E}$. The ratio of this to the total work done per cycle is called the coefficient of fluctuation of energy, and will be denoted by κ. The value of κ will be considerably influenced by the number of cylinders and arrangement of the cranks, but for a given engine is constant. The size of the fly-wheel, as expressed by its moment of inertia Σmr^2, to fulfil certain conditions, is obtained thus—

Let $\mathcal{E}$ denote the total energy stored in the fly-wheel when rotating uniformly at normal speed, $\delta\mathcal{E}$ the fluctuation of flywheel energy, σ the specified co-efficient of speed variation, and W the work done by the engine during one cycle—*i.e.* a period of torque oscillation.

Now, $$\mathcal{E} = \tfrac{1}{2}\Sigma mr^2\omega_m^2$$

$$\text{and } \delta\mathcal{E} = \frac{\Sigma mr^2}{2}(\omega_1^2 - \omega_2^2) = \frac{\Sigma mr^2}{2}(\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$\text{Since } \frac{(\omega_1 + \omega_2)}{2} = \omega_m$$

$$\delta\mathcal{E} = \frac{\Sigma mr^2}{2} \cdot 2\,\omega_m(\omega_1 - \omega_2) = \Sigma mr^2\omega_m^2\left(\frac{\omega_1 - \omega_2}{\omega_m}\right)$$

$$= \Sigma mr^2\omega_m^2\,\sigma$$

$$\text{But } \frac{\delta\mathcal{E}}{W} = \kappa = \text{coefficient of fluctuation of energy}$$

$$\text{and } \frac{\delta\mathcal{E}}{W} = \frac{\Sigma mr^2\omega_m{}^2\sigma}{W}$$

$$\text{Hence } \Sigma mr^2\omega_m{}^2 = \frac{\kappa W}{\sigma}, \ i.e. \frac{\kappa W}{\sigma\omega_m{}^2} = \Sigma mr^2$$

Since the mean power developed by the engine $=$ HP $\times$ 76 kilogramme metres per second, the work done per cycle is

$$W = 76 \text{ HP} \times T_2 \text{ kilogramme metres} \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

T_2 is the time of a cycle, and for an engine having n_c cycles per revolution is given by $T_2 = \frac{60}{2Rn_c} = \frac{30}{Rn_c}$. The moment of inertia Σmr^2 having been determined, the diameter and mass should be proportioned so as to give a convenient mass of wheel.

Effect of Synchronising Torque upon the Displacement resulting from Forced Oscillations:—In deriving the formula for the displacement θ_e of the field system, it has been assumed that the resisting torque is that due to the load only, and is therefore constant. But it has been shown in a previous section of this chapter, that when the rotor of the alternator working in parallel with others deviates from the synchronous position, a torque is set up tending to pull the field system back to the mean position. The synchronising torque will vary with the displacement, being alternately positive and negative according as the rotor lags or leads: hence, if there be no damping, the oscillations in the synchronising torque will be in phase with the oscillations in the driving torque. When the synchronising torque corresponding to any oscillating displacement of amplitude θ_e is superimposed upon the oscillating torque due to the prime mover (Figure 275), the result is that the amplitude of the oscillating torque is augmented, thus increasing the acceleration, the variable velocity, and the displacement beyond the original values. This will in turn give a larger synchronising torque and correspondingly increased displacement, the cycle of operations being repeated until a final displacement θ_o is reached. The diagrams in Figure 275 are therefore not quite correct, and should be rectified step by step by aid of the combined diagram until the ultimate displacement θ_o is obtained. The determination of the final value of the synchronising torque in this manner would be exceedingly laborious. The following graphical construction,* however, forms a basis for a more rapid solution of the problem.

Referring to the vector diagram of Figure 276, let OA represent the

* E. Rosenberg, *Electrotechnische Zeitschrift* (1902), vol. xxiii p. 450.

amplitude value of the oscillating torque as obtained from the crank effort diagram. This vector will be supposed to make one revolution in the time T_2 (= time of one cycle), its projection on the vertical axis giving the instantaneous value of the variable torque. The second vector OB, lagging 90 degrees with regard to OA, represents the amplitude value of the oscillating speed. Now, since the oscillating displacement lags 90 degrees behind the oscillating speed and 180 degrees behind the initial oscillating torque, its amplitude value can be represented by the vector OC. Each vector must, of course, be drawn to its proper scale, so that there are actually three scales, but each bearing a definite ratio to the scale of the preceding vector. The scale for OC should, further, be such that OC in the scale of the driving torque represents the synchronising torque due to the displacement θ_r.

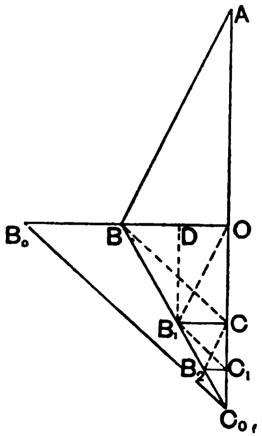


FIG. 276.

With an original displacement OC, then, owing to the reaction of the alternator upon the prime mover, the actual vector of variable torque is not OA but OA + OC. The increased oscillating torque produces, of course, an increased displacement, which goes on until a final value θ_o is attained. The latter is found as follows:—Draw OB_1 parallel to AB; CB_1 parallel to OB; and B_1C_1 parallel to BC: then, since $\frac{CC_1}{OC} = \frac{OC}{OA}$, CC_1 will represent the additional displacement due to the synchronising torque. Repeating this construction, it will be found that the points B, B_1, B_2, etc., lie on a straight line, so that the final value OC_o will be found by going through the construction once and producing the line BB_1 until it cuts the vertical axis in C_o. The final value of the oscillating torque is then represented by $C_oA = OA + OC_o$, and if from C_o a line be drawn parallel to CB it will cut off from the horizontal the length OB_o, which represents the amplitude value of the final variable velocity. Since the triangles CB_1O and OBA are similar

$$\frac{CB_1}{OB} = \frac{OC}{OA} = \frac{\text{synchronising torque}}{\text{variable torque}}$$

This is called the *reaction quotient* and will be denoted by q, so that $CB_1 = q \,.\, OB$, and $OC = q \,.\, OA$.

Also, since the triangles C_oCB_1 and C_oOB are similar,

$$\frac{OC_o}{OC} = \frac{OB}{OB - OD} = \frac{OB}{OB - CB_1} = \frac{OB}{OB - q \,.\, OB} = \frac{1}{1 - q}$$

Hence, the final value of the synchronising torque is expressed by

$$OC_o = \frac{1}{1-q} \times OC = \frac{q}{1-q} \times OA$$

The factor $\frac{q}{1-q}$ is a very important characteristic to be taken into account when an alternator has to be operated in parallel with others. For, with a given initial oscillating torque, the final displacement θ_o will depend upon the value $\frac{1}{1-q}$. Thus, if the synchronising torque OC equals $\frac{1}{10}$, $\frac{1}{5}$, and $\frac{1}{2}$ of the initial variable torque OA, the final displacement OC_o will be $\frac{10}{9}$, $\frac{5}{4}$ or $\frac{2}{1}$ times the initial displacement OC, and the actual coefficient of speed variation is $\frac{10}{9}$, $\frac{5}{4}$, $\frac{2}{1}$ times greater than that due to the prime mover alone.

Since the scale OC also represents the angular displacement of the rotor, it follows that for an initial displacement θ_i, the final value will be expressed by

$$\theta_o = \frac{1}{1-q} \cdot \theta_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

When q is less than unity, the factor $\frac{1}{1-q}$ will have finite values, but if $q=1$ they will be infinite. That means, for any initial oscillating torque except for one of infinitesimal value, the final oscillations would be infinite when $q=1$. This is a state of affairs which is reached if the duration of an oscillating torque cycle is exactly equal to the periodic time of a free oscillation, *i.e.* when there is complete resonance. If q be greater than unity, the factor $\frac{1}{1-q}$ will again be finite, but with the sign reversed. In Figure 277, the values of $\frac{1}{1-q}$ have been plotted as a function of q, and it will be noted that near the point of resonance—*i.e.*, when $q \cong 1$—the displacement increases very rapidly. It is therefore not advisable to approach too closely the critical value q, for oscillations, when once started by any small amount of speed

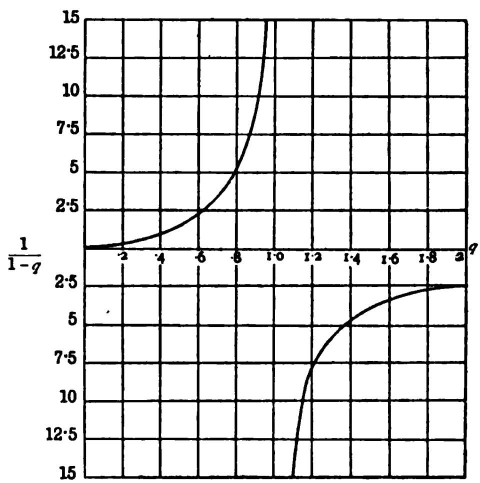


FIG. 277.

variation, may rapidly increase to such an extent that parallel running is impossible, and the alternator drops out of step. Although it is always desirable to keep the initial displacement small, the steady running of alternators in parallel does not depend so much upon this as upon the value of the reaction quotient q. This should be kept as small as possible in order to keep down the final displacement, and the resulting surging of power between the machine and the bus bars.

Damping.—In actual machines, the above-mentioned point of instability, where the value of the factor $\frac{1}{1-q}$ changes from $+\infty$ to $-\infty$ does not really exist because the free oscillations are, to a more or less extent, damped by the presence of a third torque. When the armature of a polyphase alternator supplies current, a rotating magnetic field is set up by the armature M.M.F., and should the magnet wheel not be in perfect synchronism with this field, but show oscillations superimposed upon the synchronous motion, then currents will be induced in every closed circuit that presents itself on or near the pole face. These currents are proportional to the vector of oscillating speed, and set up a torque directly opposed to it, *i.e.* they tend to damp down the oscillations. If special windings of very low resistance be imbedded in the pole shoes (see Figure 278), then the induced currents and also the torque corresponding to a given oscillating speed will be high. Should there only be solid pole shoes, the higher resistance of the iron will then reduce the damping torque for a given oscillating speed. With laminated pole shoes the torque will be still further diminished, but, as the laminations do not permit of large damping currents, the action will go deeper, and will be visible in the field coils, which, in the former cases, are protected by the currents near the pole faces. Even with laminated pole shoes a damping torque is always present, which must be taken into account in the determination of the final displacement of the rotor.

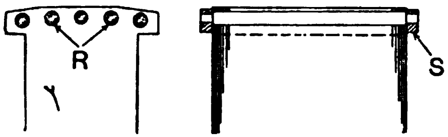


FIG. 278.—Damping coils.

When special windings, known as "amortisseurs," or damping coils, are fitted to the pole shoes they generally take the form shown in Figure 278. A number of heavy copper rods R are imbedded in tunnels close to the face of each pole, and have their ends riveted to a solid casting S of copper or bronze so as to form a grid similar to that of the squirrel cage rotor of an induction motor. By fitting such damping coils to the pole shoes the eddy currents are constrained to flow along those paths in which they will be most efficient in producing a damping torque. In some designs the amortisseurs

consist simply of gun-metal rings G (Figure 134, page 196), which also serve to retain the field coils in position.

For single-phase alternators the action of the damping currents are somewhat different and are not so effective as in the case of polyphase machinery. For the magnetomotive force of the armature sets up an alternating magnetic field which can be resolved into two components,—(1) a synchronous field and (2) an inverse field. The former induces currents in the amortisseurs when the state of steady running is disturbed, whilst the latter, moving backwards relative to the poles with double the synchronous speed, induces an alternating current having twice the machine frequency in the damping coils. Owing to the high inductance of the circuits, the currents induced in them by the inverse field will rarely be large, so that the damping torque is generally small.

The vector diagram of Figure 276 must now be modified so as to take into account the torque due to damping. The latter can be determined experimentally by running the machine as an induction motor, and loading it by means of a suitable brake. The slip corresponding to any particular load is measured, and the ratio of brake torque to slip will then be the same as the ratio between damping torque and oscillating speed, *i.e.* damping torque = brake torque × $\frac{\text{oscillating speed}}{\text{speed of slip}}$. The vector OD (Figure 279) representing the damping torque is then drawn in opposition to the vector of variable velocity, the scale for the oscillating speed being so chosen that the length of the speed vector is identical with the length of the damping torque vector corresponding to this speed.

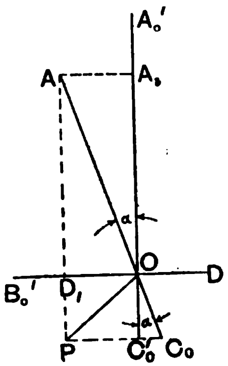


FIG. 279.

Now, since it must have a component equal and opposite to the damping moment, the primary oscillating torque will no longer be exactly opposite in phase to the displacement, but will lead by an angle of less than 180 degrees as indicated by OA. The component of the torque, which is effective in producing an initial displacement, is therefore less than OA, and is represented by $OA_1 = OA \cos \alpha$, where α is called the angle of damping.

The damping torque has therefore the effect of decreasing the initial displacement, but, unless it has a value nearly equal to the primary oscillating torque the action of the damping coils will have very little effect in diminishing the mechanical oscillations. For example, when the damping torque equals 50 per cent. of the primary torque OA (Figure 279) the effective component OA_1 is only reduced to 0.866 OA. In order that OA_1 may be one-half of the primary

oscillating torque, the damping torque must be 87 per cent. of the primary torque OA.

In a damped machine the oscillations in output are due not only to the synchronous oscillating torque OC_o' but also to the damping torque which is maintained by currents taken from or supplied to the network. Hence the oscillating power superimposed upon the average output of the machine consists of a synchronous component OC_o' in phase with and in the same direction as the displacement, and an asynchronous component OD_1 in phase with and in the same direction as the oscillating speed. The resultant of OD_1 and OC_o', *i.e.* OP, will then represent the oscillating torque from which arise the oscillations of power between the machine and the network.

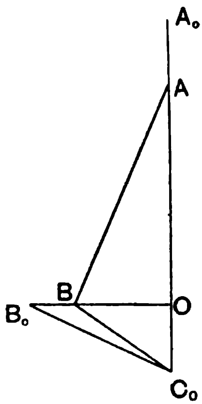


FIG. 280.

Besides showing the effect of damping upon the mechanical oscillations, the diagrams in the last two figures also provide a means whereby the oscillations of power in a damped machine can be compared with the oscillations in a machine having negligible damping. Since the damping torque $= OD_1 = OA \sin \alpha$ and $OC_o' = OC \cos \alpha$ the oscillating torque transmitting power to the network is

$$OP = \sqrt{OC_o'^2 + OD_1^2} = \sqrt{OC_o^2 \cos^2 \alpha + OA^2 \sin^2 \alpha}$$

From Figure 279 it is obvious that when $OC_o = OA$, $OP = OC_o$, and that OP will be greater or less than OC_o according as OA is greater or less than OC_o. From this it follows that when $OP = OC_o$

$$\frac{q}{1-q} = \frac{OC_o}{OA} = \frac{OC_o}{OC_o} = 1$$

(i.e. $q = 0.5$)

But when OP is greater or less than OC_o, $\frac{q}{1-q}$ is smaller or greater than 1. Though damping in all cases decreases the mechanical oscillations, the power oscillations decrease only when $\frac{q}{1-q}$ is more than 1. When $\frac{q}{1-q} = 1$ the power oscillations are unaffected by the damping torque. For values of $\frac{q}{1-q}$ less than 1 the damping actually increases the oscillations of power; but this should give no trouble provided the fluctuations do not exceed about 25 per cent. of the normal output.

Permissible Values of Phase Displacement and Cyclic Irregularity.—For the successful working of alternators in parallel, the initial displacement θ_e of the field system from the synchronous position due to the cyclic irregularity of the engine must be kept within certain limits, determined by the nature of the load connected to the bus bars. Since the initial displacement is expressed in equation 97 by

$$\theta_e = \frac{\sigma}{2\,n_c} \cdot \frac{360}{2\,\pi} \cdot p \text{ electrical degrees}$$

$$\sigma = 0.035 \cdot \theta_e \cdot n_c \cdot \frac{1}{p}$$

where n_c denotes the number of cycles per revolution and p the pairs of poles. Hence, so far as parallel working is concerned, the permissible speed variation or cyclic irregularity should, for a given phase displacement θ_e, be directly proportional to the number of cranks (not placed 180° apart), and inversely proportional to the number of poles. Now experience shows that to obtain good results the maximum permissible initial displacement should be limited to ± 3 electrical degrees,* *i.e.*, $\pm 3/p$ mechanical degrees. For instance, in the case of a 3-crank engine direct coupled to a 50 ~ alternator running at 250 R.P.M., $p = 12$, so that the permissible cyclic irregularity should not exceed

$$\sigma = 0.035 \times 3 \times 3 \times \tfrac{1}{12} = 0.026 = \tfrac{1}{40}$$

If the same type of engine were employed to drive an alternator of the same frequency at say 100 R.P.M., then $p = 30$ and

$$\sigma = 0.035 \times 3 \times 3 \times \tfrac{1}{30} = 0.0105 = \tfrac{1}{95}$$

Again, if similar engines were used for driving 25 ~ alternators, the permissible cyclic irregularity could for the same initial displacement be as high as $\frac{1}{20}$ and $\frac{1}{48}$ respectively. These examples show that in so far as parallel running is concerned a low frequency is an advantage, for a higher coefficient of speed variation will then be permissible, thus diminishing the fly-wheel mass.

In settling the value of the permissible speed variation it should further be remembered that the driving torque of the prime mover is subject to continued changes, which are due to variations in the steam pressure or in gas engines, changes in the composition of the gas, varying loads, etc. The fluctuations of energy due to these changes may be greater than those of the normal diagram, but they are not connected with the coefficient of speed variation. A gas-engine or single-cylinder steam-engine with a large cyclic irregularity will take scarcely any notice of these fluctuations, as the fly-wheel mass will be sufficient to store the excess of energy without the least trouble. But in a multi-crank steam-engine set, with a high degree of uniformity,

* Recommended by Engineering Standards Committee on Reciprocating Steam Engines, 1909.

the mean driving torque will differ comparatively little from the maximum; the fly-wheel mass, if settled solely from the permissible value of speed variation, would then be comparatively small, with the result that the above-mentioned irregular changes would, besides interfering with the free and forced oscillations of the alternator, put severe strains upon the engine.

For these reasons the engine builders invariably give sufficient fly-wheel mass to limit the cyclic irregularity to about $\frac{1}{100}$ in the case of single-crank engines. In multi-crank sets a much smaller coefficient of speed variation is desirable for purely mechanical reasons, so that the cyclic irregularity will seldom be more than $\frac{1}{150}$, whilst $\frac{1}{300}$ should be considered the limit in the other direction, as otherwise the cost of the fly-wheel becomes unnecessarily large. Hence, except in the case of very slow speed machines, the engine builder provides a greater fly-wheel mass than is actually necessary for electrical purposes.

Critical Periodic Time of an Oscillation.—Let S_{max}, as before, denote the maximum value of the displacement, then from page 363

$$S_{max}=\omega_{max}\frac{T_2}{2\pi}, \text{ but } \omega_{max}=\gamma_{max}\frac{T_2}{2\pi} \text{ and } \gamma_{max}=\frac{m_{max}}{\Sigma mr^2}$$

so that $S_{max}=\frac{m_{max}}{\Sigma mr^2}\ \frac{T_2^2}{4\pi^2}$ mechanical radians

$$=\frac{m_{max}}{\Sigma mr^2}\cdot\frac{T_2^2}{4\pi^2}\cdot p \text{ electrical radians}=\theta_s$$

Now, from equation 95, page 357, the synchronising power of an m-phase alternator is

$$W_s=mEI_o\theta_s=mEI_o\cdot\frac{m_{max}}{\Sigma mr^2}\cdot\frac{T_2^2}{4\pi^2}\cdot p$$

Let T_r denote the time of a revolution, and M_s the synchronising torque in kg. metres, then

$$W_s=M_s\cdot 9.81\cdot\frac{2\pi}{T_r} \text{ watts}$$

$$\text{or } M_s=\frac{W_s}{9.81}\times\frac{T_r}{2\pi}=\frac{mEI_o}{9.81}\cdot\frac{m_{max}}{\Sigma mr^2}\cdot\frac{T_rT_2^2}{8\pi^3}\cdot p \text{ kg. metres}$$

Since the reaction quotient is the ratio of synchronising torque to oscillating torque

$$q=\frac{M_s}{m_{max}}=\frac{mEI_o}{9.81}\cdot\frac{1}{\Sigma mr^2}\cdot\frac{T_rT_2^2}{8\pi^3}p$$

Now, when $q=1$, the synchronising torque is equal to the initial oscillating torque and the oscillations increase indefinitely. For such a condition

$$M_s=m_{max}=\frac{mEI_o}{9.81}\cdot\frac{m_{max}}{\Sigma mr^2}\cdot\frac{T_rT_2^2}{8\pi^3}\cdot p \text{ kg. metres}$$

so that the critical periodic time of an oscillation is

$$T_{crit} = 2\pi\sqrt{\frac{S_{max}}{M_s/\Sigma mr^2}} = 2\pi\sqrt{\frac{9.81 \cdot \Sigma mr^2 \cdot 8\pi^3}{m \cdot E \cdot I_o T_r \cdot p}}$$

If K_g denote the weight in kilogrammes of the rotating masses, and D the diameter of gyration in metres, then $\Sigma mr^2 = K_g \dfrac{D^2}{4g}$ and

$$T_{crit} = 2\pi\sqrt{\frac{K_g D^2}{mEI_o} \cdot \frac{2\pi^3}{T_r} \cdot \frac{1}{100\,p}} = 2\pi\sqrt{\frac{K_g \cdot D^2 \cdot 2\pi^3}{mE \cdot \frac{I_o}{I} \cdot I \cdot \frac{6000\,p}{R}}}$$

$$= 0.0026\sqrt{\frac{K_g D^2}{\text{K.V.A.} \sim \cdot \frac{I_o}{I}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

where I is the full-load current per phase, and K.V.A. the output in kilo-volt-ampères. From page 365 the periodic time of a forced oscillation is

$$T_2 = \frac{30}{Rn_c}$$

Experience shows that the periodic time of a forced oscillation should not approach too closely the critical value T_{crit}, and, damping neglected, the usual practice is to make T_{crit} at least 1.4 times T_2; that is, the natural frequency of the alternator should not exceed 70 per cent. of the frequency of the impulses impressed by the prime mover. Now, in practice, the fly-wheel is generally designed by the engine builder for a standard cyclic irregularity. $K_g D^2$ being thus fixed the electrical designer has only to check whether the given value is likely to cause trouble or not. In medium speed sets the fly-wheel effect necessary for mechanical reasons will, as already stated, generally be ample to keep the critical periodic time well above that mentioned above. In slow-speed machines, with a fixed fly-wheel effect, the critical value could be avoided by decreasing the radial depth of the air-gap so as to decrease the apparent impedance of the armature, and therefore the value of the short-circuit current I_o. Such a procedure may, however, seriously affect the inherent regulation of the machine, and in general cannot be applied to any great extent. In cases where the critical value is likely to be approached, the most satisfactory commercial result is obtained by a compromise between a large fly-wheel effect and a small short-circuit current. Of course, when near to resonance, amortisseurs must be fitted into the pole shoes, and, provided that the primary oscillations are kept very small, these may be sufficient to prevent hunting even should complete resonance occur.

When calculating the value of T_{crit} it must be remembered that neither the voltage E of the machine nor the short-circuit current

I_o are constant but vary with the load of the station. In most supply systems the voltage at the consumer's terminals is kept approximately constant, so that it is necessary to generate sufficient voltage to compensate for the drop in the feeders. The usual values of I_o would range from about 2.5 to 3.5 times the normal current according as the station load is small or large.

Influence of Shape of E.M.F. Waves.—So far it has been assumed that the E.M.F. waves, for all the alternators connected to the same network, are of the same shape, but when, as will often be the case, the form of the E.M.F. curves are dissimilar, there will always be a resultant E.M.F. acting round the circuits between the various armatures. For instance, consider two single-phase alternators whose respective E.M.F. waves can be expressed by the equations

$$E_A = E_1 \sin pt$$
$$\text{and } E_B = E_1 \sin pt + E_3 \sin 3\,pt$$

When these machines are worked in parallel, the resulting E.M.F. acting through the circuits of the two armatures will be

$$E_R = E_B - E_A = E_3 \sin 3\,pt$$

This resultant E.M.F. will set up a synchronising current having a frequency three times that of the main current. Should the wave forms be very dissimilar, the resulting oscillations of current between the machine and the bus bars might be so great as to reinforce the free oscillations of the machines, and so produce hunting, and thus render parallel working impossible. Alternators intended for parallel running should always be designed to give an E.M.F. wave which approximates very closely to a sine curve. For, although the oscillations of power from this cause may not be large enough to cause unstable working, they increase the copper loss in the armature; and further, should the periodic time of the forced oscillations approach too closely the natural period of the machine,

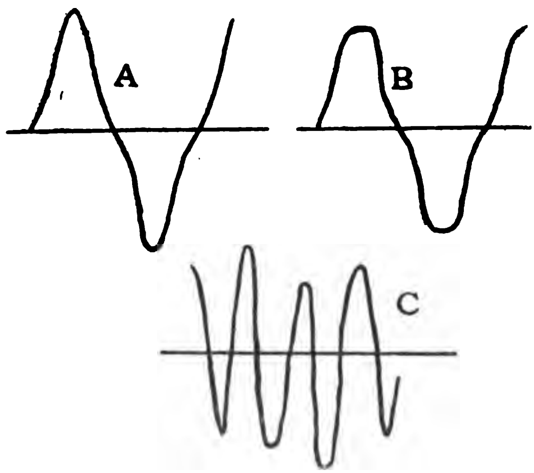


Fig. 281.

the interference due to dissimilar wave shapes might tend to produce resonance when large fluctuations in the load take place.

That the synchronising current due to this cause may, under certain conditions, be by no means a negligible factor will be obvious from the curves shown in Figure 281. A and B are the open-circuit E.M.F. curves for two 600-K.V.A. 2000-volt single-phase alternators which are connected to the same bus bars. The triple-frequency curve C is that of the synchronising current passing between the two machines when operated in parallel on a steady load and with their fields normally excited. The R.M.S. value of the synchronising current in this case is about 15 ampères.

Three-Phase Alternators with Earthed Neutrals.—In order to prevent undue rise of potential in a 3-phase transmission system supplied with current from star-connected generators, the practice of earthing the neutral point of each winding is sometimes adopted. If, with earthed neutrals, the phase E.M.F. wave forms of the various machines differ from each other, then the resultant E.M.F. will set up local currents through the earthing wire between the various generators, as shown in Figure 282. From the latter it will be observed that between any two machines there are three circuits in parallel, with one earth return. The resultant E.M.F. due to the fundamental waves tending to send current through this earth circuit will be zero, but, since the third, ninth, etc., harmonics are, for a symmetrical winding, in phase with each other (see page 133), they will produce currents in this circuit. The triple-frequency currents are, especially on full load, the most important, and experience shows that they are the only ones that need be taken into consideration.

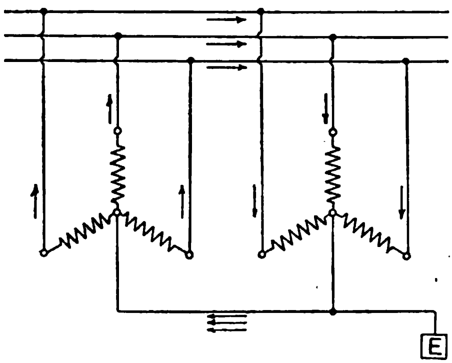


FIG. 282.

Referring to the 3-phase winding shown diagrammatically in Figure 283, if each phase carry a triple-frequency current of equal magnitude, then at the instant of maximum values the direction of these currents will be indicated by the crossed and dotted circles. Since adjacent coils neutralise each other, the demagnetising action of the armature M.M.F. upon the main magnetic circuit will be almost negligible, so that the apparent impedance of the armature winding in so far as these currents are concerned will be very small, unless the true self-inductance of the windings has an abnormally

high value. This explains why very large triple-frequency currents are observed in practice, even when the wave forms of the phase E.M.Fs. of the alternators contain relatively small third harmonics. Owing to the wide slots, long air-gaps, and small number of conductors, turbo-alternators have a much lower reactance voltage than slow or medium speed machines designed for the same output. Hence, these triple-frequency currents will be more pronounced in turbo-alternators than in slow speed sets, and this has been amply verified in practical working.

Professor Marchant and J. K. Catterson-Smith * have recently

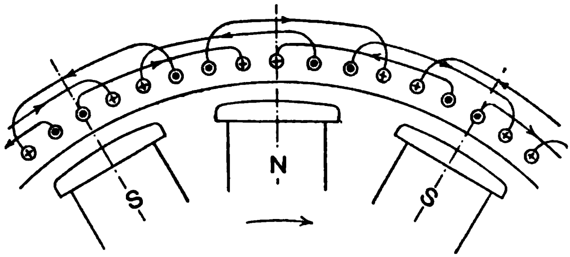


FIG. 283.—Showing direction of triple-frequency currents.

investigated the nature of these local currents when two types of 2000-K.V.A. turbo-alternators by different manufacturers were run in parallel. The phase E.M.F. wave forms were very nearly sinusoidal, and when analysed were found to satisfy the following equations:—

Type A $e_a = 100 \sin pt - 7 \sin 3pt - 9 \sin 5pt$
Type B $e_b = 100 \sin pt - 3 \sin 3pt + 4 \sin 5pt$

the normal full-load current per phase being 200 ampères. When two machines of the same type were in parallel the current circulating in the neutral was found to be very small, varying from 5 to 6 ampères. When, however, a machine of Type A was run in parallel with a machine of type B, the triple frequency component of the E.M.F. was sufficient to give a current of 60 ampères through the neutral wire (see Figure 284). It was also found that the value of the neutral line current could be increased to 120 ampères, if the excitation of the two machines were such as to make their loads of different power factors.

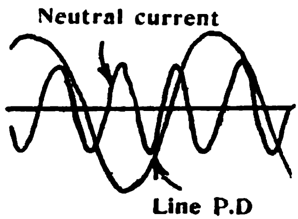


FIG. 284.

Besides causing additional heating in the armatures of the machines working in parallel, these currents, being practically watt-

* *Electrician* (1909), vol. lxiii. p. 674.

less, reduce the power factor of the machines through which they circulate. When a power factor meter is connected to a machine supplying these currents, its reading may be considerably lower than that corresponding to the real angle of lag between the fundamental wave of terminal voltage and the fundamental wave of current. This may cause difficulty in parallel working, as the alternators would have their excitations adjusted so as to give the same reading on the power factor meters, and this adjustment may not correspond to minimum circulating current. The method of determining the best power factor for any individual machine coupled to the bus bars would be to find the excitation for which the current flowing to it through the neutral wire is a minimum.

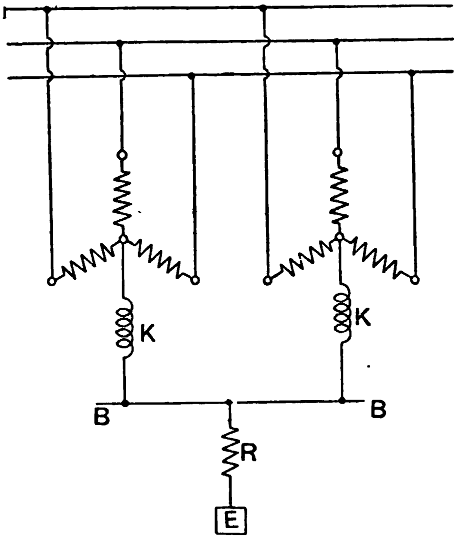


FIG. 285.

Various methods have been adopted for getting rid of the triple-frequency currents in an earthed 3-phase system. One method (see Figure 285) is to have the star-point of each alternator connected through choking coils KK to a bus-bar BB, which is earthed through a low resistance R. The impedance of the coils K to the triple-frequency currents will be three times that offered to a current of normal frequency; it is therefore equivalent to a resistance which falls to one-third its previous value, whenever it has to carry an out-of-balance current due to a fault, and so will not interfere with the proper operations of the circuit breakers. In another method, due to J. H. Rider,* a switch is designed so as to automatically connect the neutral point of one, and only one, of a number of parallel machines to earth. This method eliminates the triple-frequency earth currents entirely, but it is open to one objection, namely, that the neutral points of the unearthed machines may rise to fairly high potentials, the magnitude of which depends upon the value of the triple-frequency component of E.M.F. in the wave of phase voltage.

* *Journ. of the Inst. of Elect. Engineers* (1909), vol. xliii. p. 261.

CHAPTER XII

DESIGN OF ALTERNATORS

ALTERNATING current generators have, in nearly every case, to be designed for a given frequency; hence, if the speed of the engine or turbine is known, the number of pairs of poles p is fixed by the equation

$$\sim = \frac{R.p}{60}$$

$$i.e., \text{ number of poles} = \frac{120 \sim}{R}$$

where R denotes the speed of the rotor in revolutions per minute. The value of R must be such that the number of pairs of poles is a

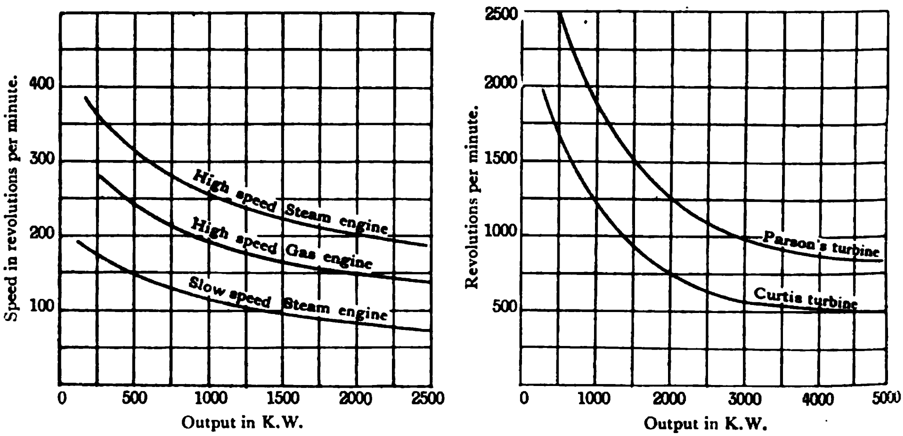


FIG. 286.—Speeds of reciprocating engines and turbines.

whole number. The curves of Figure 286 give the approximate speeds of reciprocating engines and steam turbines for various rated outputs up to 6000-K.V.A. These curves show that the speed is some inverse function of the output. In general, the higher the speed of an alternator for a given output the more satisfactory will be the design.

SIZE OF ARMATURE AS AFFECTED BY RATED OUTPUT AND SPEED

In designing an electric generator the problem before the designer is to obtain the maximum output for a given amount of material,

subject to certain conditions which the machine has to fulfil. In the case of alternators, the output to be obtained from the armature is limited by the considerations of heating and voltage drop on load. In slow and medium speed machines it is the permissible regulation (or, in the case of a synchronous motor, the change of power factor) rather than the temperature rise which determines the size of the armature; for, if this restriction were removed, the output per unit weight of material could easily be increased by simply decreasing the amount of copper on the armature.

Output Coefficient.—The number of poles being fixed, the first step is to determine the diameter and axial length of the armature core. These dimensions, for a machine of given output and speed, are connected by the formula

$$\xi = \frac{\text{K.V.A.}}{\text{D}^2 \cdot \text{L}_g \cdot \text{R}} \quad \text{or} \quad \text{D}^2\text{L}_g = \frac{\text{K.V.A.}}{\xi \cdot \text{R}}$$

where ξ = a coefficient generally designated the "output coefficient."
K.V.A. = rated output in kilo-volt-ampères.
D = diameter of armature at air-gap in cms.
L_g = gross length of armature core between flanges in cms.

The value of ξ ranges from 10^{-6} in small alternators to 2.5×10^{-6} in very large ones of 6000-K.V.A. or more. The equation for the output coefficient may be derived as follows:—

From equation 55, page 217, the E.M.F. induced in each phase winding is

$$\begin{aligned} E_t &= 4.44\, k_2 \cdot T \cdot \sim \cdot \Phi \cdot 10^{-8} \text{ volts} \\ &= 4.44\, k_2 \cdot T \cdot \frac{Rp}{60} \cdot \Phi \cdot 10^{-8} \text{ volts} \end{aligned}$$

Multiplying both sides of the equation by I_a, where I_a denotes the full-load current per phase,

$$E_t I_a = 0.74\, k_2 \cdot I_a \cdot T \cdot R \cdot p \cdot \Phi \cdot 10^{-9}$$

Further, let

B_g = maximum flux density in the air-gap.
AC = ampère-conductors per cm. of armature periphery.
m = number of phases.
σ = the ratio of pole arc to pole pitch.
l_i = ideal length of armature core, $\cong L_g$,

$$\text{then,}\quad B_g \cong \frac{2p\Phi}{\pi D \cdot \sigma \cdot L_g},\ \textit{i.e.},\ p\Phi = \frac{\pi}{2} D \cdot \sigma \cdot L_g \cdot B_g.$$

$$\text{and}\quad AC = \frac{2m \cdot I_a \cdot T}{\pi D},\ \textit{i.e.},\ I_a T = \frac{\pi D \cdot AC}{2m}$$

When these values for $p\Phi$ and $I_a . T$ are substituted in the equation for $E_i I_a$

$$E_i I_a = 0.74\ k_2 . \frac{\pi D . AC}{2m} . R . \frac{\pi D . \sigma . L_g . B_g}{2} . 10^{-9}$$

$$i.e.,\quad mE_i I_a = D^2 . L_g . R . B_g . AC . k_2 . \sigma \times \frac{\pi^2 \times 0.74}{4} . 10^{-9}$$

Since the terminal voltage E will be very little different from the induced voltage E_i, the equation to the output coefficient can be expressed thus,

$$\xi = \frac{m . E . I}{D^2 . L_g . R} \times 10^{-3} = \frac{\text{kilowatts output}}{D^2 . L_g . R} = B_g . AC . k_2 . \sigma . 1.8 \times 10^{-9}$$

$$i.e.,\quad D^2 L_g = \frac{\text{K.V.A.}}{R . B_g . AC . k_2 . \sigma} . 5.5 \times 10^9 \quad . \quad . \quad . \quad (101)$$

The equation for ξ shows that for a machine of given output and speed, the size of the armature, as expressed by D^2L_g is dependent upon two factors: (1) the *specific electric loading* as expressed in ampère-conductors per cm. of armature periphery; and (2) the *specific magnetic loading*, expressed as the maximum flux density in the air-gap. The higher the value of either or both of these factors, the smaller will be the machine. In order to determine the value of D^2L_g, it is necessary to know from experience the proper value to assign to AC and B_g.

In general, the economy of an alternator design for a given frequency and speed is governed by two laws: (1) The output is directly proportional to the product of the specific electric loading and the specific magnetic loading, *i.e.* the output of an armature of given diameter and gross length, assuming equal specific use of the materials, can be increased directly as the armature ampère-conductors. (2) The cost of a design varies almost directly as the length and as the square of the diameter. The diameter of the machine should therefore just be sufficient to accommodate the poles and the exciting coils, due consideration being, of course, given to the question of ventilation, and to the mechanical difficulties introduced by an excessive width of machine.

Against these requirements for minimum cost, there has also to be taken into account the ratio $\dfrac{\text{field ampère-turns}}{\text{armature ampère-turns}}$, as this has a most important influence on the regulation of the machine. For good regulation it is necessary to choose a small number of armature ampère-conductors. This, however, is in conflict with the first law of design; for, by reducing the turns on the armature, the value of B_g has to be proportionately increased so as to obtain the desired output. Now, to carry the larger flux, the active material, both iron and copper, must also be increased, the increase in copper being due to the longer mean

length of turn for stator and rotor as the coils have to enclose a larger area. An increase in the field ampère-turns will be against the second law, as it will entail an increase in the diameter so as to give the necessary space for the augmented field copper. Thus, whenever close regulation is required, the design of alternator finally selected must be a compromise between both these conflicting conditions.

The field ampère-turns, and consequently the energy required for excitation, will increase as B_g and AC increase.

In machines for low outputs the excitation loss will form a greater percentage of the total loss than in the case of large alternators; hence, in the interest of efficiency, it is expedient to design small alternators with lower values of B_g and AC than would be desirable for large machines.

Values of AC.—From the above considerations it will be obvious that for an economical design the specific electric loading should be as high as is consistent with the specified regulation. In Table XX. approximate values are given of AC for 2- and 3-phase alternators designed for a regulation of from 15 to 20 per cent. when supplying current to a load of approximately 0.8 power factor.

TABLE XX.—VALUES OF AC FOR TWO- AND THREE-PHASE ALTERNATORS

Output in K.V.A.	AC for Slow and Medium Speeds.		AC for Steam Turbine Speeds.	
	50 ~	25 ~	50 ~	25 ~
100–250	120	150	...	...
250–500	150	190	...	...
500–1000	180	210	170	190
1000–2000	200	230	190	210
2000 upwards	220	250	210	230

With lower power factors, AC must be reduced in proportion to $\cos \Phi$ if the voltage drop has to be maintained at the same percentage value as on the power factors of 0.8 to unity. However, for values of $\cos\Phi$ less than 0.8, it is usual to permit a wider range of regulation, in which case, the values of AC given in the above table may be quite suitable. With turbo-alternators, the specific electric loading is, for machines of the same rated output, less than in slow and medium speed alternators. For, owing to the small dimensions in high speed machines, considerable difficulty is experienced in finding sufficient space for the field copper. Weaker fields and correspondingly weaker armature ampère-turns must therefore be employed for a given specification as regards pressure regulation. In a single-phase machine, the specific electric

loading does not exceed 70 per cent. of the values recommended for polyphase machines.

Values of B_g.—The essential part of an electric generator is the region occupied by that part of the armature winding which is the seat of the induced E.M.F. This region may be termed the "active belt"* of the armature. It is a cylindrical belt of active material around the armature periphery consisting of copper conductors with the necessary insulation embedded in slots, together with the iron teeth which carry the magnetic flux. Since, for a given number of slots, the tooth density is directly proportional to the air-gap density, it follows that the higher the value of B_g, the greater will be the cross-section of iron required; and consequently, there will be less space for copper, air-ducts, and insulation. The permissible value of B_g will, therefore, for the same flux density in the teeth, and current density in the copper, depend upon the voltage of the machine. The higher the voltage, the lower must be the slot space factor; consequently, high voltage machines will have lower air-gap densities than those of lower voltages. The usual values of B_g for slow, medium, and high speed alternators are set forth in Table XXI. The values tabulated refer to 50 ∼ machines. For 25 ∼ the air-gap flux densities can be made some 15 per cent. greater, owing to the lower specific loss in the teeth, thus permitting a higher tooth density.

TABLE XXI.—AIR-GAP FLUX DENSITIES B_g IN 50 ∼† MACHINES

Output in K.V.A.	Slow and Medium Speeds.		Steam Turbine Speeds.
	500–3000 Volts.	3000–11000 Volts.	
0–500	6000–7500	5500–7000	...
500–1000	7500–8000	7000–7500	5000–6500
1000–2000	8000–8500	7500–8000	6500–7000
2000 upwards	8500	8000	7000–7500

From this table it will be noticed that the values of B_g are lower for turbo-alternators than for slow and medium speed machines. This is because the design of alternators for high speeds is a more difficult problem, and larger dimensions than would otherwise be the case are called for, chiefly to keep down the specific iron loss and hence the heating.

Values of the Output Coefficient.—When, for a machine of specified output, voltage, and regulation, the magnitude of the specific

* A term due to H. A. Mavor.

† For 25 ∼ these values can be 15 per cent. greater.

electric and magnetic loadings are known, then the output coefficient can be calculated from equation 101. For 3-phase alternators of a salient pole type, the ratio $\sigma = \frac{\text{pole arc}}{\text{pole pitch}}$ is generally made equal to 0.65, while for single- and 2-phase machines, σ may sometimes be as low as 0.6. The breadth coefficient k_2 will, on an average, have a value 0.96. In working out the design of a new machine, the best practice is to use as a starting-point the output coefficients derived from previously built machines of the same type. The curves of Figure 287 give the value of ξ as a function of the output for 2- and 3-phase alternators of modern design.

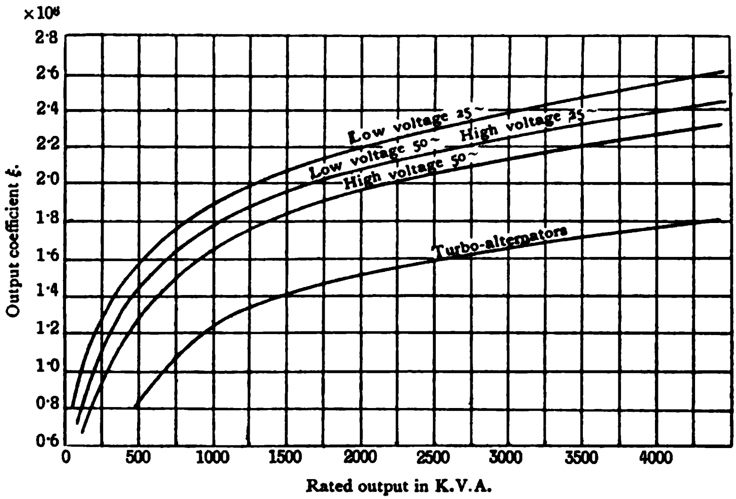


FIG. 287.

From equation 101, page 380, it will be noted that the greater the value of the specific electric and magnetic loadings, the smaller the dimensions of the machine as expressed by D^2L_g. It must, however, be remembered that the machine with the highest value of ξ is not necessarily the cheapest, as the expression for ξ does not involve the ratio of copper to iron, which is a most important factor in determining the cost of a machine.

Calculation of Main Dimensions.—For an alternator of specified output, speed, and frequency, a trial value of ξ can be obtained from the curves in Figure 287, and thence the approximate value of D^2L_g computed from the equation

$$\xi = \frac{\text{K.V.A}}{D^2L_gR} \quad i.e. \quad D^2L_g = \frac{\text{K.V.A}}{\xi \,.\, R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

D^2L_g having been obtained, the determination of the best diameter and length of armature core can only be effected by a method of trial and error. Since R is fixed, the peripheral speed, which is usually regarded as permissible in the desired type of machine, gives an approximation to a reasonable diameter of core. With alternators of ordinary construction the peripheral speed ranges from 24 m/sec in small machines to 40 m/sec in the largest sizes, whereas with turbo-alternators the speed ranges from 60 to 90, or even 100 m/sec. The usual values of peripheral speeds for various outputs are given in Table XXII.

TABLE XXII.—PERIPHERAL SPEED AT POLE FACE IN M/SEC.

Output in K.V.A.	Slow and Medium Speeds.	Turbo-speeds.
0–100	20–24	...
100–250	24–26	...
250–500	26–28	...
500–1000	28–32	60–70
1000–2000	32–35	70–80
2000 upwards	35–40	80–90

If the diameter D of the armature be expressed in cms. then since the pole face diameter of rotor $\fallingdotseq$ D, the peripheral speed in m/sec is

$$v=\frac{\pi DR}{6000}=\frac{\pi Dp\,R}{p.6000}=\frac{\tau\sim}{50}$$

where τ denotes the pole pitch in cms. Hence, in a $50\sim$ alternator the peripheral speed in m/sec is equal to the pole pitch in cms.

From a knowledge of the peripheral speed which is assumed as permissible, the diameter D and the length L_g are obtained from equations

$$D=\frac{6000\,v}{\pi R}\text{ cms. and }L_g=\frac{D^2L_g}{D^2}\text{ cms.}$$

Knowing D, and the pairs of poles p, the pole pitch τ and pole face arc b_s can next be determined.

The ratio of armature core length to pole face arc, *i.e.*, $\frac{L_g}{b_s}$ is an important factor to be considered when getting out the design of a new machine. The aim of the designer should generally be to keep this ratio as low as possible without exceeding the limit of peripheral speed, for it practically determines the shape of the section of the magnet core on which depends the weight of the field copper. The latter, for a given number of ampère-turns and a given magnetic cross-section of pole, is a minimum when the pole is of circular cross-section, and if the

pole is of a rectangular section, the weight is smaller the nearer the shape of the section approximates to a square, *i.e.* the nearer the ratio $\frac{L_g}{b_s}$ is to unity. With square poles, $\frac{L_g}{b_s}$ would be less than unity, but to obtain this the gross length of core would have to be cut down, and, for a given value of D^2L_g, the diameter increased proportionally, so that the limit of peripheral speeds set forth in Table XXII. would be exceeded. With slow and medium speed machines the ratio $\frac{L_g}{b_s}$ ranges from 1.1 to 2.0. In 25 ~ alternators, the pole face is approximately square so that it is quite possible to obtain the lower values 1.1. With alternators designed for 50 ~, the larger number of poles necessitates a shorter length of pole arc, so that higher values of this ratio, up to 1.8 or 2.0, are unavoidable.

In turbo-alternators, owing to the restricted diameter settled by mechanical requirements, the length of core is great compared with the diameter; hence the value of $\frac{L_g}{b_s}$ is seldom less than 2, and in many designs may be as high as 2.5 or 3.

Number of Ventilating Ducts.—In slow and medium speed alternators the width of a duct ranges from 1.0 to 1.5 cm.; and the distance between them from 6 to 8 cms. In turbo-alternators, owing to the small over-all dimensions, the radiating surface is relatively small, but the total losses are about the same as for a medium-speed machine of the same output and efficiency, with the result that the specific iron and copper losses are greater. Instead of increasing the dimensions or reducing the losses, which latter may be unnecessary so far as the efficiency is concerned, the heating of the armature is kept within reasonable limits by providing better ventilation than would be required in the case of slow- and medium-speed machines. The general practice is to provide a 1.25 cm. duct for every 5 or 6 cms. of gross length.

Calculation of Armature Winding.—When the main dimensions D, L_g, and σ have been provisionally determined, the next step is to settle the number of turns per phase of the armature winding. Selecting from Table XX. a suitable value for AC, the total number of ampère-conductors round the armature periphery $= \pi D \cdot AC$. Now, if I_a denote the current per phase and T the number of turns in series per phase, then, in an *m*-phase alternator

$$\pi D \cdot AC = I_a \, 2 \, T \cdot m$$

$$\textit{i.e.}, \text{ turns per phase} = T = \frac{\pi D \cdot AC}{2 \, I_a m}$$

The precise number of turns must ultimately be chosen so as to give a whole number of conductors per slot.

25

Number of Slots.—The number of slots is fixed chiefly by considerations of slot pitch and slot insulation. The greater the number of slots per pole the nearer will the curve of induced E.M.F. approach to a sine wave; but, on the other hand, the greater will be the amount of winding space occupied by insulation. In 500- volt, medium-speed alternators the insulation will not occupy much space, so that as many as 4 slots per pole per phase may be employed. In alternators designed for higher voltages and having 4 slots per pole per phase the space occupied by insulation would be excessive, so that considerations of space factor permit of only 3 or 2 slots per pole for each phase.

Alternators designed for a phase pressure of 3000 volts and upwards have, as a rule, only 2 slots per pole per phase.

In turbo-alternators the fewer number of poles and larger pole pitches necessitate a greater number of slots per pole per phase, which latter range from 5 in 6600 volt alternators to 8 in low-voltage machines. If q denote the number of slots per pole per phase, then the number of conductors per slot is

$$C_s = \frac{2\,T}{2\,p \cdot q} = \frac{T}{p \cdot q}$$

In order that the leakage reactance of the embedded portion of the armature winding may be kept within reasonable limits, the M.M.F. or ampère-conductors per slot must not be too great. The ampère-conductors per slot ($= C_s \cdot I_a$) should not exceed the following values:—

300–500 in high-voltage machines up to 400 K.V.A.
400–600 in low-voltage machines up to 400 K.V.A.
600–1000 in machines above 400 K.V.A.

Size of Conductors.—The size of conductor depends upon the allowable current density, which latter is ultimately determined by the permissible copper loss. In alternators the voltage drop due to ohmic resistance forms only a small percentage of the total drop. The armature conductors can therefore be worked at a high-current density ranging from 250 to 400 ampères per sq. cm. Owing to there being a larger loss from eddy currents in conductors of large cross-sectional area than in small round wires, the latter can, for the same specific copper loss, be worked at a somewhat higher density. Again, in high-voltage machines the greater thickness of slot insulation retards considerably the transference of heat from the interior of the slot, so that the higher the voltage the lower must be the current density for the same temperature rise. The usual values of current density would be somewhat as set forth in Table XXIII.; the lower values being those applicable to high-voltage machines.

TABLE XXIII.—CURRENT DENSITIES IN ARMATURE CONDUCTORS.

Range of Current in Ampères.	Current Density in Ampères per Cm.^{2}
up to 10	400–350
10– 25	350–320
25– 50	320–300
50–150	300–280
150–300	280–250
300–800	250–220

When the cross-sectional area of each conductor does not exceed 0.228 cms.^{2}—*i.e.* No. 5 S.W.G.—solid wire of circular cross-section should be used. Wire of larger cross-section is difficult to bend, so that an armature turn requires to be made up of two or more loops connected in parallel, the sum of the areas of cross-section giving the requisite equivalent sectional area of a single conductor. By employing round wire for low-voltage windings a large amount of space is wasted by interstices between individual wires. In order to reduce the size of slots to a minimum, wires of rectangular section should always be used when the sectional area of any one conductor exceeds 0.5 cm.^{2} Details of copper wires of circular cross-section are set forth in Table on page 492. When employed for armature windings they would be insulated with a double or triple covering of cotton. With rectangular wire there would be an external covering of braided cotton in addition to the double cotton, the total thickness of the insulation ranging from 0.4 to 0.6 mm.

Size of Slots.—In settling the dimensions of the slots the following considerations require to be taken into account: (1) The space allotted to the conductors and the necessary insulation must be such that the flux density at the section of minimum thickness of the teeth is not too great. (2) The ratio of depth to width of slot should be such that the permeance of the path of leakage flux is as small as is consistent with a suitable value of tooth density. From page 235 the flux density at the section of minimum thickness is

$$B_{max} = \frac{\Phi_a \cdot t_p}{L_n \cdot t_1 \cdot b_i}; \text{ hence } t_1 = \frac{\Phi_a \cdot t_p}{L_n \cdot B_{max} \cdot b_i}$$

where $t_p = \frac{\pi D}{n_s}$, n_s denoting the total number of slots. The value of the armature flux Φ can be calculated from the expression

$$\Phi = \frac{E_t \times 10^8}{4.44\, k_2 \cdot T \cdot \sim},$$

E_t being the induced voltage per phase. At this stage an exact calculation of E_t at full-load cannot be made, but its value can be approximately obtained by adding about 5 or 6 per cent. to the

terminal voltage E. In the expression for t_1 all the factors can therefore be calculated except B_{max}, and to determine t_1 a suitable value must be assigned to the former. For standard machines the tooth density B_{max} is usually about 18,000 or 19,000 lines per cm.[2], a value which is lower than that for direct-current machines. In an alternator there is no particular reason why a very high flux density should be employed, and although it might appear that the diameter could be reduced by selecting a smaller tooth pitch, on further consideration it will be evident that the diameter cannot be reduced without unduly restricting the space available for the field magnet poles and the exciting coils. For machines with rotating field systems it is the space required for the poles and the field winding that really determines the smallest diameter possible, and this generally leaves more room for the teeth and slots than is actually required. Once t_1 has been calculated the slot width is settled and the dimensions of slots and arrangement of conductors can then be fixed. The best dimensions of slots and arrangement of conductors can only be finally settled by a method of trial and error. If the tooth saturation works out to too high a value, and all the other dimensions are right, then a diminished tooth density could be obtained by slightly increasing L_g, or *vice versa.* From considerations of leakage reactance, the slot depth should not exceed about three times the slot width.

Armature Flux per Pole.—Having determined the dimensions of the slots, teeth, and armature conductors, the voltage drop due to ohmic resistance and leakage reactance can be calculated in the manner discussed on pages 253 to 260. The induced E.M.F. at full-load is then obtained from the equation

$$E_i = E + e_r \cos \phi + e_x \sin \phi$$

In slow- and medium-speed alternators the reactance voltage ranges from 5 to 8 per cent. of the E.M.F. generated, whereas with turbo-alternators reactance voltages as low as 2 or 3 per cent. are usual with machines of normal design. The pressure drop due to ohmic resistance ranges from 0.5 to 2 per cent., the lower value being that generally met with in high-speed designs. Knowing the total induced voltage at no-load and full-load, the flux per pole can be calculated from the equation

$$\Phi_a = \frac{E_i \times 10^8}{4.44 \cdot k_2 T \cdot \sim}$$

where T = turns in series per phase and the value of k_2 is obtained from Table XVI. page 218.

Radial Depth of Armature Core below Teeth.—For a given armature flux Φ the radial depth of armature core below teeth, as expressed by the equation

$$h_c = \frac{\Phi}{2 L_n \cdot B_c}$$

will depend upon the permissible flux density B_c in the core, which in turn is settled with reference to the specific iron loss of the core. So far as the excitation is concerned, the armature core could be economically worked at a density of about $B_c = 16{,}000$, but in order to keep the specific iron loss within reasonable limits a much lower density has to be adopted. For a given grade of iron, the higher the frequency the lower must be the flux density for the same specific loss. In Table XXIV. there are given the usual values of flux density for a range of frequency between $\sim = 15$ and $\sim = 60$.

TABLE XXIV.—FLUX DENSITIES IN ARMATURE CORES.

Frequency in Cycles per Sec.	Flux Density B_c in Lines per $Cm.^2$
15–25	13000–10000
25–40	10000–8000
40–50	8000–7000
50–60	7000–5000

Radial Depth of Air-gap.—Since the ampère-turns required to send the magnetic flux across the two air-gaps is expressed by

$$AT_g = 1.6\, k_8 \,.\, \delta \,.\, B_g,$$

it follows that for a given flux density B_g the ampère-turns AT_g will be directly proportional to the radial depth of the air-gaps δ. Hence it would appear that to economise in field copper δ should be reduced to the mechanical limit. Another advantage of a small air-gap is that the leakage factor σ will be less, thus resulting in a smaller cross-section of pole core and diminished mean length of the winding per turn. On the other hand, a strong M.M.F. is required in the field to reduce the effect of the demagnetising and distorting M.M.F. of the armature and so lead to better regulation when the load is inductive. The reluctance of the magnetic circuit therefore requires to be large to keep down the flux density which would be produced in the iron by the strong M.M.F. of the field with a small air-gap. The gap is therefore made large to obtain the necessary reluctance. To obtain the regulation mentioned on page 251, for machines of normal design, the field ampère-turns must be from 1.5 to 2 times the number of the armature ampère-turns. In modern alternators about 65 or 70 per cent. of the field ampère-turns are required for the air-gap, so that the permissible depth of air-gap under centre of pole will depend greatly upon the closeness of regulation required.

The curve in Figure 288 gives, for slow- and medium-speed machines, the average value of the depth of air-gap δ as a function of the rated output. If, after calculating the field ampère-turns on the

basis of this curve, it is found that the specified regulation is exceeded, then the latter may be improved by increasing δ, and, consequently, the ratio of field ampère-turns to armature ampère-turns.

A characteristic of high-speed machines with few poles is that the magnetic circuit is cramped, the length of the iron parts being short owing to the small dimensions. For this reason a greater proportion of the field ampère-turns has to be expended over the air-gap than would be the case with slower speed machines. This leads to longer air-gaps, ranging from 1 to 4 cms.

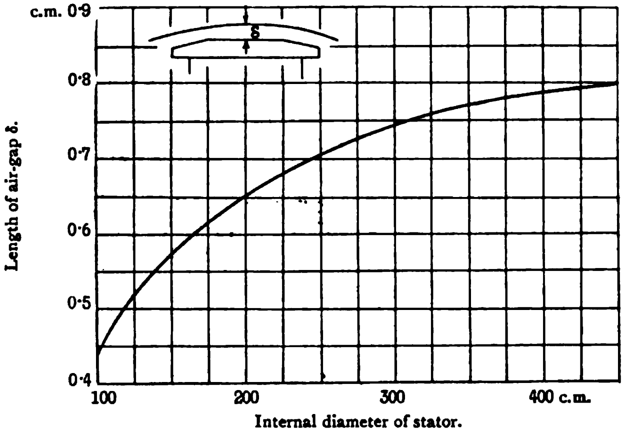


FIG. 288.—Radial depth of air-gap.

Magnet Core and Pole Shoes.—For wrought-iron or cast-steel poles the flux density at full-load should not exceed about 16,000 lines per cms.², but it is advisable to work as near to these densities as possible. In order to collect the fringing field from the flanks of the pole, the length of pole shoe is sometimes made from 2 to 4 cms. less than the gross length of the armature core. The radial depth of the field copper should, for reasons already discussed, be limited to about 4 or 5 cms., and the width of the pole core should be made less than that of the pole shoe, so that the projection of the latter serves to retain the field coils in position. The axial length of the magnet cores will depend upon the space required for the field copper, and a value must first be assumed from experience with other machines. The usual length of pole shoe in machines of standard design ranges from 15 to 20 cms. The ultimate length which is fixed upon should be such that the radiating surface is sufficient to prevent the heating of the coils exceeding the specified limit. The radial depth of the pole shoe at the centre ranges from 2.5 to 4 cms.

To obtain the flux to be generated in each pole it is necessary to assign a suitable value to the leakage factor σ. Since the dimensions

of the pole core and yoke would not at this stage be settled, a value of σ must be assumed, and in this respect the tabulated values on page 401 for various sizes of machines should be useful in obtaining a first approximation. After the dimensions of the magnetic circuit are fixed and the ampère-turns for air-gap, teeth, and armature core calculated, the value of σ should be determined more exactly.

In the design of turbo-alternators it is a somewhat difficult problem to obtain sufficient space for the field copper without undue heating and high mechanical stresses. It is therefore a good practice to ascertain, at an early stage in the design, whether there will be sufficient space for the requisite amount of field copper on the poles. If this question be left to a later stage, when the final calculations of the field winding are made, the whole design may have to be modified considerably. If the available space is not large enough, it will be necessary either to decrease the armature strength by employing fewer turns per phase, or to increase the diameter of the field system so as to obtain a greater radial depth of pole core. Both procedures reduce the value of the output coefficient. The former modification entails an increase in the flux per pole, and hence an increased length of core L_g. In order to check the space available for the winding, the total field ampère-turns can be approximately determined by adding about 15 per cent. to the ampère-turns calculated for the air-gap.

Magnet Ring.—To obtain a large fly-wheel effect and steadiness of running, the magnet ring of slow- and medium-speed machines is generally designed for a larger cross-sectional area than is actually necessary for magnetic purposes. When of cast steel, the flux density ranges from 6000 to 10,000 lines per cm.^{2}; while for cast-iron rims the densities would be somewhat lower. In high-speed machines fly-wheel considerations are not so important, with the result that flux densities as high as 14,000 or 15,000 are usually employed.

Calculation of Exciting Coils.—When the magnetic flux per pole and dimensions of the magnetic circuit have been settled, the field ampère-turns per pole are then calculated and expressed thus:—

$$AT = 0.5\{AT_c + AT_t + AT_g + AT_p + AT_y\} + AT_{DM}$$

where AT_{DM} denotes the field ampère-turns to compensate for the demagnetising M.M.F. of the armature, and may be determined as set forth on page 269. The size of wire with which the field coils are wound will be calculated by aid of the formula

$$W = \frac{2 \times 10^{-6} \cdot L \cdot (AT)^2}{A_x \cdot F_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (115)$$

where

W = watts wasted per coil at normal working temperature.

L = mean length per turn in metres.

A = ampères per coil.
T = turns per coil.
A_x = cross-section of winding space per coil in square decimetres.
F_c = space factor of winding.

This formula may be derived as follows:—

Let l = length of wire, in metres, for each coil.
a = cross-sectional area of wire in cms.^{2}
ρ = specific resistance of copper at normal temperature $= 2 \times 10^{-6}$ at 60° C.

Then the resistance per coil is expressed by

$$R = \frac{\rho \times l \times 100}{a} = \frac{2 \times 10^{-6} \times l \times 100}{a}$$

Now, $\qquad l = LT \quad \text{and} \quad a = \dfrac{A_x F_c \times 100}{T}$

$$\therefore \quad W = A^2R = \frac{2 \times 10^{-6} L . (AT)^2 \times 100}{A_x F_c \times 100}$$

$$\text{i.e.,}\ W = \frac{2 \times 10^{-6} . L . (AT)^2}{A_x . F_c}$$

When the watts have been calculated, and the voltage E per coil is known, the exciting current, number of turns, and cross-sectional area of wire is determined thus:—

$$\text{Exciting current} = A = \frac{W}{E}$$

$$\text{Number of turns per coil} = T = \frac{AT}{A}$$

$$\text{Cross-sectional area of conductor} = a = \frac{A_x . F_c}{T} \times 100 \text{ sq. cms.}$$

Design of Slow- and Medium-speed Alternators.

600-K.V.A., 50 ~ 3-phase Alternator.

Specification—

Rated output = 600 K.V.A.
Terminal pressure = 3300 volts..
Frequency = 50.
Speed = 250 R.P.M.
Regulation for cos $\Phi = 0.8$, not to exceed 16 per cent. (rise).
Exciter voltage = 125.

Main Dimensions—

$$\text{Number of poles} = \frac{120 \sim}{R} = \frac{120 \times 50}{250} = 24.$$

From Figure 287 the output coefficient for a 600-K.V.A., 50 ~ alternator will be about 1.5×10^{-6}; hence

$$D^2L_r = \frac{\text{K.V.A.}}{\xi \times R} = \frac{600}{1.5 \times 10^{-6} \times 250} = 16 \times 10^5$$

Assuming a peripheral speed of 29 m/sec.,

$$\text{internal diameter of armature} = D = \frac{60 \times 29 \times 100}{\pi \times 250} = 220 \text{ cms.}$$

$$\text{Gross length of armature core} = L_g = \frac{16 \times 10^5}{220^2} = 33 \text{ cms.}$$

Pole pitch $= \tau = 28.8$ cms.
Pole arc $= b_s = 0.66 \times 28.8 = 19$ cms.

$$\frac{L_g}{b_s} = \frac{33}{19} = 1.75.$$

The core will be built with four ventilating ducts, each 1.5 cm. wide. Nett length of iron in core is therefore

$$L_n = \{33 - (4 \times 1.5)\} \times 0.9 = 24 \text{ cms.}$$

Gross length of iron between ducts $= l = 5.4$ cms.

Armature Windings and Slots—

Assuming the stator to be star connected,

$$\text{Current per phase} = I_a = \frac{\text{K.V.A.} \times 1000}{\sqrt{3}\,E} = \frac{600 \times 1000}{\sqrt{3} \times 3300} = 105 \text{ ampères.}$$

From Table XX., the specific electric loading $= AC = 180$.
Hence number of turns in series per phase

$$= T = \frac{\pi D \, . \, AC}{2\, I_a \, . \, m} = \frac{\pi \times 220 \times 180}{2 \times 105 \times 3} = 196$$

Conductors per phase $= 2 \times 196 = 392$.

An alternator of this voltage would be designed with either 2 or 3 slots per pole per phase. Assuming 2 slots per pole per phase, the number of slots $= 3 \times 2 \times 24 = 144$,—*i.e.* 48 slots per phase. With 392 conductors per phase the number of conductors per slot $= 8.1$. Taking the nearest integer, the conductors per slot $= 8$. There will therefore be $8 \times 48 = 384$ conductors per phase. This corresponds to 175 ampère-conductors per cm. length of periphery,—*i.e.* $AC = 175$. The ampère conductors per slot $= 8 \times 105 = 840$, a value which is within the limits mentioned on page 386.

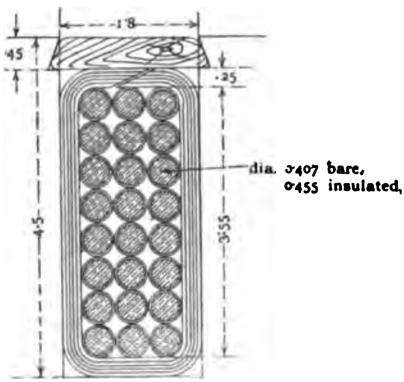


FIG. 289.—Slot for 600 K.V.A. 3300-volt alternator (full size).

For a current of 105 the current density will, from Table XXIII., be about 290 ampères per cm.² Adopting this figure as a basis, the cross-sectional area of each conductor $= 105/290 = 0.36$ cm.² Supposing each conductor to

be made up of 3 wires in parallel, then cross-sectional area of wire $\cong 0.36/3 = 0.12$ cm.^{2}

The nearest standard wire gauge (S.W.G.) to this is No. 8, which has a cross-sectional area of 0.1295 cm.^{2}, corresponding to a current density of 270 ampères per cm.^{2}

Diameter of No. 8 S.W.G. = 0.407 cm. (bare)
= 0.445 cm. (triple cotton covered).

There will be 24 wires per slot, disposed in 8 layers of 3 wires side by side. For a phase pressure of 1900 volts a micanite tube 0.25 cm. in thickness will be necessary for lining the slots. Allowing for some margin in the winding, the slot may be made 1.80 cm. wide by 4.5 cms. deep, the depth of slot including an opening at the top of 0.45 cm., which is closed by a hard-wood wedge. A detail drawing of the slot is given in Figure 289.

$$\text{Tooth pitch at air-gap} = \frac{\pi \times 220}{144} = 4.8 \text{ cms.}$$

$$\text{Width of tooth at air-gap} = t_1 = 4.8 - 1.8 = 3.0 \text{ cms.}$$

$$\text{Tooth pitch at bottom of slots} = \frac{\pi \times 229}{144} = 5.0 \text{ cms.}$$

Width of tooth at root = 3.2 cms.

Induced E.M.F. and Flux per Pole.—From page 319 the effective resistance per phase

$$r_a = \frac{1.7 \times 10^{-6} . k . l_a . T(1 + .004\, T^c)}{a_a} \text{ ohms.}$$

$k = 1.6.$

T = turns in series per phase = 192.

a_a = area of wire = $3 \times 0.1295 = 0.3885$ cm.^{2}

l_a = mean length per turn $\cong 170$.

Hence for a rise in temperature of 40° C. the effective resistance is

$$r_a = \frac{1.7 \times 10^{-6} \times 1.6 \times 170 \times 192\ (1 + 0.004 \times 40)}{0.3885} = 0.27 \text{ ohm.}$$

Volts drop per phase due to resistance

$$= e_r = I_a r_a = 105 \times 0.27 \cong 30$$

From example on page 260 the reactance voltage per phase

$$= e_x = 144$$

The E.M.F. to be induced in each phase at full-load (see p. 277)

$$= E_i = E + e_r \cos\phi + e_x \sin\phi$$
$$= 1900 + 30 \times 0.8 + 144 \times 0.6 \cong 2000 \text{ volts.}$$

The flux per pole at full-load ($\cos\phi = 0.8$) is

$$\Phi_{a1} = \frac{2000 \times 10^8}{4.44 \times 0.96 \times 192 \times 50} = 4.9 \times 10^6 \text{ lines.}$$

Also flux per pole at no-load

$$\Phi_{a2} = \frac{1900 \times 10^8}{4.44 \times 0.96 \times 192 \times 50} = 4.65 \times 10^6 \text{ lines.}$$

Dimensions of Magnetic Circuit.—For a 50 ~ alternator the flux density in core should ≅ 7000 lines per cm.[2] Hence radial depth of iron below teeth is

$$d = \frac{\Phi_{a1}}{2 \cdot B_c \cdot L_n} = \frac{4.9 \times 10^6}{2 \times 7000 \times 24} \cong 15 \text{ cms.}$$

From Figure 288, radial depth of air-gap under centre of pole = say 0.65 cm.

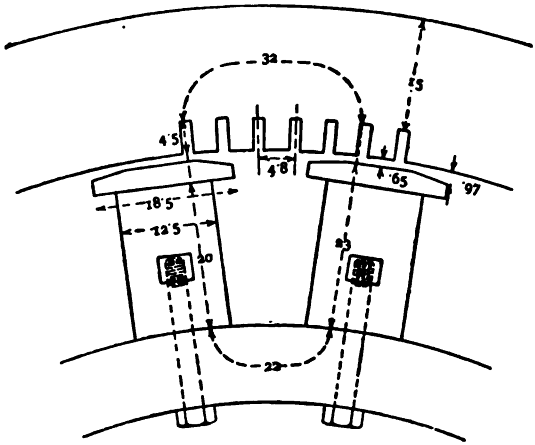


FIG. 290A.

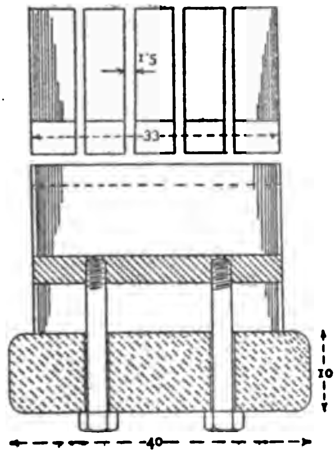


FIG. 290B.

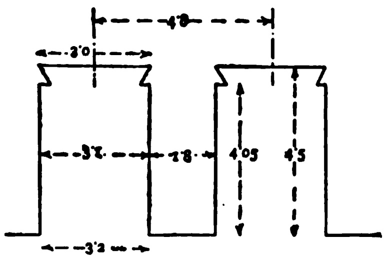


FIG. 290C.

Since the winding is placed in open slots, the pole shoes must be laminated. In this design the pole core and shoes will be in one piece, and built up from sheet-steel laminations 1 mm thick, the pole shoes being bevelled as in Figure 138. A dimensioned drawing of the magnetic circuit is given in Figure 290A. Allowing 5 per cent. for

insulation, sectional area of pole core $= 12.5 \times 33 \times 0.95 = 400$ cms.^{2} Assume for the present a leakage factor $\sigma = 1.25$, then the flux density in pole core at full-load is

$$B_p = \frac{4.9 \times 10^6 \times 1.25}{400} = 15{,}000 \text{ lines per cm.}^2$$

The rim of the magnet wheel will be of cast steel. With a radial depth of 10 cms. the corresponding flux density $= 7600$ lines per cm.^{2}

Saturation Curve.—The ampère-turns to drive the full-load flux of 4.9×10^6 lines through the magnetic circuit has been worked out on

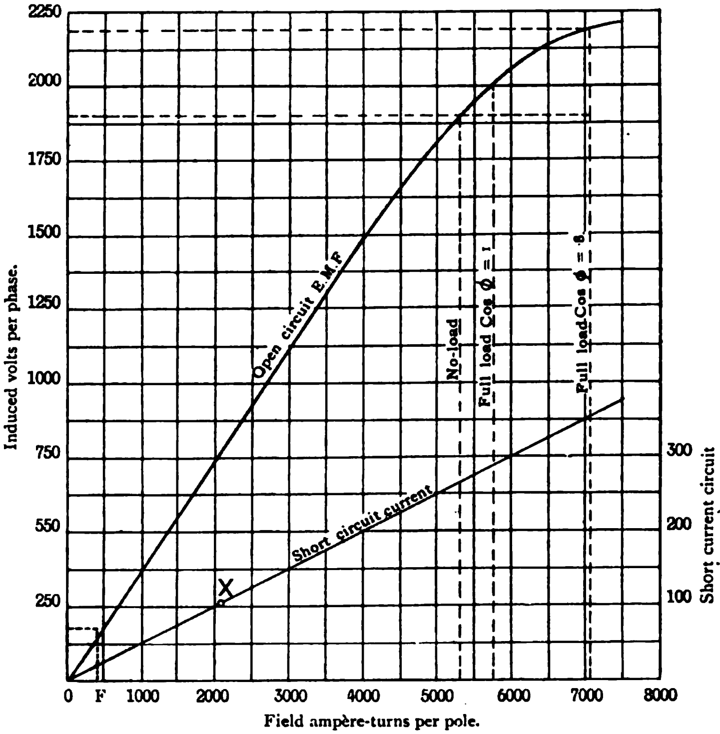


FIG. 291.—Saturation curve and short circuit current of 600-K.V.A. alternator.

pages 242 to 243, and $= 5750$ per pole. By similar calculations the ampère-turns necessary to maintain six other values of the flux ranging from 1.2 to 5.4 megalines have been determined and are set forth in Table XXV. The magnetisation curve (Figure 291) is then obtained by plotting the induced E.M.F. as a function of the calculated ampère-turns per pole.

[TABLE

TABLE XXV.—DATA FOR OPEN-CIRCUIT SATURATION CURVE.

Induced E.M.F. in Volts.	Flux per Pole $=\Phi$.	Ampère-turns per Pair of Poles.						Ampère-turns per Pole.
		Armature		Air Gap.	Magnet		Total.	
		Core.	Teeth.		Core.	Yoke.		
500	1.2×10^6	30	20	2500	70	40	2660	1330
1000	2.45×10^6	50	40	5000	140	80	5310	2655
1500	3.65×10^6	70	80	7500	280	120	8050	4025
1900	4.65×10^6	75	250	9500	600	140	10565	5280
2000	4.9×10^6	80	520	10000	750	150	11500	5750
2150	5.25×10^6	100	950	10700	1150	170	13070	6535
2200	5.4×10^6	120	1700	11000	1400	200	14420	7210

Leakage Factor.—Having determined the ampère-turns to drive the flux Φ through the magnetic circuit, the leakage factor σ should now be estimated. From the example on page 249 $\sigma=1.25$, a value which is in close agreement with that assumed for the magnetic circuit calculations.

Inherent Regulation and Short-circuit Current.—The inherent regulation can now be estimated from the saturation curve of Figure 291. To maintain a phase pressure of 1900 volts at full non-inductive load there must be induced in each phase an E.M.F. of 2000 volts, and to drive the necessary flux through the magnetic circuit 5750 ampère-turns are required per pole. For an inductive load corresponding to $\cos\phi=0.8$, the demagnetising ampère-turns of the armature $=$ 1350 (*vide* p. 276). Hence for full inductive load and $\cos\phi=0.8$ there is required an excitation of $5750+1350=7100$ ampère-turns per pole in order to give a terminal pressure of 1900 volts per phase. When full-load is thrown off the terminal voltage will rise to 2190. This corresponds to a regulation of $\frac{2190-1900}{1900}\cdot100=15$ per cent., a value which is just within the limit specified.

When full-load current flows through the armature on short circuit, the voltage consumed in armature impedance is

$$e_z=\sqrt{e_r^2+e_x^2}=\sqrt{30^2+144^2}=147 \text{ volts.}$$

From Figure 291 the excitation necessary to induce this voltage $=$ OF $=400$ ampère-turns. The internal phase angle on short-circuit

$$=\phi_1=\sin^{-1}\frac{e_x}{e_z}=\sin^{-1}\frac{144}{147}=\sin^{-1}0.98$$

The corresponding demagnetising armature ampère-turns are (see p. 276)

$$AT_{DM}=0.45\times0.96\times3\times16\times105\times0.98\times0.83=1770$$

The excitation necessary to maintain full-load current through the armature on short-circuit is therefore $400 + 1770 = 2170$ ampère-turns. This gives the point X on the short-circuit curve. Joining OX and producing the line OX until it cuts the ordinate corresponding to full-load excitation, it will be found that the normal short-circuit current = 350 ampères = 3.3 times normal full-load current.

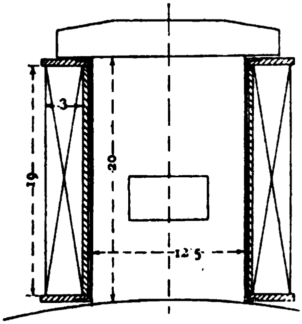


FIG. 292.

Field Coils.—The bobbins for the field coils have the dimensions shown in Figure 292. Cross-section of winding space = 19×3 cm.^{2}

Mean length per turn = 105 cms.,—*i.e.* L = 1.05 metre.

Field ampère-turns at full-load = AT = 7100.

Cross-section of winding space = A_x = 0.57 dcm.^{2}

In revolving field alternators, except in small sizes, the field coils are always wound with wire of rectangular section, the radial depth of the winding ranging from 2.5 to 3.5 cms. When copper strip, wound on edge, is used the space factor will be fairly high $\cong 0.8$, but for cotton insulated wire the value of F_c may be as low as 0.5 or 0.6.

Winding will be of copper strip; hence space factor = $F_c = 0.8$.

$$\text{Watts per coil} = W = \frac{2 \times 10^{-6} \times 1.05 \times 7100^2}{0.57 \times 0.8} = 232.$$

Exciter voltage = 125.

Leaving 15 volts spare, the volts per coil = E = 110/24 = 4.6.

Exciting current = 232/4.6 = 50.5 ampères.

Number of turns per coil = 7100/50.5 = 140.

$$\text{Sectional area of copper strip} = \frac{19 \times 3 \times 0.8}{140} = 0.326 \text{ cm.}^2$$
$$= 3 \text{ cm.} \times 0.11 \text{ cm.}$$

Current density = 150 ampères per cm.^{2}

Thickness of paper insulation between layers = 0.025 cm.

Losses and Heating at Full-load Cos ϕ = 0.8 — (1)

Armature—

Mass of armature teeth = $3.1 \times 4.5 \times 24 \times 144 \times 0.0078 = 375$ kgs.

Mean flux density (p. 242) = 17,000.

From curves in Figure 241, $\epsilon = 0.22$ and $\beta = 2.1$.

Iron loss in teeth = $375 \times 50 \times 2.1 \times 0.22 = 8700$ watts.

$$\text{Mass of armature core} = \frac{\pi}{4}(260^2 - 230^2) \times 24 \times 0.0078 = 2160 \text{ kgs.}$$

$$\text{Flux density} = \frac{4.9 \times 10^6}{2 \times 24 \times 15} = 6800$$

$\epsilon = 0.05$ and $\beta = 2.1$.

Iron loss in core $= 2100 \times 50 \times 2.1 \times 0.05 = 11{,}000$ watts.

Total iron loss $= 20{,}000$ watts $= 20$ K.W.

Effective armature resistance $= r_a = 0.27$ ohm per phase.

Armature copper loss $= m \,.\, I_a^2 \,.\, r_a = 3 \times 105^2 \times 0.27 = 9000$ watts $= 9$ K.W.

Copper loss within slots $= 9000 \times \dfrac{66}{170} = 3500$ watts.

Total loss between core flanges

$$= W_a = 20{,}000 + 3500 = 23{,}500 \text{ watts.}$$

Cooling surface of armature

$$= A_a = \frac{\pi}{4}(260^2 - 220^2)\, 6 + 33 \times \pi\,(260 + 220) = 140{,}000 \text{ cms.}^2$$
$$= 1400 \text{ dcm.}^2$$

Probable temperature rise $= T^\circ = K_a \,.\, \dfrac{W_a}{A_a}$

$$\cong 2.3 \times \frac{24000}{1400} \cong 40^\circ \text{ C.}$$

Field Winding—

Watts per pole $= W_m = 232$.

Total excitation loss $= W_{ce} = 232 \times 24 = 5600$ watts $= 5.6$ K.W.

Cooling surface per coil $= 24$ dcms.^{2}

Probable temperature rise $= T^\circ = K_m \,.\, \dfrac{W_m}{A_m(1 + 0.1\, v_m)}$

$$= 9 \,.\, \frac{232}{24(1 + 0.1 \times 30)} = 24^\circ \text{ C.}$$

Efficiency at Full-load Cos $\phi = 0.8$—

Armature iron loss	=	20,000 watts.
Armature I^2R loss	=	9,000 "
Excitation loss	=	5,600 "
Friction and windage (assumed)	=	5,000 "
Total loss	=	40,000 "
Power output	=	480,000 "
Power input	=	520,000 "
Efficiency	=	92 per cent.

Weights—

Armature copper	=	340 kgs.	1320 kgs.
Field copper	=	980 "	
Armature laminations	=	2600 kgs.	5900 "
Magnet poles	=	1800 "	
Magnet ring	=	1500 "	
Weight of active material per K.W. output	=	14.8 "	

TABLE XXVI.—DESIGNS FOR SIX MEDIUM-SPEED ALTERNATORS.

(All Dimensions in Cms.)

Design Number	1	2	3	4	5
Drawing on Plate	VI.	VII.	VIII.	IX.	X.
Manufacturer	Vickers Sons & Maxim.	British Electric Plant Company.	Crompton & Company.	General Electric Company.	Electric Construction Company.
Specification—					
Output in K.V.A.	285	440	625	800	1250
Normal rating K.W.	200	375	500	600	1000
Normal rating cos Φ	0.7	0.85	0.8	0.75	0.8
Number of phases	Three	Three	Three	Three	Two
Frequency	25	50	25	50	50
Speed in R.P.M.	300	375	150	300	200
Number of poles	10	16	20	20	30
Terminal volts	2000	600	5300	220/230	3300
Star or delta connection	Star	Star	Star	Star	...
Current output in ampères	82	425	68	2100	190
Stator Core—					
External diameter	168	190	290	260	384
Internal diameter	137	160	240	220	336
Gross length between flanges	31.5	27	50	35	25
Number of vent ducts	1	4	6	6	2
Nett length of iron	27	22	37	25	20
Number of slots	90	240	180	120	240
Slots per pole per phase	3	5	3	2	4
Stator Winding—					
Conductors per slot	11	1	10	½	5
Turns in series per phase	165	40	300	15	300

Size of conductor (bare)	Diameter 0.43 two in parallel	1.8 × 0.6	0.54 diameter	4(0.35 × 2.5)	2(0.505 × 0.405)
Current density in ampères per cm.^{2}	280	390	300	300	460
Slot space factor	0.46	0.5	0.25	0.60	0.37
Mean length per turn	210	160	230	190	160
Air-gap—					
Radial depth	0.76	0.6	0.4	0.7	0.75
Field Magnet Iron—					
Pole pitch	43	31.5	37.5	34.5	35
Pole arc ÷ pole pitch	0.65	0.66	0.65	0.66	0.65
Length of pole shoe parallel to shaft	31.5	25.5	50	32	25
Radial depth of pole + shoe	20	19	24	24	24
Cross-sectional area of magnet ring	430 cms.^{2}	175 cms.^{2}	600 cms.^{2}	840 cms.^{2}	1400 cms.^{2}
Poles of	Laminated steel	Cast steel	Laminated steel	Cast steel	Cast steel
Magnet ring of	Cast iron	Cast steel	Cast steel	Cast iron	Cast iron
Field Winding—					
Turns per pole	157	100	80	40	176
Size of wire (bare)	0.54 × 0.54	0.525 × 0.525	0.9 × 0.9	2.5 × 0.4	0.23 cm.^{2}
Depth of winding space	4.8	3.6	4.0	2.5	3.5
Space factor	0.65	0.6	0.8	0.85	0.65
Mean length per turn	120	100	168	120	110
Ampères at normal rating	64	83	145	220	50
Current density in ampères per cm.^{2}	215	300	180	220	220
Exciter volts (max.)	100	85	100	36	240
Magnet Data (full-load)—					
Flux in armature per pole	7.6×10^6	4.2×10^6	9.4×10^6	6.5×10^6	5.3×10^6
Leakage coefficient	1.25	1.2	1.15	1.29	1.25
Flux density in armature core	12,000	7,600	7,000	8,500	6,700
,, ,, teeth (max)	16,000	16,500	20,000	17,500	17,500
,, air-gap	8,600	9,100	8,500	7,500	9,000
,, pole core	17,500	16,000	14,000	15,000	13,000
,, magnet ring	9,000	13,500	9,000	5,000	2,500

TABLE XXVI. (*continued*)—

Design Number	1	2	3	4	5
Drawing on Plate	VI.	VII.	VIII.	IX.	X.
Manufacturer	Vicker Sons & Maxim.	British Electric Plant Company.	Crompton & Company.	General Electric Company.	Electric Construction Company.
Losses and Efficiency (full-load)—					
Cos Φ	0.7	0.85	0.8	0.75	0.8
Core loss in watts	7,000	13,000	19,000	25,000	28,500
Stator I^2R loss in watts	4,700	10,000	9,000	9,000	25,000
Excitation loss in watts	5,200	7,000	12,000	8,000	13,400
Friction and windage losses in watts	2,000	4,000	11,000	...	5,000
Efficiency	91.4 %	92 %	91 %	...	93.2 %
Heating (6 hours' test at full-load)—					
Temperature rise of stator	30° C.	22° C.	..	36° C.	30° C.
Watts loss between core flanges	8,400	16,400	23,000	28,300	36,500
Cooling surface (dcm.²)	250	800	2,200	1,780	1,660
Heating coefficient K_a	1.2	1.1	...	2.3	1.4
Temperature rise of end-connections	38° C.	21° C.	...	22° C.	...
,, ,, field coils	40° C.	21° C.	...	20° C.	34° C.
Excitation watts per coil	520	440	600	265	375
Cooling surface per coil (dcm.²)	37	37	36	26	23
Heating coefficient K_m	9	6	...	9	9
Inherent Regulation (rise)—					
Full-load cos Φ = 1	6 %	5 %	7 %	5.5 %	4.5 %
Full-load cos Φ	20 %	16.5 %	16 %	19.9 %	15 %
	0.7	0.85	0.9	0.75	0.8
Weights (in kgs.)—					
Stator copper	265	190	440	530	380

Field copper	340	400	2,000	850	1,200
Stator iron	1,400	2,740	5,500	2,700	3,700
Magnet pole	830	1,900	3,200	2,100	1,800
Magnet yoke	850	600	2,700	3,500	10,000
Total active material	3,685	5,830	13,840	9,680	17,080
Weight per K.W. output	18.4	15.5	28	16	17
Constants—					
Output coefficient	1.6×10^6	1.7×10^6	1.45×10^6	1.6×10^6	2.2×10^6
Peripheral speed in m/sec.	22	31.5	19	35	35
Ampère - conductors per cm. of armature periphery	190	200	160	180	215
Flux density in air-gap	8,600	9,100	8,500	7,500	9,000
$\frac{L_c}{b_s}$	1.1	1.3	2.1	1.45	0.9
$\frac{\text{Short-circuit current}}{\text{Full-load current}}$	...	1.6	4	3.2	2.0 [Field excited to give 3300 volts at no-load]

Examples of Designs.

In Table XXVI. there are given the principal dimensions and constants of five medium-speed alternators for outputs up to 1250 K.V.A. The detail drawings are reproduced in Plates VI. to X. Alternator No. 4, by the General Electric Company, is of special interest, as it is designed for an exceptionally low voltage, namely, 220/230. With this low voltage it was impossible to have one winding in series for each phase, so that the winding had to be divided into two paralleled halves, each half being in separate slots. The two halves per phase were perfectly balanced by arranging alternately the long and short coils to be in series in each half.

Turbo-alternators

Design of a 1000-K.V.A., 50 ~ Three-phase Turbo-alternator of the cylindrical Field Type.

Specification—

Rated output . . . . = 1000 K.V.A.
Terminal pressure . . . . . . = 6600 volts.
Frequency . . . . = 50.
Speed of rotor . . . . . . = 1500 R.P.M.
Exciter voltage . . . . . . = 75.

Main Dimensions—

Number of poles = 4.

Output coefficient (Figure 287) = 1.24×10^{-6}.

$$D^2L_g = \frac{1000}{1.24 \times 10^{-6} \times 1500} = 5.4 \times 10^5.$$

Peripheral speed = 70 m/sec.

$$\text{Internal diameter of stator} = D = \frac{60 \times 7000}{\pi \times 1500} = 90 \text{ cms.}$$

$$\text{Gross length of armature core} = L_g = \frac{5.4 \times 10^5}{90^2} = 70 \text{ cms.}$$

Pole pitch = τ = 70 cms.

Number of 1.2 cm. ventilating ducts = 12.

Nett length of stator iron = L_n = 50 cms.

Armature Winding and Slots—

For star-connected winding current per phase = 87 ampères.

From Table XX. ampère-conductors per cm. = 180.

$$\text{Turns in series per phase} = \frac{\pi \times 90 \times 180}{2 \times 87 \times 3} = 100.$$

Conductors per phase = 2 × 100 = 200.

Assume 5 slots per pole per phase.

Number of slots per phase = 4 × 5 = 20

Total number of slots = 60.

Conductors per slot $= \frac{200}{20} = 10$.

Ampère-conductors per slot $= 870$.

For a current density $= 300$ ampères per cm.^{2} cross-sectional

area of conductor $= \frac{87}{300} = 0.3$ cm.^{2}

Size of conductor $= 0.6 \times 0.25$ (bare) – two in parallel.

Dimensions of insulated wire $= 0.67 \times 0.32$.

The dimensions of the slots, teeth, insulation, and arrangement of conductors are shown in Figure 293.

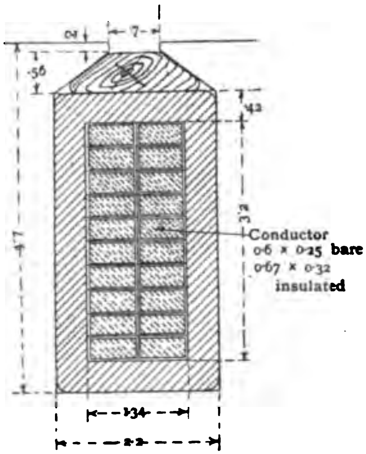


FIG. 293A.

Induced E.M.F. and Flux per Pole—

Mean length per armature turn $= 400$ cms.

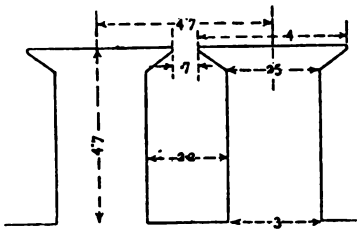


FIG. 293B.

Effective resistance per phase

$$= r_a = \frac{1.7 \times 10^{-6} \times 1.6 \times 400 \times 100\ (1 + 0.004 \times 45)}{0.3} = 0.43 \text{ ohm.}$$

Volts drop per phase $= e_r = 87 \times 0.43 = 38$.

$$\text{Slot inductance} = L_s = 0.4\ \pi \frac{C^2}{q} \cdot l_i \left(\frac{d}{3s} + \frac{d_4}{s} + \frac{2\ d_3}{s + s_o} + \frac{d_2}{s_o} \right) \times 10^{-8}$$

(see p. 255)

$$= 0.4\ \pi \times \frac{50^2}{5} \times 110 \left(\frac{3.2}{3 \times 2.2} + \frac{0.42}{2.2} + \frac{2 \times 0.56}{2.2 + 0.7} + \frac{0.1}{0.7} \right) \times 10^{-8}$$

$$= 8.3 \times 10^{-4} \text{ henries.}$$

$$\text{Tooth head inductance} = L_a = 0.92 \cdot \frac{C^2}{q} \cdot l_i\ \Delta' \times 10^{-8}$$

$$\Delta' = \log\left(1 + \frac{\pi t_p}{s_o}\right) + 2.22 + 5 \log \frac{r}{4 t_p}$$

$$= \log\left(1 + \frac{\pi \times 4.7}{0.7}\right) + 2.22 + 5 \log \frac{47}{4 \times 4.7} = 5.6.$$

$$\left[r = \tau \cdot \frac{p}{p+1} = 70 \times \frac{2}{3} = 47 \right]$$

Hence $L_a = 0.92 \times \frac{50^2}{5} \times 110 \times 5.6 \times 10^{-8} = 28 \times 10^{-4}$ henries.

Inductance due to end leakage $= L_e = 0.46\, l_e\,.\, C^2\left(\log\frac{l_e}{d_s} - 0.5\right)10^{-8}$

$$= 0.46 \times 290 \times 50^2\left(\log\frac{290}{7} - 0.5\right)10^{-8} = 37 \times 10^{-4} \text{ henries.}$$

Reactance volts per phase $= e_x = 2\,\pi \sim n_c\,.\,\{L_s + L_a + L_e\}\, I_a$

$$= 2\,\pi \times 50 \times 2\,\{(8.3 + 28 + 37)10^{-4}\}\,87 = 400 \text{ volts.}$$

Induced E.M.F. per phase at full-load ($\cos\phi = 0.8$)

$$= E_t = E + e_r \cos\phi + e_x \sin\phi$$
$$= 3800 + 38 \times 0.8 + 400 \times 0.6 = 4070.$$

Flux per pole at full-load ($\cos\phi = 0.8$)

$$= \Phi_{a1} = \frac{4070 \times 10^8}{4.44 \times 0.95 \times 100 \times 50} = 19.3 \times 10^6 \text{ lines.}$$

Flux per pole at no-load

$$= \Phi_{a2} = \frac{3800 \times 10^8}{4.44 \times 0.95 \times 100 \times 50} = 18.0 \times 10^6 \text{ lines.}$$

Magnetic Circuit.—To obtain a flux density of 7000 lines per cm.^{2}, radial depth of armature core below teeth

$$= d = \frac{19.3 \times 10^6}{2 \times 7000 \times 50} = 27.3 \text{ cms.}$$

Hence external diameter of stator iron = 154 cms.

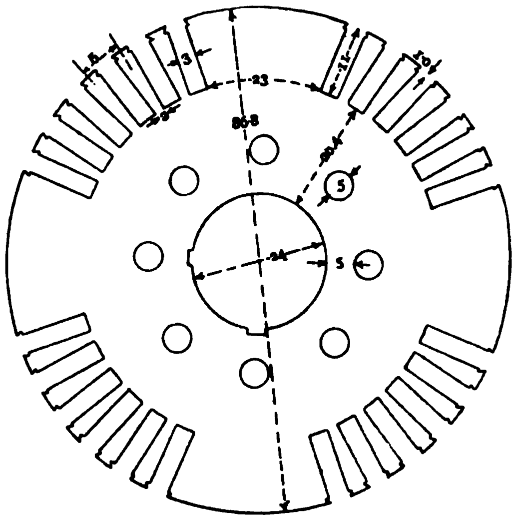


FIG. 294.

Assume (provisionally) a radial depth of air-gap = 1.6 cm. The rotor will be built up from 1 mm. sheet steel laminations, keyed

directly upon the shaft and assembled as in Figure 297. For ventilation there are provided eleven radial ducts equally spaced between those of the stator, the cooling air entering these ducts through four longitudinal holes in the laminations. The rotor laminations are dimensioned as shown in Figure 294, the field coils being wound in 24 slots,—*i.e.* six slots per pole.

Size of slots	$=3 \times 11$
Depth of wedge	$= 1.0$ cm.
Width of iron at tooth root	$= 2$ "
Width of iron at root of pole	$= 23$ "
Gross length of core	$= 66$ "
Width of each duct	$= 1.0$ "
Nett length of rotor iron	$= 50$ "
Cross-section of iron per pole at tooth root	
$= (5 \times 2 + 23)50$	$= 1650$ cms.^{2}
Assumed leakage factor	$= \sigma = 1.15$.

The ampère-turns necessary to drive six values of the armature flux

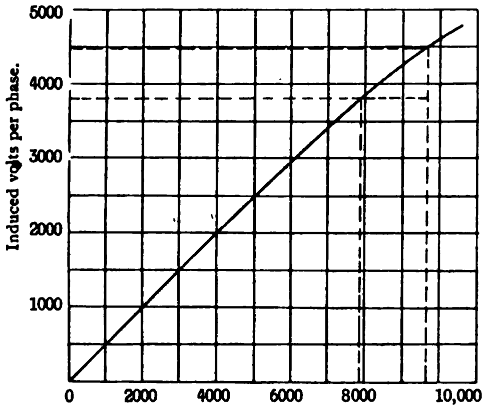


FIG. 295.—Saturation curves of 1000-K.V.A. turbo-alternator. Air-gap, 1.6 cm.

ranging from 4.8 to 21.3 megalines have been calculated as set forth in Table XXVII. From this data the saturation curve of Figure 295 has been plotted.

[TABLE

TABLE XXVII.—DATA FOR SATURATION CURVE.

Induced Volts per Phase.	Magnetic Flux per Pole.	Ampère-turns per Pair of Poles.						Ampère-turns per Pole.
		Armature Core.	Armature Teeth.	Air-gap.	Rotor Poles.	Rotor Yoke.	Total.	
1000	4.7×10^6	80	20	3,900	40	50	4,100	2,050
2000	9.5×10^6	180	40	7,800	90	100	8,200	4,100
3000	14.2×10^6	240	60	11,700	140	170	12,300	6,150
3800	18.0×10^6	280	130	14,600	260	360	15,630	7,820
4070	19.3×10^6	300	150	15,600	350	500	16,900	8,450
4500	21.3×10^6	450	250	17,400	500	1000	19,600	9,800

Demagnetising Armature Ampère-turns—

$$\frac{E_{CM}}{\cos \phi_1} = k_0 \,.\, 10^{-8} \,.\, m \,.\, \frac{T^2}{p} \,.\, \sim \,.\, \frac{b_i \,.\, l_i}{\delta} \,.\, I_a$$

$$= 1.03 \times 10^{-8} \,.\, 3 \,.\, \frac{100^2}{2} \,.\, 50 \,.\, \frac{45 \times 70}{1.6} \times 87 = 130.$$

[Ideal pole breadth $= 0.65\ \tau = 0.65 \times 70 = 45$ cms.]

$k_3 \cong 1$

$$\phi_1 = \tan^{-1}\left(\frac{E \sin \phi + e_x + \dfrac{E_{CM}}{\cos\phi_1}}{E \cos \phi + e_r}\right)$$

$$= \tan^{-1}\left(\frac{3800 \times 0.6 + 400 + 130}{3800 \times 0.8 + 38}\right) = \tan^{-1} 0.91 = 42 \text{ degrees.}$$

$\sin 42$ degrees $= 0.67$: $k_2 = 0.96$: $T_1 = \dfrac{100}{4} = 25$

$$\text{and } \frac{\sin \dfrac{\sigma}{2}}{\dfrac{\pi\sigma}{2}} = 0.63.$$

Demagnetising ampère-turns per pole

$$= AT_{DM} = 0.45\ k_2 \,.\, m \,.\, T_1 \,.\, I_a \,.\, \sin \phi_1 \,.\, \frac{\sin \dfrac{\pi\sigma}{2}}{\dfrac{\pi\sigma}{2}}$$

$$= 0.45 \times 0.96 \times 3 \times 25 \times 87 \times 0.67 \times 0.63$$
$$= 1200.$$

Regulation.—In order to obtain the rated output at 0.8 power factor the field excitation must $= 8450 + 1200 = 9650$ ampère-turns per pole. When full-load is thrown off and the excitation maintained

constant, the terminal voltage per phase rises from 3800 to 4475. The inherent regulation therefore $= \frac{675}{3800} \times 100 = 18.0$ per cent. This will be about the value required from a turbo-alternator, but should a closer degree of regulation be called for, then it would be necessary to increase the air-gap depth to 1.7 or 1.75 cm.

Short-circuit Current.—Volts consumed in armature impedance with full-load current

$$= e_z = \sqrt{38^2 + 400^2} = 400 \text{ volts.}$$

From saturation curve ampère-turns to induced this voltage

$$= 900.$$

On short-circuit internal phase angle

$$= \phi_1 = \sin^{-1} \frac{400}{400} = \sin^{-1} 1.$$

Hence when the armature is short-circuited and the excitation adjusted to give full-load current, the armature demagnetising ampère-turns per pole

$$= AT_{DM} = 0.45 \times 0.96 \times 3 \times 25 \times 87 \times 1 \times 0.63 = 1800.$$

The field excitation necessary to send full-load current through the armature when short-circuited is therefore $= 900 + 1800 = 2700$ ampère-turns per pole. Assuming the short-circuit curve to be a straight line, the short-circuit current at full-load excitation $= \frac{9650}{2700} = 3.5$ times full-load current.

Field Windings.—Space available per slot for wires and insulation $= 10$ cms. $\times 3$ cms. The rotor slots will be lined (see Figure 296) with press-spahn or leatheroid to a thickness of 0.15 cm. Space available for wires $= 9.7$ cms. $\times 2.7$ cms. The field coils will be wound with wire of rectangular section, and insulated with a covering of cotton and braid. The space factor F_c for the winding space will be $= 0.65$.

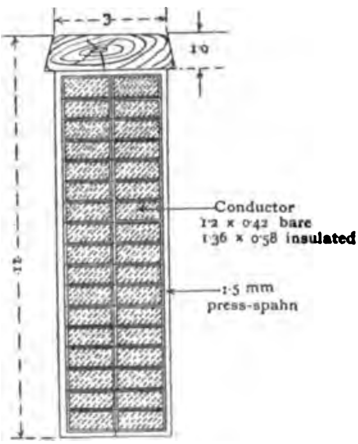


FIG. 296.—Rotor slot.

Ampère-turns per pole $= AT = 9650$.

Mean length per turn $= L = 4.0$ metres.

Cross-section of winding space $= A_x = 9.7 \times 2.7 \times 3 = 0.79$ sq. dcm.

$$\text{Watts per coil} = W = \frac{2 \times 10^{-6} \,.\, L \,.\, AT^2}{A_x \,.\, F_c}$$

$$= \frac{2 \times 10^{-6} \times 4.0 \times 9650^2}{0.79 \times 0.65} = 1450 \text{ watts.}$$

Exciter voltage = 75.

Leaving 15 spare, the voltage per pole = 15 ; hence exciting

$$\text{current} = \frac{1450}{15} = 96 \text{ ampères.}$$

$$\text{Number of turns per pole} = \frac{9650}{96} = 102.$$

$$\text{Conductors per slot} = \frac{100}{3} = 34.$$

Conductor, 2 abreast and 17 deep.

Thickness of insulation round conductor = 0.08 cm.

Hence width of conductor = 1.2 cm.

Thickness of conductor = 0.42 cm.

Actual space factor = 0.64.

$$\text{Current density} = \frac{96}{1.2 \times 0.42} = 190 \text{ ampères per cm.}^2$$

Losses and Efficiency for Full-load and Cos $\phi = 0.8$—

Mass of armature teeth = 300 kgs.

Mean flux density = 14,500.

From Figure 241 : $c = 0.17$ and $\beta = 2.1$.

Iron loss in teeth = $300 \times 50 \times 0.17 \times 2.1 = 5350$ watts.

Mass of armature core = 4300 kgs.

Flux density = 7000.

$c = 0.054$ and $\beta = 2.1$.

Iron loss in core = $4300 \times 50 \times 0.054 \times 2.1 = 24{,}500$ watts.

Total iron loss = 30,000 watts.

Armature resistance = $r_a = 0.43$ ohm.

Armature copper loss = $m \,.\, I_a^2 \,.\, r_a = 3 \times 87^2 \times 0.43 = 10{,}000$ watts.

Excitation loss = $1450 \times 4 = 5800$ watts.

Total loss (excluding friction and windage) = 45,800 watts.

Efficiency (excluding friction, etc. loss) = 95.6 per cent.

Heating of Armature—

$$\text{Copper loss within slots} = 10{,}000 \times \frac{140}{400} = 3500 \text{ watts.}$$

Total loss between core flanges = $W_a = 30{,}000 + 3500 = 33{,}500$ watts.

Cooling surface of armature

$$= A_a = \frac{\pi}{4}(154^2 - 90^2)\ 14 + 55 \times \pi\ (154 + 90) = 214{,}000 \text{ cms.}^2$$
$$= 2140 \text{ dcms.}^2$$

$$\text{Temperature rise} = T^\circ = K_a \,.\, \frac{W_a}{A_a} = 2.3 \times \frac{33{,}500}{2140} = 36^\circ \text{ C.}$$

Weights—

Armature copper	=	320 kgs.	1045 kgs.
Field copper	=	725 „	

Armature iron	= 4600 „	} 6600 kgs.	
Rotor iron	= 2000 „		
Weight of active material per K.W.	= 7.65 „		

An outline drawing of the machine is given in Figure 297.

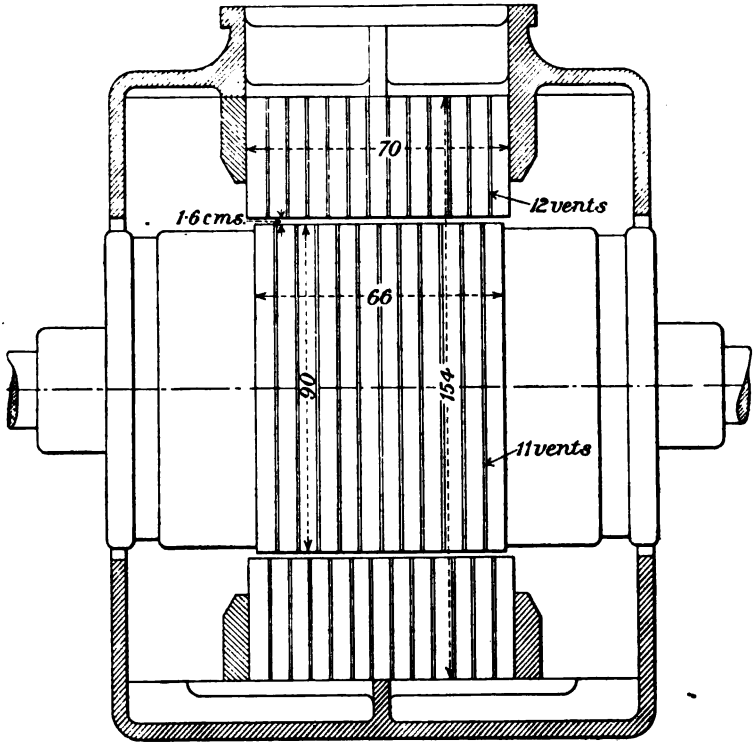


FIG. 297.—1000 K.W. turbo-alternator, 1500 revs. per min., 50 ~ 6600 volts. Dimensions in cms.

EXAMPLES OF DESIGNS.

In Table XXVIII. there is set forth the design data of two 3-phase, 50 ~ turbo-alternators having cylindrical rotors. Design No. 6 is for an output of 560 K.V.A at 500 volts, and is totally enclosed. Air is drawn in from an opening in the base of the end-covers, and after being propelled through the rotor and stator ducts leaves the machines by openings in the stator frame. This alternator is designed with two poles. The drawings are given in Plate XI. and a photograph of the machine while on the test bed is reproduced in Figure 298.

Alternator No. 7 (Plate XII.), by the Electric Construction Company, is designed for a continuous output of 1200 K.V.A. at 6600 volts and

an overload output 1800 K.V.A. for two hours. The rotor body is of forged steel, and carries a distributed winding in open slots. A special feature of this design is that the ordinary practice of radial slots is not adopted, the slots being machined out in the manner shown in the drawing. The advantage of this method is that former-wound coils can be placed directly in the parallel slots without disturbing the form of the coil. The winding in the two inside slots is divided into two or three sections according to the depth of the slot, and each section is kept in place by phosphor-bronze wedges. The end-connections are secured by strong steel plates and phosphor bronze keys, as shown in the drawing: each element is thus supported individually. The coils are wound with flat strip on edge, and compressed after having been placed in position. The armature construction is of the usual standard design, and the windings are held in position by strong brass clamps. The stator frame is provided with air-suction and delivery openings, cold air being drawn in at the base of the machine and discharged from chimney-shaped openings at the top.

TABLE XXVIII.—DESIGN DATA OF THREE-PHASE, 50~ TURBO-ALTERNATORS WITH CYLINDRICAL ROTORS.

Design Number	6	7
Drawing on Plate	XI.	XII.
Manufacturer	Brush Electrical Engineerin Co.	Electric Construction Co.
Specification—		
Output in K.V.A.	560	1200
Normal rating K.W.	450	1000
Normal rating Cos ϕ	0.8	0.85
Speed in R.P.M.	3000	1500
Number of poles	2	4
Terminal volts	500	6600
Connections of stator winding	Star	Star
Current at normal rating (ampères)	650	110
Stator Core—		
External diameter	35	140
Internal diameter	60	88
Gross length between flanges	74	85
Number of ducts	17	10
Nett length of iron	50	66
Number of slots	36	60
Slots per pole per phase	6	5
Stator Windings—		
Conductors per slot	2	8
Turns in series per phase	12	80
Size of conductor bare	2 (0.405 × 1.27)	2 (0.305 × 0.51)
Ampères per cm.^{2}	320	350
Space factor	0.46	0.25
Mean length per turn	350	370

TABLE XXVIII. (*continued*).

Design Number	6	7
Drawing on Plate	XI.	XII.
Manufacturer	Brush Electrical Engineering Co.	Electric Construction Co.
Air-gap—		
Radial depth	1.4	1.6
Rotor Iron—		
External diameter	57.2	84.8
Internal diameter	32	22
Pole pitch	90	67
Gross length of iron	70	81
Number of ducts	18	6
Number of slots	64 44 wound	24
Material	0.5 mm. laminated steel	Forged steel
Rotor Winding—		
Turns per pole	96	132
Size of wire (bare)	0.38 × 1.4	0.45 cm.^{2}
Width of slot	1.52	3.0
Depth of slot	4.45	8, 12, and 19
Space factor	0.65	0.6
Mean Length per turn	350	300
Ampères at normal rating	205	110
Ampères per cm.^{2}	380	240
Exciter voltage (max.)	43	75
Magnetic Data (Full-load)—		
Flux in armature per pole	26 × 10^6	20 × 10^6
Leakage coefficient	1.03	1.15
Flux density in armature core	8,000	6,500
,, armature teeth (max.)	12,000	14,500
,, air-gap	7,000	6,000
,, rotor teeth	13,000	12,000(rotor pole)
,, rotor body	...	12,000
Losses and Efficiency (Full-load)—		
Cos ϕ	0.8	0.85
Core loss in watts	17,000	32,000
Stator I^2R loss in watts	3,000	9,700
Excitation loss in watts	4,500	8,300
Friction and windage in watts	...	20,000
Efficiency	95.5 % (excluding friction, etc.)	93.5 %
Heating of Stator—		
Temperature rise of stator	35° C.	36° C.
Watts lost between flanges	18,260	37,000
Cooling surface (dcm.)^2	2,650	1,700
Heating coefficient K_a	0.5	1.7
Regulation—		
Full-load cos ϕ = 1	7 %	6 %
,, ,, = 0.85	...	18 %
,, ,, = 0.8	16 %	...

TABLE XXVIII. (*continued*).

Design Number	6	7
Drawing on Plate	XI.	XII.
Manufacturer	Brush Electrical Engineering Co.	Electric Construction Co.
Weights (in Kgs.)—		
Stator copper	140	240
Field copper	360	700
Stator iron	4,300	4,750
Rotor iron	800	2,500
Weight per K.W. output	12	8
Constants—		
Output coefficient	0.7×10^{-6}	1.27×10^{-6}
Peripheral speed in m/sec.	93	70
Ampère-conductors per cm. of periphery	250	190
Flux density in air-gap	7,000	6,000
$\frac{\text{Short-circuit current}}{\text{Full-load current}}$	2.3	1.6 (field excited for 6000 volts at no-load)

3200-*K.V.A., Three-phase Turbo-alternator, with definite Pole Rotor, and cooled by Forced Draught.*

This alternator, the drawings of which are given in Plate XIII., was built for an output of 3200 K.V.A. at cos $\phi = 0.8$, the speed of the turbine being 980 revolutions per minute. With six poles this corresponds to a frequency of 49 cycles per second. The terminal pressure is 3200 volts, so that with a star-connected armature the current per phase = 578 ampères.

MECHANICAL CONSTRUCTION

Stator.—The construction of the stator has been carried out so that all parts, where heat is generated, shall be thoroughly ventilated. The armature core is built of 0.5 mm. stampings arranged in nine segments per layer, and the laminations are separated at intervals so as to form eleven equally spaced radial air ducts 1.5 cm. wide. The iron distance pieces, riveted to plates of somewhat greater thickness than the normal, are of circular shape in the body of the core, and U-shaped at the teeth. When assembled the laminations are clamped between the cast-iron end-flanges by 36 steel bolts of $1\frac{1}{16}$-inch diameter. On account of the great length of the core a supporting flange is built in so as to take up the stress in the middle part of the frame. To avoid crossing of the various phases the end-connections are arranged in three planes, and the clamps for their support are fixed to bolts screwed into the end-flanges.

Rotor.—The six steel pole cores together with the hexagonal

yoke form a single casting which is machined all over so as to give a perfect mechanical balance. The field coil for each pole consists of 86 turns of 2 × 45 mm. copper strip wound on edge, adjacent turns being insulated from each other by 0.2 mm. press-spahn strip. The pole shoes are of laminated steel, and each is secured to its pole by two dovetails and six 2¼-inch bolts. The pole shoes are shaped to give an almost sine distribution of magnetic flux. The middle of the pole shoe surface is turned to a diameter of 127 cms. concentrically with the armature, while at the ends they are bevelled, the depth of air-gap increasing from 2.5 cms. under centre of pole to 3.7 cms. at extreme edge. As is usual in this type of construction, the component of centrifugal force tending to make the coils bulge sidewise is resisted by four V-plates of bronze bolted between adjacent coils. The cooling air from the fan reaches the pole core through six axial air channels which are bored through the spider. From thence the air passes through ten radial ducts in each pole towards the rotor periphery. At each side all the pole shoes are connected together by a bronze ring, to which is bolted a bronze plate shaped so as to propel the air through the ventilating channels in the spider. The slip rings, fitted to the shaft on either side of the field system, are of bronze and bolted to the six arms of a cast-steel spider. They are insulated from the latter by micanite bushes, and current is led into and out of the rings through copper platted carbon brushes.

Ventilation.—Three independent air passages have been provided for cooling the stator core, stator end-connections, and field magnet respectively. The air is forced through the machine from a fan placed in a recess in the foundations. The fan has a maximum output of 700 cubic metres per minute, and is driven by a direct-coupled 30-B.H.P., 3-phase motor running at 720 revolutions per minute. The cooling air passes to all the lower parts of the machine through a sheet-iron trough, and from here three passages are open to it. One passage is round the outer part of the frame and through the air ducts of the stator. The second air passage round the flanks of the stator and past the end-connections consists of two paths placed symmetrically with regard to the centre of the machine. The remaining path is through two channels formed in the protecting end-covers on either side of the rotor, and from there through the axial channels in the spider to the sides of the armature core. The five heated air currents meet at the top of the frame, and from there escape through a vent in the top of the frame.

ELECTRICAL AND MAGNETIC DATA.

Stator Core—

External diameter	196	Nett length of iron	85
Internal diameter	132	Number of slots	162
Gross length between flanges	110	Slots per pole per phase	9

Stator Winding—

Conductor per slot	1	Ampères per cm.^{2}	250
Turns in series per phase	27	Slots space factor	0.5
Size of conductor (bare)	2.26 cms.^{2}	Mean length per turn	450

Air-gap—

Radial depth 2.5

Field Magnet Iron—

Pole pitch	66.5	Length of pole shoe	112
Pole arc ÷ pole pitch	0.65	Radial depth of pole and shoe	28.5

Field Magnet Winding—

Turns per pole	86	Mean length per turn	270
Size of wire	0.2 × 4.5	Ampères at normal rating	220
Depth of winding space	4.5	Ampères per cm.^{2}	240
Space factor	0.85	Exciter volts (max.)	70

Magnetic Data (Full-load Cos ϕ = 0.8)—

Flux in armature per pole	35×10^6	Flux density in teeth (max.)	19,500
Leakage coefficient	1.15	Flux density in air-gap	7,500
Flux density in armature core	7,000	Flux density in pole core	16,000

Losses and Efficiency (Full-load Cos ϕ = 0.8)—

Core loss in watts	44,000	Exciter loss in watts	1,500
Stator I^2R loss watts	18,500	Fan motor	20,000
Excitation watts	14,000	Efficiency	96.0 %
Friction and windage watts	36,500		

Heating (12 Hours Test at Full-load)—

Temperature rise of stator	26° C.	Temperature rise of field coils	32° C.
Watts loss between flanges	53,000	Excitation watts per coil	2,330
Cooling surface (dcm.^{2})	3,240	Cooling surface per coil (dcm.^{2})	58
Heating coefficient, K_a	1.6	Heating coefficient, K_m	6

Inherent Regulation (Rise)—

Full-load cos ϕ = 1	5.2 %
Full-load cos ϕ = 0.8	16.7 %
Full-load cos ϕ = 0.7	18.5 %

Weights—

Stator copper	960 kgs.	Stator iron	10,500 kgs.
Field copper	1,170 ,,	Rotor iron	7,100 ,,

Total active material	19,730 ,,
Weight per K.W. output	7.5 ,,
Total weight, including bed-plate and exciter	approx. 40,000 ,,

Constants—

Output coefficient	1.7×10^6
Peripheral speed m/sec.	66
Ampère-conductors per cm. of armature periphery	220
Flux density in air-gap	7,500
L_g/b_e	2.5
$\dfrac{\text{Short-circuit current}}{\text{Full-load current}}$	3

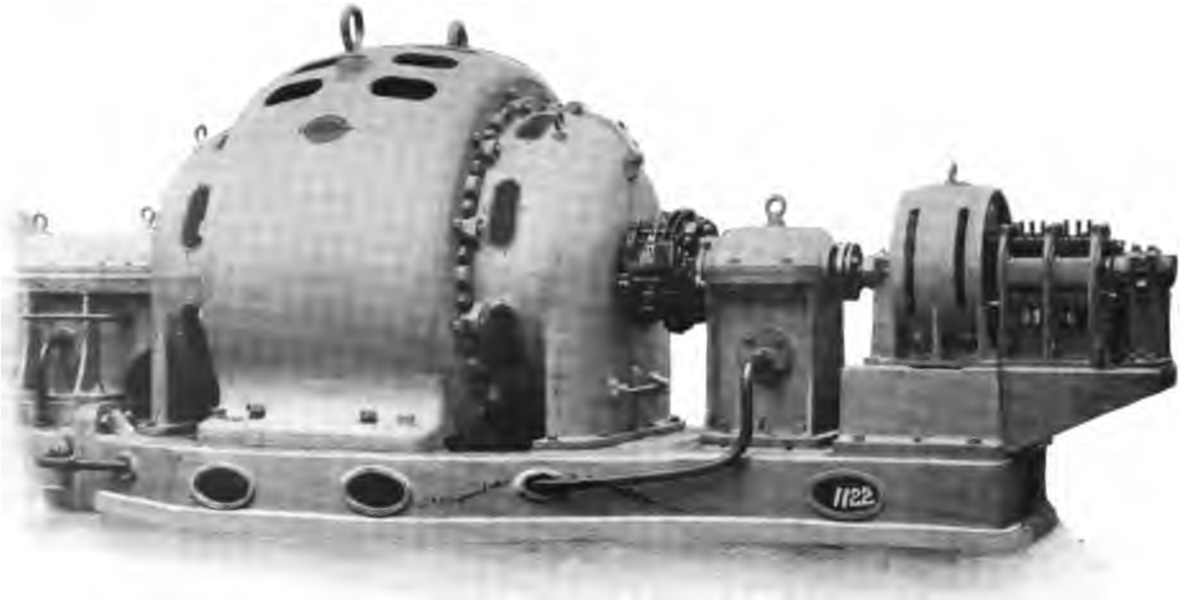


FIG. 298.

[To face p. 416.

6000-K. V.A., Three-phase Turbo-alternator, with definite Pole Rotor. Constructed by Parsons & Co. Ltd.

Plate XIV. shows the general design of a 6000-K.V.A. turbo-alternator built by Parsons & Co. Ltd. for the Electric Supply Station at Sidney, N.S. Wales. The rotor is of the same construction as that shown in Figure 143A, except that the solid steel pole shoes are not ribbed at the top, but so bevelled as to give a wave form approximating very closely to a sine curve. The field coils are each completely closed in sheet steel cases, all the joints of which are soldered up while the ends of the coils are carried by a projecting portion of the pole piece. The slip rings are shrunk direct on the shaft. The leading data and constants of this machine are given in the following table:—

Specification—	
Output in K.V.A.	= 6000
Normal rating K.W.	= 5000
Normal rating $\cos\phi$	= 0.85
Number of phases	= 3
Frequency	= 25
Speed in R.P.M.	= 750
Number of poles	= 4
Terminal volts	= 6600
Current output in ampères	= 520
Dimensions—	
External diameter of stator	= 270
Internal diameter of stator	= 152
Gross length of core between flanges	= 115
Number of ventilating ducts	= 18
Number of slots	= 48
Slots per pole per phase	= 4
Conductors per slot	= 5
Radial depth of air-gap	= 2.86
Constants—	
Output coefficient	= 3.0×10^{-6}
Peripheral speed m/sec.	= 60
$\dfrac{\text{Short-circuit current}}{\text{Full-load current}}$	= 2.3
Full-load efficiency	= 95.6 per cent.

CHAPTER XIII

ROTARY CONVERTERS:—TRANSFORMATION RATIO—ARMATURE REACTION AND WINDINGS

In modern supply systems the usual practice is to erect the generating station on a site where coal is cheap and where an abundant water supply can be had for the boilers and condensers. This economic consideration often means locating the generating plant at some considerable distance from the area of supply, and it then becomes necessary to transmit the power by means of polyphase current at a pressure of 5000 or more volts between lines according to the distance and amount of power to be transmitted. For certain purposes, however, a direct current is either the only one that it is possible to use, or, as in the case of electric railways and tramways, it is more advantageous than an alternating current. Machines and apparatus must hence be provided by means whereby the high pressure alternating current may be transformed to a direct current of moderate voltage. Machines for effecting this transformation may take one of three forms, namely:—

1. Motor generators.
2. Rotary converters.
3. Cascade converters.

When a motor generator is used, the motor part may consist of a polyphase synchronous or induction motor, while the generator is simply an ordinary direct current dynamo. Instead of there being two separate machines as in the case of a synchronous motor set, the alternating current winding and the direct current winding could be placed on the one armature, thus eliminating one field system, and one or two bearings. In such a combination the alternating current necessary to drive the machine as a motor would be supplied through slip rings mounted on the armature shaft and connected to the alternating current winding, while the direct current would be collected from the commutator of the second winding. The great drawbacks to such an arrangement are that the armature would have to be unduly large in order to carry the windings, and that the insulating of the two windings from one another is extremely difficult. A further simplification is, however, made possible by combining the two windings in one, so

that the alternating and direct currents traverse the same conductors. The winding of such an armature would be similar to that of a direct current machine, but in addition to the commutator there would also be slip rings connected to certain points in the armature winding. Alternating current is delivered to the armature winding through brushes bearing on these slip rings, thus causing it to run as a synchronous motor, while direct current is taken from the commutator in the usual way by another set of brushes. A machine operated in this manner is known as a "rotary converter." A rotary converter may also be run as a motor from the direct current side, and alternating current withdrawn from the slip rings, in which case it is referred to as an "inverted rotary." Since, as is afterwards shown, the alternating current volts are always lower than the direct current volts, which latter usually range from 200 to 600, it follows that the alternating current pressure to be supplied to the slip rings of the rotary is too low for long-distance transmission, and it becomes necessary to insert transformers to reduce the high voltage of the transmitted current to a pressure suited to the rotary convertor. Step-down transformers are therefore nearly always used in conjunction with rotary convertors.

Ratio of E.M.F.'s and Currents.—In a direct current dynamo, the currents flowing in the armature coils are alternating currents which, by the action of the commutator, are made to flow through the external circuit in one direction. Considering a 2-pole direct current ring-wound armature, as represented in Figure 299, let it be assumed that the magnetic flux is distributed along the armature periphery according to the sine law, then, as the armature rotates, the sinusoidal E.M.F.'s induced in consecutive coils will differ in phase by $\frac{\pi}{x}$ radians, where x denotes the number of armature coils per pole. In order to determine the alternating voltage existing between any two points (say, x_1 and x_2) on the winding, it is necessary to add vectorially the E.M.F.'s induced in all the coils situated between these points. Thus, taking the two points x_1 x_2, which correspond to a span equal to the pole pitch and embrace twelve armature coils, there is obtained the open polygon OAB of Figure 300, the closing side OB of which is the resultant E.M.F. of the group of coils considered. If each of the vectors be taken to represent the maximum value of the alternating E.M.F., then

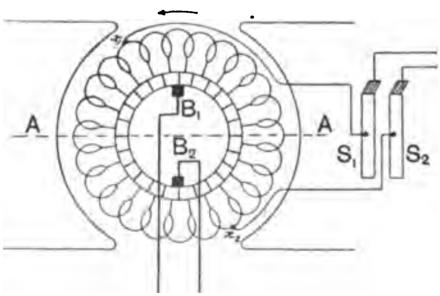


FIG. 299.—Single-phase converter.

the vector OB will, for a group of coils extending over one pole pitch, represent the maximum value of the E.M.F., which is evidently the same as the direct E.M.F. between the two brushes bearing on the commutator. For any smaller group of coils, such as that made up of coils 1 to 7, there is obtained for the maximum value of the E.M.F. a vector OA. Now if, as would invariably be the case, the number of armature coils between two brushes is of the order of 50 or more, the regular polygon (OAB in Figure 300) which results from the vectorial addition of the E.M.F.'s of consecutive coils, may, for all practical purposes, be considered as equivalent to a circular arc. Hence, in order to find the amplitude value of the alternating voltage between any two points on the armature winding, the following construction may be adopted:—Referring to Figure 301, on the line OA as diameter—

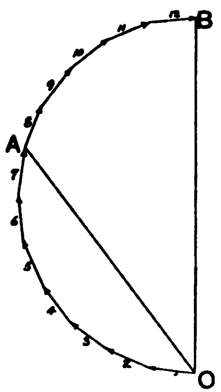


FIG. 300.

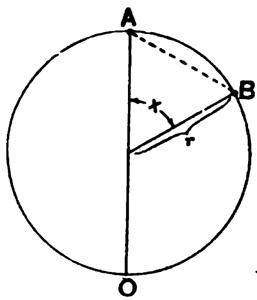


FIG. 301.

the length of OA representing to a suitable scale the voltage between two consecutive brushes of the commutator—describe a circle OBA. The maximum value E_m of the alternating voltage existing between any two points on the armature winding, separated by a distance equivalent to a phase angle x, is given by the chord AB: hence,

$$E_m = AB = 2r \sin\frac{x}{2} = E \sin\frac{x}{2}$$

where r denotes the radius of the circle, and E is the direct current voltage. Having obtained this ratio, the voltage and current relations between the direct and alternating current sides of single and polyphase converters can now be investigated.

For the purpose of studying the theory, rotary converters may be considered either as used to convert direct currents into alternating, or to convert alternating into direct. The latter function is the more frequent occurring in practice, but the former is the more easy to follow out in thought.

1. *Single-phase.*—The principle of the single-phase rotary converter will be explained by aid of Figure 299, the armature of which is supposed to be driven as a motor from a direct current supply. The commutator brushes B_1 B_2 are fixed in such a position that the armature coils undergoing commutation are midway between the poles. If the two points x_1 x_2 on the armature winding, situated 180 space degrees apart, be connected to the slip rings S_1 S_2, then it is obvious that the E.M.F. between these points, and thus between the slip rings S_1 S_2, will be a maximum at that instant when x_1 and x_2 coincide with the brushes B_1 and B_2 respectively, and is at that instant equal to the direct current voltage E of the machine. As the points x_1 and x_2 move away from this position, the difference of potential between the slip rings S_1 and S_2 decreases, and becomes zero at the instant when x_1 and x_2 coincide with the plane AA. At this moment the difference of potential between x_1 and x_2 reverses and increases again, attaining the value $-E$ when x_1 and x_2 coincide with the brushes B_2 and B_1 respectively. It will thus be seen that between the slip rings S_1 and S_2 there is produced an alternating E.M.F., whose amplitude is equal to that of the direct current voltage E. In a 2-pole machine, a complete period of the alternating E.M.F. will correspond to one revolution of the armature, and in general, if there be p pairs of poles, and the armature speed be R revolutions per minute, the frequency of the alternating voltage will be

$$\sim = \frac{Rp}{60}$$

Since the maximum value of the alternating voltage is equal to that of the direct current side, and assuming the alternating electromotive force to vary according to the simple sine law, the effective value of the alternating voltage across the slip rings will be

$$E_s = \frac{E}{\sqrt{2}} = 0.707\ E$$

That is, if a machine such as is represented in Figure 299, have its direct current side connected to a suitable supply of voltage E, it will run as a motor from the D.C. side and produce at the slip rings an alternating E.M.F. $= \frac{1}{\sqrt{2}}$ times the direct current voltage; also, since every alternating current generator is reversible, if the slip rings of such a machine be connected to an alternating supply of voltage $= \frac{1}{\sqrt{2}} \times$ the direct current, and frequency $= \frac{Rp}{60}$, the machine if duly synchronised will run as a synchronous motor, or, if at the same time supplying direct current, as a synchronous or rotary converter. Supposing the machine to be devoid of losses, and assuming the alternating current

and induced voltage to be in phase, then the output in K.V.A. from the direct current (D.C.) side must equal the input in K.V.A. to the alternating current (A.C.) side. From this it follows that if I denote the value of the current taken from the D.C. side, the current input at the slip rings will be

$$I_{so} = \sqrt{2}I = 1.414 \text{ I}$$

2. *Two- or 4-phase.*—Suppose the armature of Figure 299 to be provided with another pair of slip rings (Figure 302), which are connected to points x_3 and x_4 on the armature winding midway between x_1 and x_2, there will then exist between these additional slip rings an alternating voltage of the same magnitude and frequency as between the slip rings S_1 S_2 in Figure 299. The E.M.F.'s between the respective pairs of slip rings will, however, be in quadrature, for it is obvious, from what has been said above, that when the voltage between x_3 and x_4 is a maximum, that between x_1 and x_2 must be zero. A 2- or 4-phase converter is thus obtained by connecting four equidistant points x_1 x_2 x_3 and x_4 of a 2-pole direct current generator to four slip rings, which are supplied with current from a suitable 2-phase supply. The ratio of the E.M.F.'s is given by the equation

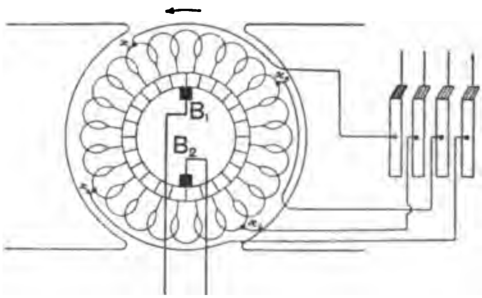


FIG. 302.—Four-phase converter.

$$E_s = \frac{E}{\sqrt{2}} = 0.707 \text{ E}$$

and since (assuming 100°/$_\circ$ efficiency and unity power factor) the power input to the AC side must equal the power output from the D.C. side, *i.e.*, $2E_s\, I_{so} = EI$, the current per phase is

$$I_{so} = \frac{\sqrt{2}\,.\,I}{2} = \frac{I}{\sqrt{2}} = 0.707 \text{ I}$$

3. *Three-phase.*—A 3-phase converter is obtained by connecting three equidistant points on the armature winding of a direct current machine to three slip rings, as shown in Figure 303. Since each section of the winding has the same number of turns, and occupies the same space on the periphery of the armature core, the potential difference between any two of the three slip rings must be the same, but the induced E.M.F. in the three sections will rise and fall at intervals corresponding to one-third of a complete period. In Figure 304 the E.M.F.'s between the respective slip rings are repre-

sented by the vectors E_1E_2, E_2E_3, and E_3E_1, which form the side of an equilateral triangle $E_1E_2E_3$. If the centre O of the circumscribed circle be joined to the points E_1, E_2 and E_3 respectively, then the vectors OE_1, OE_2, and OE_3 will represent the E.M.F.'s between the three slip rings and the neutral point of the 3-phase system. Since the angle $x = 120°$, the effective voltage between any two slip rings is

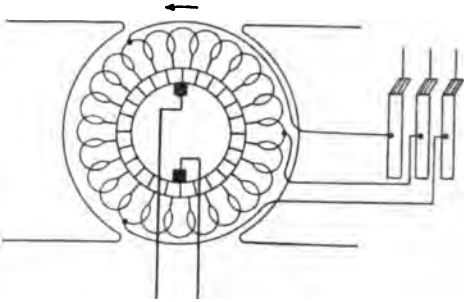


FIG. 303.—Three-phase converter.

$$E_s = \frac{E_1E_2}{\sqrt{2}} = \frac{E_2A}{\sqrt{2}} \sin\frac{x}{2} = \frac{E}{\sqrt{2}} . \sin\frac{x}{2} = \frac{E}{\sqrt{2}} . \sin 60° = 0.612 \text{ E}$$

Let I_{so} denote the line current, then since the total 3-phase power $\sqrt{3} . E_s . I_{so}$ must, on the same assumptions as above, equal the total direct current power EI, the current per slip ring is expressed by

$$I_{so} = \frac{EI}{\sqrt{3} . E_s} = \frac{EI}{\sqrt{3} \times 0.612 \text{ E}} = 0.94 \text{ I}$$

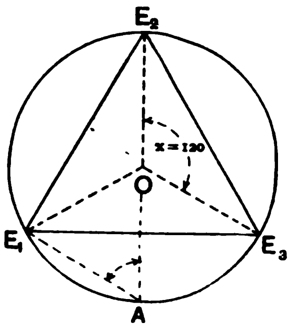


FIG. 304.—Vector diagram for 3-phase converter.

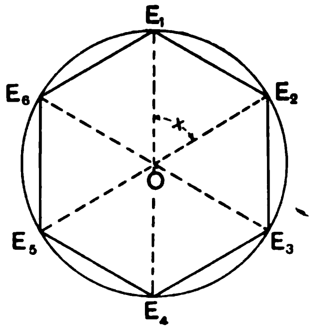


FIG. 305.—Vector diagram for 6-phase converter.

Further, if I_{ao} denotes the alternating current in each phase of the winding at $\cos\phi = 1$ and 100 % efficiency, then since

$$3 \, I_{ao}E_1 = EI$$

$$I_{ao} = \frac{EI}{3 \times 0.612 \text{ E}} = 0.545 \text{ I}$$

m-slip rings.—In the general case of a converter in which *m* equidistant points on the armature winding are connected to *m* slip rings,

the angular distance between adjacent tappings will be $2\pi/m$, and the corresponding slip ring voltage is

$$E_s = \frac{E}{\sqrt{2}} \cdot \sin\frac{\pi}{m}$$

Now, from Figs. 304 and 305 it is obvious that, independent of the number of phases, the voltage E_n between any slip ring and the neutral point of the system $= \frac{E}{2\sqrt{2}}$; hence, since I_{so} denotes the line current,

$$mE_nI_{so} = EI$$

$$\text{and } I_{so} = \frac{EI}{mE_n} = \frac{2\sqrt{2} \cdot I}{m} \quad \ldots \ldots \quad (116)$$

Further, the armature current per phase is obtained thus,

$$mI_{ao}E_s = EI$$

$$\left.\begin{aligned} &i.e.,\ I_{ao} = \frac{EI}{mE_s} = \frac{EI\sqrt{2}}{mE\sin\frac{\pi}{m}} = \frac{\sqrt{2}\,I}{m\sin\frac{\pi}{m}} \\ &\text{or, } I_{ao} = \frac{\sqrt{2} \cdot m \cdot I_{so}}{2\sqrt{2} \cdot m \cdot \sin\frac{\pi}{m}} = \frac{I_{so}}{2\sin\frac{\pi}{m}} \end{aligned}\right\} \quad \ldots \quad (117)$$

Assuming the voltage and current at the D.C. side to be unity, the alternating voltages and currents for converters having different number of slip rings would then be as follows:—

TABLE XXIX.—CONVERTER RATIOS (THEORETICAL VALUES).
Efficiency = 100 Per Cent. Power Factor Cos $\phi = 1$.

	Direct Current.	Single-phase.	Three-phase.	Two- or Four-phase.	Six-phase.	Twelve-phase.	m-phase.
Volts between slip rings and neutral point of the system E_n . . .	1	0.354	0.354	0.354	0.354	0.354	0.354
Volts between slip rings E_s	1	0.707	0.612	0.5	0.354	0.183	$\frac{\sin\frac{\pi}{m}}{\sqrt{2}}$
Current per slip ring or line current I_{so} . .	1	1.414	0.943	0.707	0.472	0.236	$\frac{2\sqrt{2}}{m}$
Current in armature per phase I_{ao} . . .	1	0.707	0.545	0.5	0.472	0.455	$\frac{\sqrt{2}}{m\sin\frac{\pi}{m}}$

Owing to the voltage consumed by the current in overcoming the resistance of the armature winding, the ratio between the terminal P.D. on the D.C. side and the alternating voltage applied to the slip rings, will in practice differ somewhat from these theoretical values. The method of making the necessary correction, which is comparatively small, is described on page 441.

In obtaining the current ratios I_{so} and I_{ao} given above, the power factor on the alternating current side was taken as unity, while the small power component of the current supplying the losses in the converter was neglected. To allow for these let I_{ao} denote the value of the alternating current when these factors are neglected, hI_{ao} the power component of the current when allowance has been made for the iron, friction, and shunt excitation losses of the machine, and kI_{ao} the wattless current of the converter, then the total current taken by the converter $= I_a = I_{ao}\sqrt{h^2+k^2}$. That is, the current ratios given above have to be multiplied by the factor $H_x = \sqrt{h^2+k^2}$,—where h is a quantity slightly greater than unity, according to the losses in the converter, and k is the ratio of the wattless current to the watt current I_{ao}; *e.g.* if the iron, friction, and excitation losses of the converter be 4 per cent. of the D.C. output, while the wattless current = 20 per cent. of the watt current, then 1.04 and 0.2 are the values that have to be assigned to h and k respectively.

Factors which affect the Converter Ratios.—The converter ratios given in Table XXIX. have been deduced on the assumption that the flux density along the armature periphery varies according to the sine law. In a machine of standard construction the air-gap would be of uniform depth over the greater part of the pole face, and this would give rise to a flat-topped curve of induced E.M.F. In general, therefore, the sine wave hypothesis will only be approximately correct, and though one would expect the actual values of the voltage ratio to be somewhat different from those derived on the sine wave hypothesis, still it is found as a result of the distributed winding that these values are sufficiently exact for most practical purposes. In a given machine the ratio of pole arc to pole pitch is the most important factor influencing the relation between the alternating and direct current voltages. By altering this ratio the curve of flux distribution is changed, and hence also the ratio between the maximum and effective values of alternating E.M.F. The ratio of the direct and alternating voltages can be determined graphically for various values of pole arc ÷ pole pitch, when once the curve of flux distribution is known. In Table XXX. there are set forth the voltage and current ratios, as obtained by Arnold and La Cour,[1] the drop due to ohmic resistance being neglected. The results in this table show that when the length of the pole shoe arc is about

[1] *Die Wechselstromtecknik*, vol. iv. pp. 686, 688.

70 per cent. of the pole pitch the conversion ratios are approximately the same as for sine law distribution.

TABLE XXX.—CONVERTER RATIOS AS DEPENDENT ON POLE ARC/POLE PITCH.

Pole Arc ÷ Pole Pitch.	0.8	0.75	0.7	Sine Distribution.	0.65	0.6
Ratio of line volts A.C./D.C.—						
Direct-current	1	1	1	1	1	1
Single-phase	0.67	0.69	0.707	0.707	0.73	0.75
Three-phase	0.59	0.60	0.62	0.612	0.64	0.66
Four-phase	0.48	0.49	0.50	0.5	0.52	0.53
Six-phase	0.34	0.347	0.354	0.354	0.367	0.377
Twelve-phase	0.177	0.182	0.185	0.185	0.192	0.197
Ratio of line currents A.C./D.C.—						
Direct-current ($\cos \phi = 1$)	1	1	1	1	1	1
Single-phase	1.50	1.45	1.41	1.414	1.37	1.33
Three-phase	1.00	0.97	0.94	0.943	0.915	0.89
Four-phase	0.75	0.73	0.707	0.707	0.69	0.67
Six-phase	0.50	0.48	0.47	0.472	0.46	0.44
Twelve-phase	0.25	0.24	0.24	0.236	0.23	0.22
Ratio of armature current to D.C. current—						
Three-phase	0.565	0.550	0.545	0.545	0.52	0.51
Four-phase	0.53	0.51	0.5	0.5	0.485	0.47
Six-phase	0.49	0.48	0.472	0.472	0 455	0.445
Twelve-phase	0.48	0.46	0.455	0.455	0.435	0.425

The ratio between the direct and alternating current voltages varies also with the wave shape of the alternating E.M.F. applied to the slip rings. For the direct-current voltage bears a definite relation not only to the maximum value of the alternating voltage, but to the effective value, as read on a voltmeter, only in so far as the latter depends upon the former, being $= \frac{1}{\sqrt{2}}$ the maximum value with a sine wave. Thus if an E.M.F. wave having a different ratio of maximum to effective value be impressed upon the slip rings, the ratio between direct and alternating voltage is altered in the same proportion as the ratio of maximum to effective value. For example, with a flat-topped wave of impressed E.M.F. the maximum value, and thus the direct-current voltage depending thereon, is lower than with a sine wave E.M.F. having the same effective value. On the other hand, with a highly peaked wave the voltage ratio may be lower by as much as 8 to 10 per cent.

The E.M.F. generated by a rotary converter on the A.C. side will, owing to the winding being well distributed, approximate very closely to a sine wave. Hence, with a non-sinusoidal wave of applied E.M.F.,

a current of higher frequency will flow through the armature of the converter due to the difference in the wave shapes of the impressed and counter E.M.F.'s. Since the resistance of the armature winding is very small in comparison with the inductance, these currents will be practically wattless, and will therefore have a negligible influence upon the voltage ratio of the converter. These super-currents, nevertheless, result in an increased loss in the armature winding, and also tend to facilitate hunting. Hence it is always preferable to obtain an applied E.M.F. whose wave form is as near to a sine curve as possible.

Armature Heating and Output.—One of the most important problems to be considered in connection with rotary converters is how the current, in the process of being transformed from alternating to direct, or *vice versa*, is distributed in the windings of the armature. Consider the general case of a revolving armature which at the same time is traversed by an alternating current to drive it as a motor, and by a direct current which it supplies as a generator. The current which the armature receives as a motor must be flowing against the electromotive force induced in virtue of its rotation, while the current which it gives out as a generator must be in the same direction as the induced E.M.F. The current in the armature conductors will therefore be the difference between the alternating current input and the direct current output. There will now be investigated the relative copper losses in the armature of a direct-current machine when (1) it is fitted with m-slip rings and used as a single or polyphase rotary converter, and (2) when used as an ordinary direct-current dynamo giving the same output.

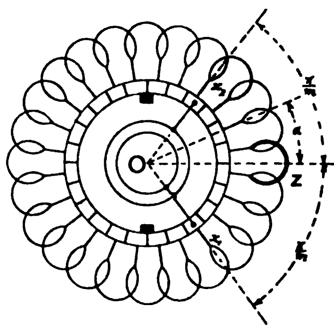


FIG. 306.

For the sake of simplicity a 2-pole machine with a ring-wound armature will again be considered, and it will be assumed that the alternating currents and E.M.F.'s follow a sine law. Both assumptions may not be valid in actual practice, but the error thus introduced will be quite small.

Referring to Figure 306, let the coils between the tappings x_1 and x_2 represent the winding belonging to one phase of an m-slip ring converter, then for a 2-pole machine the angle subtended between x_1 and x_2 is $2\pi/m$ radians. Further, let oa be a plane bisecting that section of the winding lying between x_1 and x_2 into two equal parts, such that $aox_1 = aox_2 = \pi/m$. Assuming for the present a power factor $\cos\phi = 1$, the alternating E.M.F., between x_1 and x_2, and hence also

the alternating current flowing in the armature section x_1 x_2, will attain its maximum value when the considered section is midway between the brushes, as shown in the figure. Since the direct-current changes its direction in every armature coil at the moment when the coil passes under the commutator brushes, it is therefore really an alternating current of nearly rectangular wave form, as indicated by curve I_c in Figure 307. If the direct-current output from the commutator be denoted by I, then for a 2 pole machine, that in each armature conductor will be I/2. At the instant when the alternating-current input is a maximum, that armature coil situated midway between x_1 and x_2 will be intermediate between the brushes B_1 and B_2, and is thus in the middle of its rectangular direct-current waves. Consequently in this

Direct current I_c and alternating current I_a in coil Z (Fig. 306).

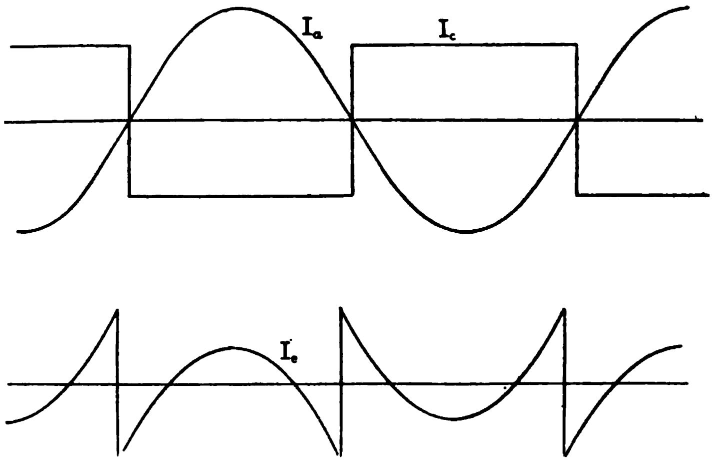


FIG. 307.—Resultant current in coil Z (Fig. 306).

coil the alternating current input and the rectangular direct-current output are in phase with each other, but opposite, as shown in Figure 307 by the curves I_a and I_c. On subtracting the ordinates of these two curves there is obtained the curve I_e, which gives the resultant current in the coil midway between x_1 and x_2.

In successive armature coils the direct-current reverses successively. That is, the rectangular currents flowing in successive armature coils are successively displaced in phase from each other, and, since the instantaneous value of the alternating current is the same for the whole section x_1 x_2, and in phase with the rectangular current in the coil z, it becomes more and more out of phase with the rectangular current from coil z towards x_1 or x_2 (Figure 308), until the maximum phase displacement $= \frac{\pi}{m}$ between alternating and rectangular current is

reached at the coils x_1 and x_2. For a single-phase converter $m = 2$, and the maximum phase displacement is therefore $\frac{\pi}{2}$ as shown in Figure 309. If a (Figure 306) denote the phase angle by which any armature coil of an m-slip ring converter is displaced from the plane passing midway between the tapping points to two adjacent slip rings, and h has the same meaning as on page 425, then the direct current in

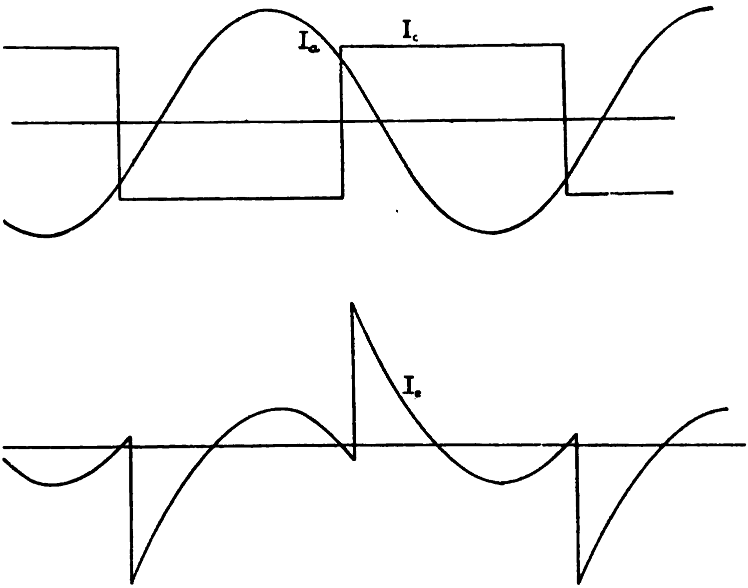


FIG. 308.—Resultant current in coils between Z and x_1 or x_2

that coil is I/2 for the half-period o to π, whilst the instantaneous alternating current is expressed by

$$i_1 = \sqrt{2}\, I_a \sin(\theta - a)$$

where $I_a = \dfrac{h \cdot \sqrt{2}\, I}{m \sin \frac{\pi}{m}}$ is the effective value of the alternating current in any one phase, such as $x_1\, x_2$, and θ is measured from the vertical axis passing through the D.C. brushes.

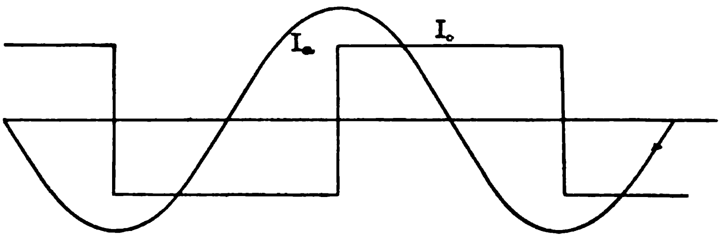
This equation for i_1 assumes that the wattless current of the converter is zero. If, however, the current lags or leads with respect to the induced E.M.F. by a given angle ϕ_1, the alternating current and direct-current in the armature coil z midway between adjacent leads will not be in exact phase opposition for the position shown in Figure 306, the resultant current wave I_e shown in Figure 307 being now obtained in that armature coil displaced from the mid coil z by an angle $= \phi_1$. When

there is a phase difference ϕ_1 between current and induced E.M.F. the equation for the instantaneous current in any armature coil becomes

$$i = \sqrt{2}\ I_a \sin(\theta - a - \phi_1)$$
$$= \sqrt{2}\ I_a \sin(\theta - a)\cos\phi_1 - \sqrt{2}\ I_a \cos(\theta - a)\sin\phi_1$$

Now, $I_a \cos\phi_1$ = energy component of the current $= hI_{ao}$, and $I_a \sin\phi_1$ = wattless component of the current $= kI_{ao}$, where $I_{ao} = \dfrac{\sqrt{2}\ I}{m \sin\frac{\pi}{m}}$

Direct current I_c and alternating current I_a in coils x_1 and x_2 (Fig. 306).



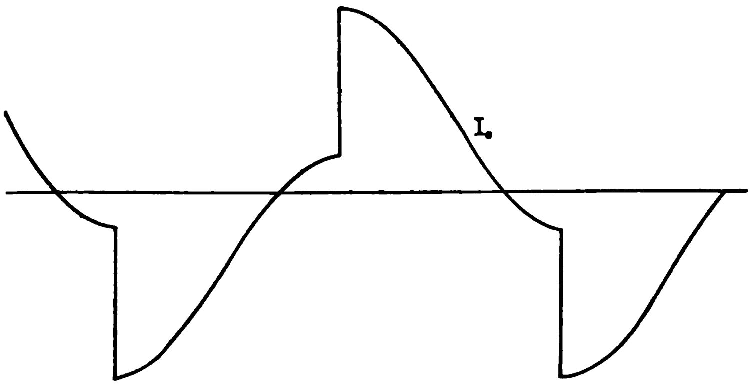


FIG. 309.—Resultant current in coils x_1 and x_2 (Fig. 306).

When these values are substituted in the equation for i there is obtained

$$i = \sqrt{2}\ I_{ao}\ h \sin(\theta - a) - \sqrt{2}\ I_{ao}\ k \cos(\theta - a)$$

The resultant instantaneous current in that armature coil displaced a radians with respect to the central plane is then

$$i_2 = i - I/2 = \sqrt{2}\ I_{ao}\ \{h \sin(\theta - a) - k \cos(\theta - a)\} - I/2$$
$$= \frac{I}{2}\left\{\frac{4}{m \sin\frac{\pi}{m}}\left[h \sin(\theta - a) - k \cos(\theta - a)\right] - 1\right\}$$

The corresponding effective value of the converter current is

$$I_c = \sqrt{\frac{1}{\pi}\int_0^\pi i_2^2 \,.\, d\theta}$$

$$= \frac{I}{2}\sqrt{1 + \frac{8(h^2+k^2)}{m^2 \sin^2 \frac{\pi}{m}} - \frac{16(h \cos a - k \sin a)}{\pi m \sin \frac{\pi}{m}}}$$

$$= \frac{I}{2}\sqrt{1 + \frac{8\,H_x^2}{m^2 \sin^2 \frac{\pi}{m}} - \frac{16\,H_x \cos(a+\phi_1)}{\pi m \sin \frac{\pi}{m}}} \quad \ldots \quad (118)$$

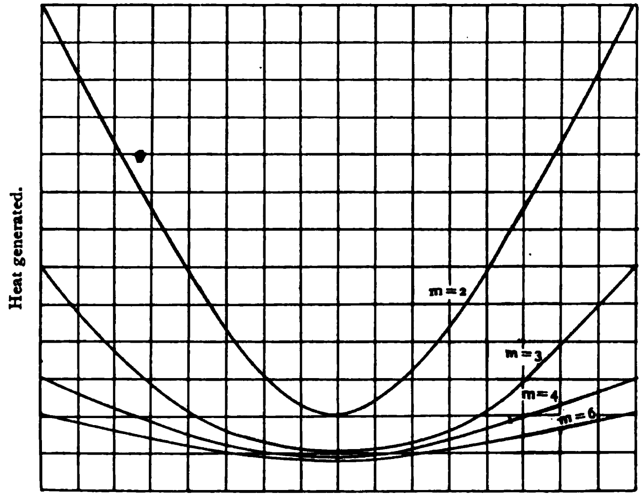


FIG. 310.—Curve showing relative heating between tapping points of converter armature having m slip rings.

where $H_x = \sqrt{h^2+k^2}$, $\frac{h}{H_x} = \cos \phi_1$, and $\frac{k}{H_x} = \sin \phi_1$; and h and k have the same meaning as on page 425. Since the production of heat in an electric circuit is proportional to the square of the current flowing, the heat generated in the considered armature coil will be given by

$$K\left[\frac{I^2}{4}\left\{1 + \frac{8\,H_x^2}{m^2 \sin^2 \frac{\pi}{m}} - \frac{16\,H_x \cos(a+\phi_1)}{\pi m \sin \frac{\pi}{m}}\right\}\right]$$

where K is a coefficient. This expression shows that, if ϕ_1 be zero, the heat produced in a coil increases as the angle a increases, and will hence be greatest at the points of the armature winding connected to the slip rings, and least in that coil midway between the tappings. If, for various numbers of slip rings ranging from $m=2$ to $m=6$, the above expression for the heat generated in a coil be plotted as a function

of α between the limits $\alpha = +\frac{\pi}{m}$ and $\alpha = -\frac{\pi}{m}$, corresponding to the distance between two adjacent tappings, curves such as are shown in Figure 310, will be obtained. The scale of ordinates is the same for all curves, but the abscissa scale is different, having been so chosen that the length of the base line in each case represents $\frac{2\pi}{m}$ radians —*i.e.* the base line in each case is proportional to the distance between adjacent tappings to the slip rings. The curves exhibit in a striking manner the rapid decrease in the total rate of heat production, and the increase in the uniformity of the heat generated in the various coils as the number of slip rings is increased. When the power factor is less than unity the coils in which the maximum heat is generated will not correspond to those directly connected to the slip rings, but, according to the angle of lag ϕ_1, will be somewhere intermediate between two tapping points.

Since I/2 is the magnitude of the current in each armature coil of a D.C. generator giving the same output as the converter under consideration, the ratio of the I^2R loss in any coil of the latter to that in the corresponding coil of the generator, and thus the ratio of coil heating, is

$$\beta_1 = \left(\frac{I_c}{\frac{I}{2}}\right)^2 = 1 + \frac{8\,H_x^2}{m^2 \sin^2 \frac{\pi}{m}} - \frac{16\,H_x \cos(\alpha + \phi_1)}{\pi m \sin \frac{\pi}{m}}$$

If this expression be integrated between the limits $\alpha = +\frac{\pi}{m}$ and $\alpha = -\frac{\pi}{m}$,—*i.e.*, over the whole phase or section $x_1\,x_2$, and divided by the base $\frac{2\pi}{m}$, there will then be obtained the ratio between the total copper loss in the armature winding of a converter with m slip rings, and that in a similar machine when giving the same output as a D.C. generator. The relative armature heating is therefore given by the expression

$$\beta = \frac{m}{\pi}\int_{-\frac{\pi}{m}}^{+\frac{\pi}{m}} \beta_1\, d\alpha$$

$$= 1 + \frac{8\,H_x^2}{m^2 \sin^2 \frac{\pi}{m}} - \frac{8\,H_x \left\{\sin\left(\frac{\pi}{m} + \phi_1\right) + \sin\left(\frac{\pi}{m} - \phi_1\right)\right\}}{\pi^2 \sin \frac{\pi}{m}}$$

$$= 1 + \frac{8\,H_x^2}{m^2 \sin^2 \frac{\pi}{m}} - \frac{16\,H_x \cos \phi_1}{\pi^2}$$

$$= 1 + \frac{8(h^2 + k^2)}{m^2 \sin^2 \frac{\pi}{m}} - \frac{16h}{\pi^2} \qquad (119)$$

Hence, if R_a denotes the ohmic resistance of the winding from the D.C. side, the I^2R loss in the armature of a converter is

$$W_{ca} = \beta I^2 R_a \quad \ldots \ldots \ldots \quad (120)$$

whilst the voltage drop in the windings due to the ohmic resistance

$$= \sqrt{\beta} \cdot IR_a \quad \ldots \ldots \ldots \quad (121)$$

If the I^2R loss, and consequently the heating in the armature winding, is made the same as in the corresponding D.C. generator, the current from the converter, and hence the output, can be increased in the ratio $1/\sqrt{\beta}$. Hence

$$\text{Output as a converter} = \frac{1}{\sqrt{\beta}} \times \text{output as D.C. generator} \quad (122)$$

The value of β, $\sqrt{\beta}$, and $\frac{1}{\sqrt{\beta}}$ have been calculated and set forth in the following table for three typical cases; namely:

(*a*) $h = 1.0 \quad k = 0$
(*b*) $h = 1.04 \quad k = 0.3$
(*c*) $h = 1.04 \quad k = 0.5$

TABLE XXXI.—CONSTANTS FOR CALCULATING THE I^2R LOSS IN CONVERTER ARMATURES.

	Number of Phases.					
	One.	Three.	Four.	Six.	Twelve.	
β	1.38	0.57	0.38	0.27	0.21	$h=1.0$
$\sqrt{\beta}$	1.17	0.75	0.61	0.51	0.45	$k=0$
$1/\sqrt{\beta}$	0.85	1.33	1.63	1.93	2.20	
β	1.66	0.71	0.49	0.36	0.29	$h=1.04$
$\sqrt{\beta}$ . . .	1.29	0.84	0.70	0.60	0.54	$k=0.3$
$1/\sqrt{\beta}$	0.77	1.18	1.43	1.67	1.87	
β	2.00	0.9	0.65	0.50	0.43	$h=1.04$
$\sqrt{\beta}$	1.40	0.95	0.81	0.71	0.65	$k=0.5$
$1/\sqrt{\beta}$	0.71	1.05	1.23	1.40	1.53	

From the values of $1/\sqrt{\beta}$ given in the above table it will be seen that the rating of a given armature when employed as a converter increases rapidly as the number of slip rings are increased, 3-phase and 6-phase converters when supplied with current at unity power factor giving respectively 33 and 93 per cent. greater output than the corresponding D.C. generator. On account of their small output arising from the excessive heating of those conductors near to the slip

28

rings, and also on account of their inferior commutating properties, single-phase converters are seldom employed in practice, motor generators being used instead. In the early days of electric traction, rotary converters were nearly always operated as 3-phasers, but in more recent practice 4- and 6-phase rotaries predominate. The former are generally supplied through step-down transformers from a 2-phase transmission system, and the adoption of 6-phase converters is rendered possible by the use of 3- to 6-phase transformers, mentioned on page 135.

In Figure 311 the values of β, as obtained for various converters from equation 119, have been plotted as a function of the wattless current expressed as a fraction of the energy current input. As the heat-

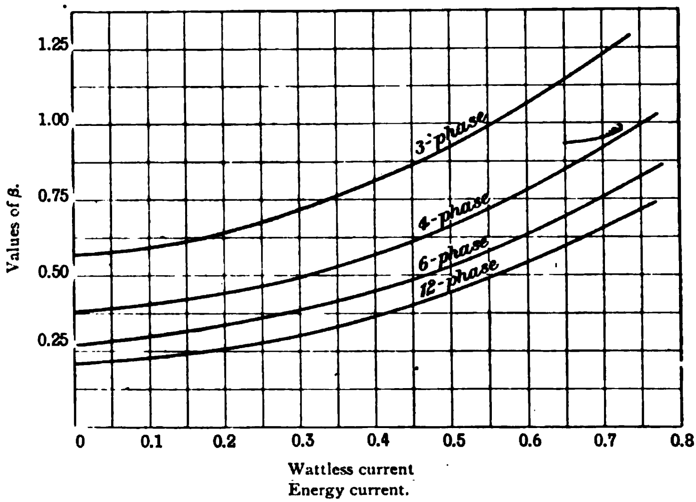


FIG. 311.—Variation B with wattless current ($h = 1.04$).

ing of the armature is directly proportional to β, these curves indicate the rapid increase in heating that occurs with increasing phase displacement, and draw attention to the necessity for keeping the power factor at full-load as near unity as possible. In the case of compound converters this means that the shunt excitation should be adjusted to give a considerable lagging current at no-load, so that the current may come into phase with the voltage when full-load or nearly full-load is reached. The adjustment of the converter field for minimum current at no-load is therefore to be avoided, as this will give a considerable leading current at full-load, and, as a consequence, unnecessary heating of the armature.

Armature Windings.—Though the preceding theory has been discussed with reference to a ring-wound armature, drum windings are

always employed in practice, and to these the conclusions arrived at above are equally applicable. In a rotary converter armature the conductors are joined up to the commutator segments exactly as for any ordinary D.C. dynamo, and on the alternating current side sections of the same phase are connected between the same pair of collector rings. When the converter is to be rated at 100 K.W. or less, the armature can be wound for two circuits independent of the number of poles, *i.e.* wave wound, but in a machine for larger outputs, multiple-circuit or lap windings should be adopted. With an m-slip ring converter the armature winding, when wound with two circuits, should be tapped in m-equidistant points and each tapping connected to a slip ring. In a lap-wound armature having p pairs of poles the armature requires to be tapped in mp equidistant points, and those taps whose angular distance from one another $= \frac{360}{p}$ space degrees joined to the same collector ring, thus connecting all sections of the same phase in parallel. Since the rules for carrying out the windings will be the same as for the windings of D.C. generators, it should here be sufficient to give only a few typical examples.

Single-Phase.—To make a 2-circuit or wave winding suitable for a single-phase converter it is necessary to tap the winding in two points,

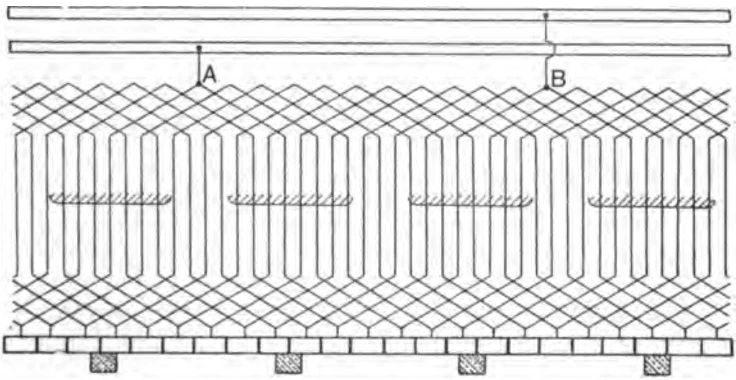


FIG. 312.—Two-circuit 4-pole armature winding tapped for single-phase, 46 conductors.

each point being connected to a slip ring. Consider, for instance, the 2-circuit winding shown in Figure 312, which has 4 poles, 46 conductors, and front and rear pitches = 11, the winding is tapped at any point A to one slip ring, and then after tracing round one-half of the conductors a tap is taken out at the point B to the other collector ring. Figure 313 illustrates how a 4-pole multiple-circuit winding having 48 conductors should be connected up as a single-phase converter. From the rule given above a multiple-circuit winding

must have one tapping to each slip ring per pair of poles,—*i.e.* in the case considered there will be 4 equidistant tappings, as at A_1, B_1, A_2 and B_2. The winding is thus divided into 4 equal sections of 12 inductors each. Since the points marked A_1 and A_2 should be at the same potential, they are connected to one slip ring, and the other points B_1 and B_2 joined to the second slip ring.

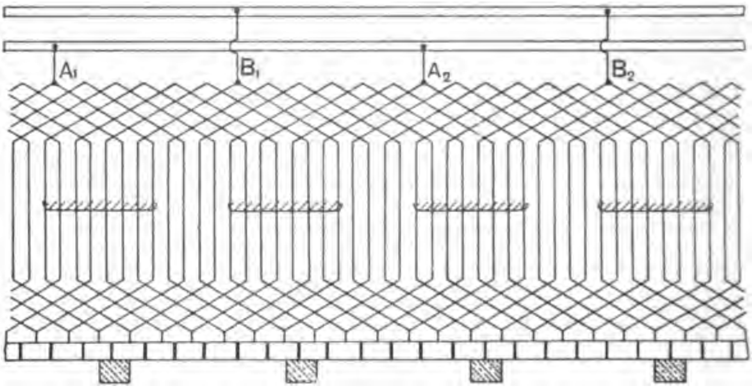


FIG. 313.—Multiple circuit 4-pole armature winding tapped for single-phase, 48 conductors.

Three-Phase.—For a 3-phase rotary having a 2-circuit winding the armature will be divided into 3 equal sections, and each of the 3 points A, B and C tapped to a slip ring as shown in Figure 314, which represents a 4-pole winding having 54 conductors and a mean winding pitch = 13. The 3 phases of the winding are distinguished by the colours black, red, and blue. For a 3-phase multiple-circuit winding there will be 3 sections per pair of poles. Thus a 6-pole machine with a lap-wound armature will have $3 \times 3 = 9$ sections. To tap such a winding for 3-phase, leads are taken out to the collector rings at equal ninths through the armature from beginning to end, 3 leads spaced 120 degrees apart being connected to each slip ring. A 6-pole multiple-circuit winding having 72 conductors is shown developed in Figure 315, where the portion of the winding corresponding to one pair of poles is divided into 3 sections and taps are taken out at the junctions of these sections, thus giving 9 taps in all.

Six-Phase.—Figure 316 is the same winding diagram as Figure 315, except that the connections on the A.C. side are made for 6 phases. The winding has now to be divided into twice as many sections as for a 3-phase converter; for instance, in Figure 316 there will be 18 tapping points as compared with 9 in Figure 315. Though generally referred to as 6-phase, this winding is really a

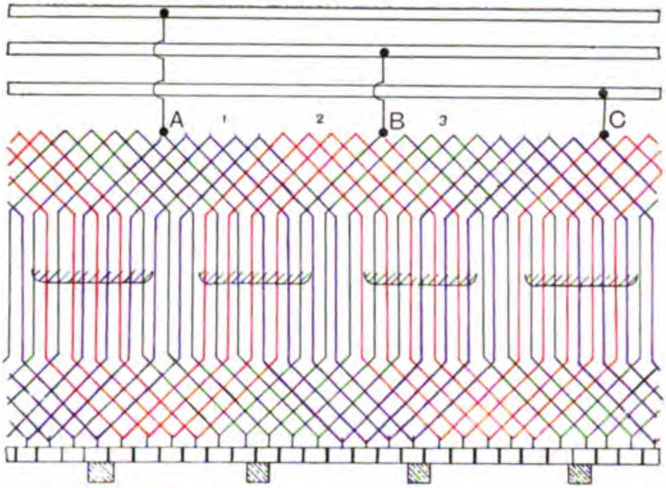


FIG. 314.—Two-circuit 4-pole armature winding tapped for three phases. 54 conductors.

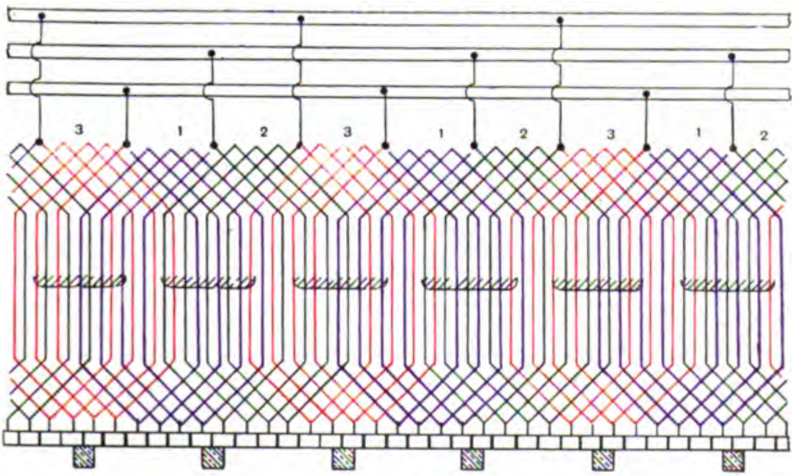


FIG. 315.—Multiple-circuit 6-pole armature winding tapped for three phases. 72 conductors.

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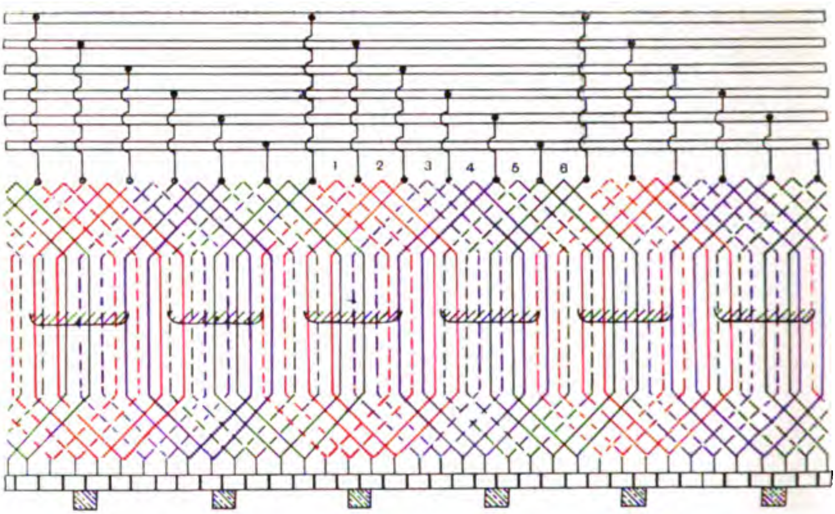

FIG. 316.—Multiple-circuit 6-pole armature winding tapped for six phases. 72 conductors.

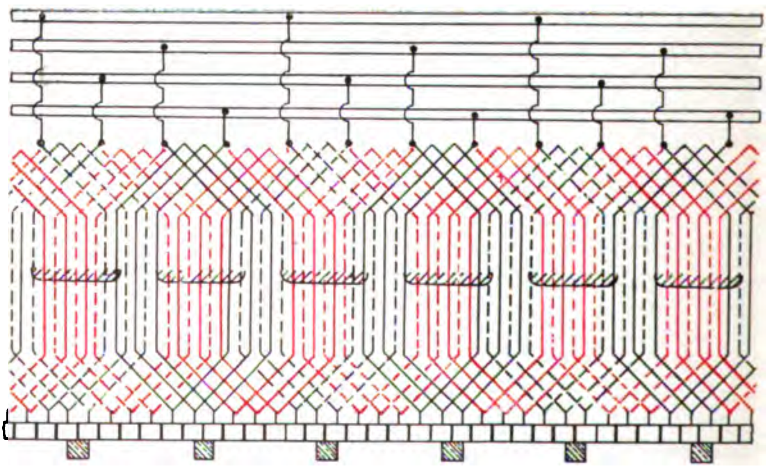

FIG. 317.—Multiple-circuit 6-pole armature winding tapped for four phases. 72 conductors.

[To face p. 437.

special case of the 3-phase arrangements previously examined. A study of Figure 316 shows that with 6 sections the first and second, the third and fourth, and fifth and sixth, taken in pairs, give a distribution of the conductors suitable for a 3-phase winding, each of the above pairs constituting a phase.

Four-Phase.—In Figure 317 there is developed a 6-pole multiple circuit winding tapped to form a 4-phase converter. In this case the winding covered by one pair of poles is divided into 4 sections, *i.e.* there are $\frac{6}{2} \times 4 = 12$ tappings. All 4-phase windings are characterised by having the conductors of each phase spread over an arc equal to 50 per cent. of the pole pitch. Sections 1 and 3, and also 2 and 4, are really in the same phase, and for this reason windings such as above are sometimes referred to as 2-phase.

Breadth Factor k_2 for Converter Windings.—In the case of a single-phase converter the conductors belonging to any one phase are distributed over one-half of the armature periphery, and the breadth factor k_2 will then have the same value as for the armature winding of a single-phase alternator with all the slots wound. For 4- and 6-phase converters the conductors of each phase are respectively distributed over one-half and one-third the periphery; hence the breadth factors will have the same values as given in Table XVI. for 2- and 3-phase windings. An inspection of Figures 314 and 315 will show that in a 3-phase converter the conductors of the various phases overlap each other, with the result that any portion of the armature winding carries conductors belonging to two phases. At one portion the conductors will belong alternately to phases 1 and 2, then to 2 and 3, and then 3 and 1, then again 1 and 2, the repetition occurring once per pair of poles. As a consequence of this the conductors of one phase are distributed over two-thirds of the entire periphery and the breadth factor will then be less than for a 3-phase alternator winding spread over one-third the periphery. With a sinusoidal field distribution or a pole arc $\psi = 0.7$, and a winding distributed in several slots per pole per phase, the values of the breadth factor k_2 for 3-, 4-, and 6-phase windings will be as follows:—

Three-phase	$k_2 = 0.83$
Four-phase	$k_2 = 0.901$
Six-phase	$k_2 = 0.956$

Armature Reaction.—The armature reaction of a rotary converter is the resultant of two reactions; namely, the reactions as (1) a direct-current dynamo and (2) a synchronous motor. Supposing the commutator brushes to be set with neither lead nor lag, so that the armature coils undergo commutation when their conductors are midway between poles, the armature reaction due to the direct current

will then be entirely cross-magnetising. If the cross M.M.F. be plotted as a function of the armature periphery the stepped curve AA of Figure 318 will be obtained, the M.M.F. being zero under the middle of the pole and reaching a maximum under the centre of the interpolar space. That is, the cross magnetomotive force due to the direct current may be considered as lagging one-fourth of a period behind the main field. If T denote the number of armature turns per pole, and I the value of the direct-current output, then, assuming a 2-pole 2-circuit armature, the amplitude value of the curve AA will be

$$A_1 = \frac{T}{2} \cdot \frac{I}{2} = 0.25 \text{ I . T ampère-turns.}$$

With the large number of slots per pole common to rotary converters the stepped curve of magnetomotive force will deviate very little

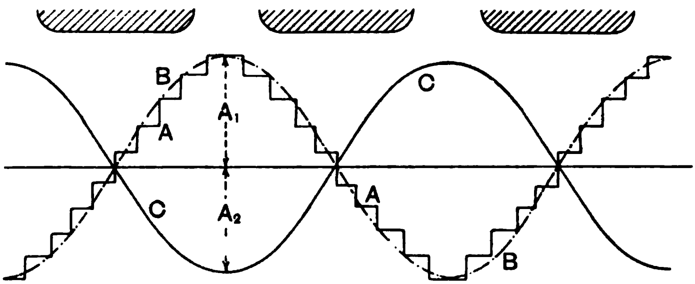

FIG. 318.—Armature reaction in a converter.

Cross M.M.F. due to direct current ————

" " " alternating current ....

from a sine curve, and, for the present discussion, can be considered as replaced by the sine curve BB having an amplitude $= A_1$.

In a polyphase converter the alternating current circulating in the armature winding gives rise to a rotating magnetomotive force which, since the armature rotates at synchronism, will remain stationary in space. The curve showing the variation of M.M.F. along the air-gap will be of rectangular shape, as explained on page 264 , and if analysed into its harmonics the fundamental will be by far the most important and the only one that need be considered. For the general case where the alternating current has a phase difference ϕ_1 with respect to the induced E.M.F., this fundamental wave of M.M.F. should be resolved into two components: one component, due to the wattless current $I_a \sin \phi_1$, which either magnetises or demagnetises the main field according as the current lags or leads; the second component, due to the energy current $I_a \cos \phi_1$, which sets up a cross magnetic field. The M.M.F. due to the energy component of the current will, for a motor, be a quarter of a period ahead of the main field, as shown

by the curve C C in Figure 318, and is directly opposed to the cross M.M.F. of the generated current. From page 270 the amplitude value of the curve C C is

$$A_2 = 0.45\, k_2 \,.\, m \,.\, T_1 \,.\, I_a \cos \phi_1$$

where k_2 = breadth factor of the winding

m = number of phases

T_1 = turns per pair of pole per phase $= \dfrac{T}{mp}$

I_a = armature current per phase.

$I_a \cos \phi_1$ denotes the energy current per phase, the values of which for a direct current = 1 and 100 per cent. efficiency are set forth in the last line of Table XXIX. page 424. Inserting these values in the equation for A_2, and multiplying by 1.04 to allow for a 4 per cent. loss due to hysteresis, eddy currents, friction, and excitation, there is obtained the following data:—

No. of Phases.	Amplitude Value of Cross M.M.F. Wave due to Direct Current.	Alternating Current.
3, 4, 6, 12	0.25 IT	0.215 IT

The amplitude values of the cross M.M.F. waves due to the direct and alternating currents are therefore almost equal, and since the two M.M.F. waves are in phase opposition the armature reaction of the direct current is practically neutralised by the armature reaction of the corresponding alternating current. The field distortion in a polyphase converter will therefore be quite negligible, and this explains why these machines can be operated from no-load to a considerable overload with the brushes fixed in that position which commutates the current in a coil when it is in the middle of the interpolar space.

The nature of the armature reaction in a single-phase converter is somewhat different from that described above. For, at the moment when the alternating current is zero, the full direct current exists; and at the moment when the alternating current is a maximum the reaction is the difference between that of the alternating and the direct current. Since the maximum alternating current equals twice the direct current, at that moment the resultant armature reaction is opposite and nearly equal to the direct-current reaction. The cross M.M.F. of a single-phase converter therefore oscillates with twice the frequency of the current, the maximum value being the same as that in a direct-current generator of the same output. Since this oscillating M.M.F. is in quadrature with the field excitation, it strives to distort the main flux in the direction of rotation, and this tends to produce sparking. Though this oscillating reaction is somewhat reduced by the damping effect of eddy currents set up in the pole face and other solid metal

parts, the commutation of a single-phase converter is not so good as in a polyphase machine operated under similar conditions, and this is another reason why its application is so limited. When single-phase converters are used they must either have their brushes fixed with a certain amount of lead or else have commutation poles fitted between the main poles. In the former case the direct-current ampère-turns embraced by twice the angle of lead of the brushes act as a demagnetising armature reaction, so that the field excitation requires to be correspondingly increased.

With a direct-current generator of given field strength the armature reaction is limited by the field distortion caused thereby, but with a polyphase converter this limitation does not exist and a much greater armature reaction, as compared with a direct-current machine, is therefore permissible. Since the heating is relatively low and the distortion of the field, which causes sparking under overloads in a D.C. generator, is negligible, the practical limit of overload will be much higher for a given machine when used as a converter than when used as a direct-current generator. With a rotary converter the theoretical limit of overload—that is, the overload at which the converter as a synchronous motor is pulled out of step and comes to rest—is for normal frequency and impressed voltage usually much in excess of the limit set by commutation and thermal considerations.

CHAPTER XIV.

ROTARY CONVERTERS—VOLTAGE REGULATION, LOSSES, HEATING, AND EFFICIENCY.

In addition to the difference already mentioned between a direct current dynamo and a rotary converter as regards loading, the latter has the further peculiarity that change of field strength does not alter to any appreciable extent the ratio of alternating and direct voltages. Thus, although the D.C. voltage will alter slightly with change of load, it cannot be corrected by merely shifting the field regulator, nor can the converter be overcompounded for traction work by merely putting series coils on the field.

Variation of D.C. Volts with the Load.—As previously mentioned, the voltage ratios given in Table XXIX. have, in a loaded converter, to be corrected to allow for the voltage that is expended in overcoming the resistance of the armature winding and also brush contacts. In virtue of this IR drop, which will increase in direct proportion to the load, the ratio between the slip ring voltage and that at the D.C. terminals will increase as the load comes on the machine. If it be assumed that the potential over the commutator is distributed according to a sine law, and that the commutator brushes stand in the neutral zone,—*i.e.* at the zero of the potential curve,—then, for a constant slip ring pressure of E_s volts, the terminal voltage on the D.C. side, when the converter is on open circuit, will be

$$E = \frac{\sqrt{2}\, E_s}{\sin \frac{\pi}{m}}$$

When the converter is loaded the current passing through the armature causes a mean voltage drop in the armature winding of $\sqrt{\beta}\,.\,IR_a$ volts. together with a corresponding drop under the brushes of Δe volts. In a loaded converter the terminal voltage on the D.C. side will then be given by

$$E = \frac{\sqrt{2}\, E_s}{\sin \frac{\pi}{m}} - \sqrt{\beta}\,.\,IR_a - \Delta e \quad . \quad . \quad . \quad . \quad . \quad . \quad (123)$$

where Δe, for the brushes of both polarity taken together, ranges from 1.5 to 2.5 volts, according to the quality of the carbons. Since the second and third terms in the above equation for E must necessarily be small in comparison with the first term, it will be obvious that the alteration of the D.C. terminal volts from no-load to full-load will be small, only amounting to about 2 per cent. of E.

Voltage Regulation. — Since, for a given applied alternating pressure E_a, the direct voltage E between the brushes of the commutator has (see equation 123) a definite value independent of the field excitation except for the variation of β which makes a very small difference, it follows that the voltage regulation on the D.C. side of a converter cannot be effected by merely altering the strength of the field current, as would be the case in an ordinary D.C. generator. Any increase or decrease of excitation,—other things remaining constant —simply causes the current to lead or lag accordingly. Hence, in order to regulate the voltage on the D.C. side of a rotary converter, either the A.C. volts applied to the slip rings must be varied or the D.C. volts must be altered by means of a direct-current booster suitably excited. The regulation of the pressure on the D.C. side is invariably accomplished by one or other of four methods.

The first method consists in varying by hand the voltage impressed upon the slip rings, and can be employed in two ways,—either the impressed voltage may be varied by altering the ratio of transformation of the step-down transformers, or it may be varied by means of an "induction regulator," the principle of which has been described on p. 142. In the first case, the step-down transformers are arranged in conjunction with a multiple contact switch, so that either the number of secondary turns or the number of primary turns can be altered by hand, thus varying the ratio of transformation. This method has several inherent drawbacks, the most important being those of first cost and difficulty of operation. If the regulation be performed on the primaries, the switches, on account of the high pressure of the circuits into which they are connected, become somewhat difficult and expensive to construct; while, if the regulation be performed on the secondary side, the expense becomes even greater, on account of the heavy currents to be controlled, and further, the difficulties which arise with the contact surfaces of all regulating switches for heavy currents must also be considered. In addition, there is the difficulty of arranging such switches to regulate gradually, and to avoid short-circuiting the sections of the transformer winding connected to the contacts of the regulator as these sections are being cut in or out. Consequently the employment of an induction regulator, which does not possess any of the above-mentioned defects, and which may be designed with a comparatively small loss, will give the best results if hand regulation of the converters is desired. In

order to avoid difficulties connected with insulation, induction regulators should preferably be worked from the secondaries of the step-down transformers.

The *second method* also varies the voltage impressed upon the slip rings, but in this case use is made of choking coils and a compound winding on the converter field, thus rendering the regulation entirely automatic. The compound winding of the converter causes the field strength to automatically increase with the load; but this, as already stated, will not be accompanied by any rise of voltage on the D.C. side unless there is a corresponding rise in the voltage across the slip rings. In order to increase the voltage at the slip rings as the load increases, a certain amount of reactance must be inserted between the slip rings and the secondary terminals of the transformer. As a rule, this reactance is obtained from choking coils connected in series with each phase of the converter, but the equivalent reactance could also be obtained by designing the step-down transformers with a much larger magnetic leakage than is usual in standard practice. The technical data of a transformer designed with this end in view is given on p. 183, the regulation at 0.8 power factor being 11.5 per cent.

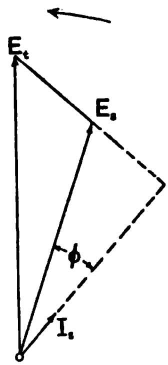

FIG. 319.

As already shown, rotary converters operate at a power factor which is determined wholly by the field excitation; for, any alteration of the latter with a given load on the machine increases or decreases the power factor on the A.C. side. For each load on the rotary there is a certain excitation which will make the power factor a maximum and the armature current a minimum.

Now, suppose the shunt excitation of a rotary converter, which is arranged with a compound winding and, if necessary, with choking coils in series between the transformers and the slip rings, be so adjusted that when operating at no-load the machine takes a certain amount of lagging current partly due to under-excitation and partly due to the reactance in each phase, then the vector diagram for such a condition of working will be as in Figure 319. If OE_s represent the P.D. at the slip rings, then the alternating current input OI_s will lag behind OE_s by some angle ϕ. The current OI_s in passing through the choking coil sets up a reactance voltage which will be in quadrature with OI_s, and in the diagram is represented by E_sE_t. The voltage required at the secondary terminals of the transformers will then be given by the resultant of OE_s and E_sE_t—*i.e.*, by OE_t. Now, supposing the transformer voltage OE_t to remain constant, then as the load increases the field is strengthened and the current comes more nearly

into phase with the slip ring voltage E_s, until when, as in Figure 320, coincidence of phase is reached and OE_s is of greater magnitude than the corresponding vector in Figure 319. Any further increase of load will over-excite the converter to such an extent that the current now leads with respect to the slip ring voltage (see Figure 321), which at the same time increases in magnitude. Hence, by suitably proportioning the. amount of self-induction in the choking coils and the number of turns on the field coils, the voltage across the slip rings will automatically increase with the load, and thus give various amounts of over-compounding up to about 10 or 15 per cent., the regulation being nearly as good as with an ordinary over-compounded direct-current dynamo. With a rotary converter of standard design the change of field flux necessary in order to get the desired amount of over-compounding is much greater than is required with the equivalent direct-current machine; for the D.C. voltage is not proportional to the field flux, but proportional to the impressed pressure on the slip rings. The only objection to this method of regulation is that the continual changing of the power factor of the alternating-current supply renders the maintenance of constant pressure at the receiving and of the transmission line exceedingly difficult. Experience shows that good results can be obtained by adjusting the shunt excitation at no-load to such a value as will make the armature current a lagging one, and equal to about 30 or 40 per cent. of the full-load current. As the machine becomes loaded the power factor improves, reaching unity at about three-quarters full-load, while for still greater loads the armature current will lead with respect to the applied E.M.F.

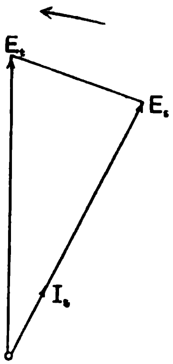

FIG. 320.

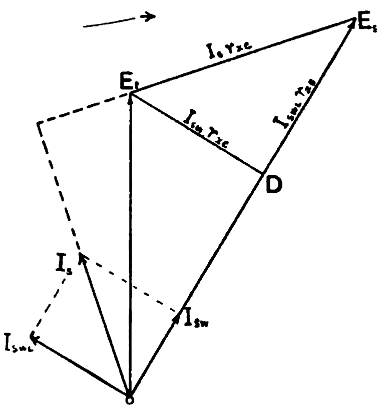

FIG. 321.

The third method regulates the voltage by means of a synchronous booster inserted between the collecter rings and the converter armature. The booster is, in fact, a small alternating current generator with a revolving armature keyed to the converter shaft, and wound for the same number of phases as the converter. The stationary field system will have the same number of poles as the converter, and will in general be excited by either the main current of the converter

or a current proportional to the output. The E.M.F. induced in the armature of the auxiliary machine will thus be zero at no-load, and will increase in direct proportion to the load, adding itself to the A.C. voltage applied between the tapping points on the armature. This arrangement does not require any additional slip rings or switches; and further, the voltage is regulated independently of the wattless current. The drawings of a 700-K.W. converter in which this method of voltage regulation is adopted are given in Plate XVI.

The fourth method of voltage regulation is carried out by means of a small direct-current booster connected in series with the D.C. leads, and having its field coils wound for series excitation. The booster in question could either be direct coupled to the shaft of the converter or driven by a constant speed motor. Since the current passing through the booster is the same as that supplied by the converter, the commutator for the booster will be about as large as that for the converter, and the resultant heavy initial cost places this method of regulation somewhat at a disadvantage.

Of the four methods just described that best adapted for the requirements of lighting work is the one which involves the employment of induction regulators, thereby allowing the pressure to be gradually varied by hand in accordance with the slowly varying load. For traction work the employment of either compound windings or an A.C. booster is to be preferred, on account of the large range of regulation required and the rapid variations of the load.

Calculation of Wattless Current, Reactance of Choking Coils, and Series Field Turns of a Compounded Converter.— Referring to Figure 321, OE_t represents the constant voltage E_t between the secondary terminals of the step-down transformer, E_tE_s the reactance voltage $= I_s r_{xe}$ induced in the choking coil by a current I_s from the transformer secondary terminals and OE_s the voltage E_s between any pair of slip rings. If from E_t there be drawn a perpendicular meeting OE_s in D, then DE_t represents the reactance voltage $I_{sw}r_{xe}$ produced by the watt component I_{sw} of the current I_s; whilst $DE_s = I_{swl}r_{xe}$ is the reactance voltage due to the wattless component I_{swl} of the current. From this construction there is obtained the equation

$$OE_t^2 = (OE_s - DE_s)^2 + DE_t^2$$

$$i.e., \quad E_t^2 = (E_s - I_{swl}r_{xe})^2 + I_{sw}^2\, r_{xe}^2$$

$$\text{hence} \quad E_t = (E_s - I_{swl}r_{xe})\sqrt{1 + \frac{I_{sw}^2 r_{xe}^2}{(E_s - I_{swl}r_{xe})^2}}$$

Expanding the square root

$$E_t \cong E_s - I_{swl}r_{xe} + \frac{I_{sw}^2\, r^2_{xe}}{2\, E_s} \quad . \quad . \quad . \quad . \quad . \quad (124)$$

The alteration of the voltage is therefore

$$(E_s - E_t) = I_{swl}r_{xe} - \frac{I_{sw}^2 r_{xe}^2}{2\, E_s}$$

Let E_{so} denote the voltage between slip rings at no-load, then, if the watt current, when the D.C. side is on open circuit, be neglected, there is obtained

$$(E_{so} - E_t) = I_{swl}' r_{xe}$$

where I'_{swl} = wattless current at no load

If it be desired that the voltage at the slip rings should increase by δE_s between no-load and full-load, then the increase is expressed by

$$\delta E_s = E_s - E_{so}$$

Subtracting the two above equations,

$$E_s - E_{so} = (I_{swl} - I_{swl}')r_{xe} - \frac{I_{sw}^2 r_{xe}^2}{2\ E_s}$$

$$= \delta I_{swl} r_{xe} - \frac{I_{sw}^2 r_{xe}^2}{2\ E_s} \quad . \quad . \quad . \quad . \quad . \quad (125)$$

where $\delta I_{swl} = (I_{swl} - I_{swl}')$ denotes the alteration in the wattless current between no-load and full-load.

From equation 125 it also follows that, for a given voltage alteration δE_s and reactance r_{xe}, the alteration in the wattless current from no-load to full-load is expressed by

$$\delta I_{swl} = \frac{1}{r_{xe}}\left(\delta E_s + \frac{I_{sw}^2 r_{xe}^2}{2\ E_s}\right) . \quad . \quad . \quad . \quad . \quad (126)$$

From equation (125) it will be seen that a considerable voltage variation can be obtained by making either δI_{swl} or r_{xe} fairly large. On account of the adverse effect it has upon the overload capacity of the converter operating as a synchronous motor it is not advisable to give the choking coils too high a reactance; whilst, on the other hand, δI_{swl} must not be too great, because of the increased armature heating which would follow. This limitation in the permissible values of r_{xe} and δI_{swl} somewhat restricts the voltage variation that can be obtained by this method, and for a given increase δE_s the values of δI_{swl} and r_{xe} decided upon must be a compromise between two conflicting requirements. In general, a 15 per cent. alteration in voltage will suffice for most practical purposes, and if the alteration of the wattless current be limited to 50 per cent. of the watt current, then

$$0.15\ E_s = 0.5\ I_{sw} r_{xe}\left(1 - \frac{I_{sw} r_{xe}}{2\ E_s}\right)$$

Neglecting the fraction $\frac{I_{sw} r_{xe}}{2\ E_s}$

$$0.5\ I_{sw} r_{xe} = 0.15\ E_s, \text{ or } I_{sw} r_{xe} = 0.3\ E_s$$

that is, the reactance voltage of the watt current will amount to 30 per cent. of the slip-ring voltage E_s.

The number of turns required for the series field winding may be

derived by the following procedure. At no-load the direct E.M.F. induced in converter armature is

$$E_{io} = E_o - \sqrt{2}\, I_{wl}' r_x$$

where E_o = direct *e.m.f.* which would be induced at no-load with no wattless, current I_{wl}' denotes the wattless current in the armature at no-load, and r_x the leakage reactance of the armature winding between two slip rings. The value of r_x would be calculated in a similar manner as the reactance per phase in an alternator. In order to produce the flux which corresponds to the voltage E_{io}, AT_{O1} ampère-turns must act on the magnetic circuit. But the ampère-turns

$$AT_M = 0.45\, k_2\, m\, T_1\, I_{wl}'\ \frac{\sin \frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma}$$

due to the wattless armature current will assist the field magnet ampère-turns, since the wattless current at no-load will be a lagging one. The ampère-turns to be produced by the field magnet coil at no-load are then

$$AT_O = AT_{O1} - AT_M$$

If at normal full-load the D.C. terminal volts be E, and the wattless current I_{wl}, then the direct E.M.F. that must be induced in the armature winding is

$$E_i = E + I\,(\sqrt{\beta}\, R_a + R_s) + \Delta e - \sqrt{2}\, I_{wl} r_x$$

where R_s denotes the resistance of the series winding. To induce this voltage, AT_1 ampère-turns must be impressed upon the magnetic circuit. Now, at full-load the wattless current I_{wl} will be a leading one, so that the demagnetising ampère-turns

$$AT_{DM} = 0.45\, k_2 m\, .\, T_1\, .\, I_{wl}\, .\, \frac{\sin \frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma}$$

produced thereby must be added to AT_1 in order to obtain the total field ampère-turns AT at full load,—*i.e.*

$$AT = AT_1 + AT_{DM}$$

In the equation for the demagnetising ampère-turns the wattless current in the armature at load $= I_{wl} = I_{wl}' - \delta I_{wl}$.

The shunt winding would be designed so that at no-load it supplies the total field ampère-turns required. At full-load the shunt ampère-turns will have increased to $AT_o\, .\, \frac{E}{E_o}$, so that the series winding should be designed to give

$$AT_s = AT - AT_o \frac{E}{E_o} \text{ ampère-turns.}$$

Hence for a normal full-load current of I ampères the number of series turns

$$= T_s = \frac{AT - AT_o \frac{E}{E_o}}{I \cdot (1 - x)}$$

where x denotes the fraction of the main current which is sent through the diverter (if any) in parallel with the series winding.

The Pulsation in the D.C. Voltage of a Converter.—The terminal voltage on the direct-current side of a converter is never absolutely uniform, but has superposed upon it an alternating voltage due to (1) the variable voltage drop in the armature winding, and (2) the higher harmonics of armature M.M.F. If the diagram of a single-, 2-, or 6-phase converter be examined it will readily be observed that, when the tapping points of one phase are directly under the commutator brushes, the current will pass direct from slip ring to commutator brush. The pressure at the D.C. terminals will then be

$$E_o - \Delta e \text{ volts.}$$

For any other position of the tapping points with respect to the brushes the terminal voltage will be less than this by an amount represented by the drop of potential in those conductors through which the alternating current passes on its way to the commutator. When a brush is mid-way between two connecting points the direct-current voltage will have its smallest value, because for this position the watt current converted into direct current has to take the longest path through the armature winding. The mean volts absorbed in the armature resistance $= IR \sqrt{\beta}$ volts, and if it be assumed that the voltage drop for each armature position varies according to the sine law the maximum voltage consumed at any instant in the armature will be

$$\frac{\pi}{2} IR_a \sqrt{\beta} \text{ volts}$$

Now, the smallest voltage drop is zero; hence the pressure fluctuation on the D.C. side due to the changing of position of the tapping points relative to the brushes is thus $= \frac{\pi}{2} IR_a \sqrt{\beta}$ volts, the period of pulsation being m times greater than the normal frequency of the converter. From Table XXXI. page 433 it will be seen that the value $\sqrt{\beta}$ diminishes rapidly as the number of phases are increased, and in multiphase converters the pulsation of voltage from this source is reduced to an almost negligible amount. For instance, in a 3-phase converter having a drop $IR_a = 2$ per cent. of the terminal voltage the fluctuation in the D.C. volts produced by the different positions of the feeding points relative to the commutator brushes $= \frac{\pi}{2} \cdot 2 \cdot \sqrt{0.71} = 2.65$ per cent.

In Chapter VII it was shown that the fifth, seventh, and eleventh harmonics of magnetomotive force set upin a polyphase armature induced E.M.F.'s of high frequency in the field winding and solid metal parts surrounding the pole pieces. These super fields also induce in the armature winding E.M.F.'s which may be of considerable magnitude, and in the case of rotary converters give rise to large fluctuations in the D.C. voltage. Pulsation of voltage from this cause will be most noticeable in single-phase converters, but can be considerably damped by the eddy currents induced in the damping coils. Voltage fluctuations are also made to occur if the E.M.F. wave applied to the slip rings deviates much from a sinusoidal curve; for the resultant higher harmonic currents arising in the converter armature set up violent field pulsations, which in turn produce fluctuations in the D.C. voltage.

Losses.—The losses occurring in a rotary converter are

1. Iron loss in armature core and teeth.
2. Armature copper loss.
3. Commutator I^2R loss.
4. Commutator friction loss.
5. Friction and I^2R loss at collector rings.
6. Excitation loss.
7. Bearing friction and windage losses.

Losses 1, 6, and 7 can be estimated in exactly the same manner as in the case of alternators, and the excitation loss will of course include the power expended in both shunt and series windings. The armature I^2R loss is, as already stated, calculated from the formula βI^2R_a, where R_a denotes the resistance of the armature if the converter were used as a D.C. generator giving the same output. The resistance R_a is determined from the formula

$$R_a = \frac{1.7 \cdot 10^{-6} k\, l_a \cdot T\,(1 + 0.004\ T^{\circ})}{qa}$$

where l_a = length of an armature turn $= 2L_{\kappa} + 2.5\tau$.
T = turns in series per circuit between + and − brushes.
T_o = temperature rise of winding at normal load.
a = cross-sectional area of conductor.
q = number of armature circuits in parallel.

The factor k is to allow for the increase in the copper loss resulting from the eddy current induced in the body of the conductors by the flux which enters the core through the slots, and also by the rotating flux set up by the harmonics in the armature M.M.F. wave. In a polyphase converter the values of k range from about 1.3 in a 25 ~ machine to 1.7 in a 50 ~ converter.

Commutator I^2R Loss.—Owing to the better commutation results and greater durability, carbon brushes are always used for

rotary converters, and the magnitude of the I^2R loss at the brush contacts will be directly proportional to the specific contact resistance. For a commutator peripheral speed between 14 and 18 metres per second the contact resistance is affected by the mechanical pressure on the brush, the current density, and the quality of the carbon from which the brushes are made. Within certain limits an increase of mechanical pressure lowers the resistance of contact between the brush and the commutator, the reduction in the I^2R loss, however, being accompanied to some extent by an increase in friction loss. When the brush pressure is increased beyond 120 grammes per cm.^{2} the increase in friction loss more than balances the decrease in I^2R loss. Hence the brushes of a converter are invariably adjusted to a standard pressure of from 0.1 to 0.12 kg. per cm.^{2}

With a given brush pressure the specific contact resistance of carbon brushes decreases rapidly as the current density is increased from zero, but at current densities greater than about 7 or 8 ampères per cm.^{2} the resistance approaches a more or less constant value, as shown by the curves in Figure 322. This phenomenon is due partly to the fact that carbon has a negative temperature coefficient of resistance by virtue of which its resistance falls as its temperature rises. When the current density is increased, any rise in temperature of the brush at once reduces the specific contact resistance, thereby checking the increase in the amount of power expended in heating the contact. The fall of resistance with increasing current density is also helped by the small carbon particles which are worn off the brushes, especially under high-current density, when a blackening of the commutator surface results. A more efficient contact between brush and commutator is obtained by this wearing away of the carbon, which proceeds rapidly when the brush is heated and, becoming softer, disintegrates more readily. Experience shows that the current density best suited for converters is one of between 5 and 6 ampères per cm.^{2}

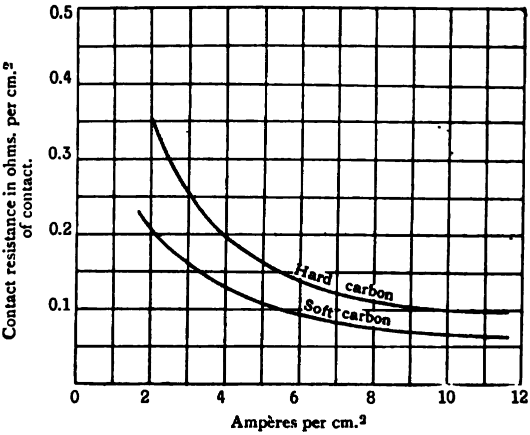

FIG. 322.—Contact resistance of carbon brushes.

With a pressure of 0.1 kg. per cm.^{2}, and a current density of about 5 ampères per cm.^{2}, the contact resistance varies greatly with different

grades of carbon, and may be taken as ranging from 0.1 ohm in brushes of soft quality to 0.15 ohm in a hard quality. In Figure 323 curves are given for the loss of volts over two sets of brushes (*i.e.* positive and negative) for two grades of carbon, soft carbon brushes being used for voltages of 250 or less. If Δe denotes the loss of volts over the

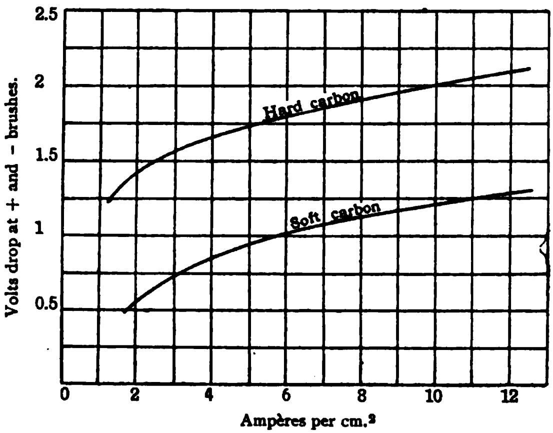

FIG. 323.—Volts drop due to contact resistance of carbon brushes (+ and −). Pressure, 100 grms. per cm.²

two sets of brushes, and I the current output from the D.C. side, then the I^2R loss at the commutator is

$$W_{cr} = \Delta e \cdot I \text{ watts.}$$

Commutator Friction Loss.—The power wasted due to friction between the brushes and the rotating commutator is expressed by

$$W_{cf} = 100 v_c \cdot 1000 \, P.A. \, \mu \text{ gramme cms. per second,}$$

where v_c = surface speed of commutator in metres per second,
P = pressure in kgs. per cm^2.
A = total brush contact area in $cms.^2$
μ = coefficient of friction.

Since 1 gramme-centimetre per second = 981 ergs per second = 981×10^{-7} watts, the brush friction loss is

$$W_{cf} = 100 \, v_c \cdot 1000 \, P \cdot A \cdot \mu \cdot 981 \times 10^{-7}$$
$$= 9.81 \, v_c \cdot P \cdot A \cdot \mu \text{ watts.}$$

For carbon brushes the coefficient of friction $\mu \cong 0.25$.

Collector Ring I^2R and Friction Loss.—Since the contact resistance of the brushes on the slip rings is not required to assist in the commutation of the current a very soft quality of carbon may

be used (see Figure 323). If $\Delta' e$ denote the volts drop per brush, and I_s the line current, then the watts lost in brush contact resistance is

$$W_{sr} = m \; \Delta' e \; I_s \text{ watts}$$

Owing to the low-contact resistance used it is permissible to work the brushes at a fairly high current density, usually about 20 ampères per cm.[2] The friction loss is, of course, given by the same equation as for the corresponding commutator loss, namely,

$$W_{sf} = 9.81 \; m \, . \, v_s \, . \, P \, . \, A \, . \, \mu$$

where m = number of rings and A = area of contact per ring and v_s = peripheral speed of ring.

Heating.—(1) *Armature.* The temperature rise of any part of a rotary converter is determined on the same principles as for alternators. In the case of the armature the highest temperature will arise in the active zone of the armature occupied by the teeth and the copper lying within the slots. I^2R loss within the slots is

$$W_c = \frac{2L_g}{l_a} \, . \, \beta \, . \, I^2 R_a \text{ watts}$$

and total loss coming into consideration for armature heating is

$$= W_a = W_c + W_t$$

where W_t denotes the iron loss in core and teeth. If A_a denotes the armature cooling surface as outlined in Figure 250, page 334, the temperature rise of the armature will be

$$T_a^\circ = K_a \, . \, \frac{W_a}{A_a \, (1 + 0.1 \, v_a)}$$

The heating coefficient K_a will have a value ranging from 10 to 11, according to the construction of the armature. (See p. 334.)

Commutator and Collector Rings.—The temperature rise of these parts is calculated from the formula

$$T_c^\circ = K_c \, \frac{W_{cm}}{A_c(1 + 0.1 \; v_c)} \text{ (for commutator)}$$

$$\text{and} \quad T_s^\circ = K_r \, \frac{W_r}{A_r(1 + 0.1 \; v_r)} \text{ (for collector rings)}$$

where W_{cm} and W_r denote the sum of the I^2R and friction losses at the commutator and slip rings respectively. The value to be assigned to the heating coefficients K_c and K_r are

$$K_c = 2.0 \text{ to } 2.1$$
$$K_r = 0.9 \text{ to } 1.1$$

Field Magnetic Coils.—If W_m denote the sum of the excitation losses, W_{sh} and W_{se} and A_m the external surface of coils, then temperature rise of field winding is

$$T_m^\circ = K_m \, . \, \frac{W_m}{A_m}$$

where the value of the heating coefficient ranges from 3.5 to 4.0.

Efficiency.—On account of the relative small armature copper loss in rotary converters their efficiency will be somewhat higher than is obtained from D.C. generators of the same output. The curve in Figure 324 gives approximate values of the efficiency at full normal load that may be expected from polyphase rotary converters. Of course, in considering the efficiency of the whole transforming apparatus the losses in the step-down transformers must also be taken into account. The losses and efficiency of a converter are determined by exactly the same tests as for D.C. machines. The only point that need be emphasised is that, in computing the armature copper losses, the current must be taken as $\sqrt{\beta}\,\mathrm{I}$, and not equal to I.

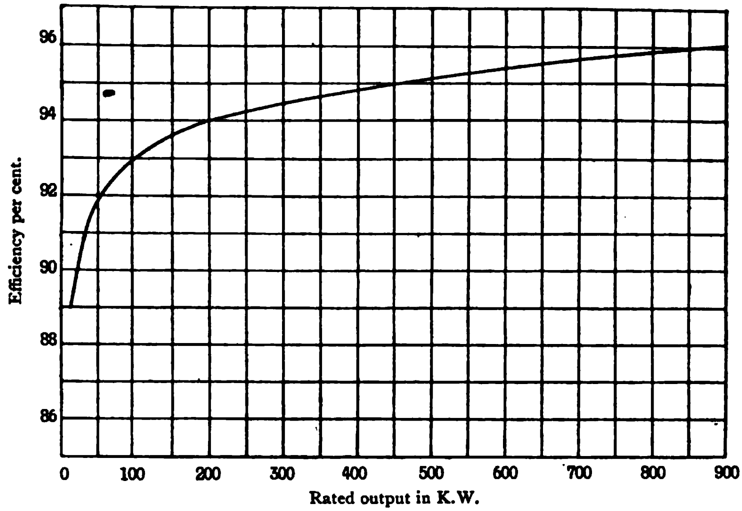

FIG. 324.—Efficiency of polyphase rotary converters.

Starting of Rotary Converters.—A rotary converter may be run up to synchronous speed from either

(1) The direct-current side, or

(2) The alternating current side, or

(3) By an auxiliary motor direct coupled to the armature shaft.

1. *Starting from the D.C. side.*—When started from the direct-current side the converter is brought up to speed by operating it as a shunt motor, the usual starting resistance being inserted in the armature circuit. The resistance in the shunt circuit is adjusted until synchronous speed is reached, and as soon as equality of phase and voltage between the mains and converter is attained, as shown by the usual phase lamps or synchroscope and voltmeter, the switch on the A.C. side is closed. As the converter would in general have to be built for very large currents, the best practice is to avoid any switch

gear between it and the transformers, and on this account the switches are usually on the high-tension side, as shown in Figure 325, which gives the diagram of connections for a 3-phase converter. Immediately synchronism is reached the main oil switch between the transformers and the high-tension feeders should be closed.

Starting a converter by this method has the disadvantage that, in the case of rapidly varying loads, such as occur in traction systems, considerable difficulty may be experienced in synchronising because of the continual fluctuations in the voltage of the direct-current side. If the converter is brought up to the proper speed from the D.C. side,

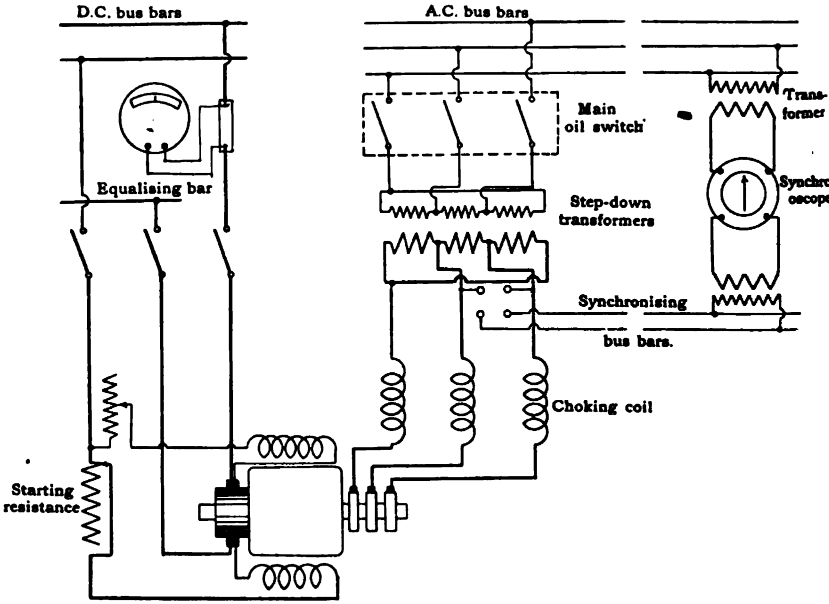

FIG. 325.—Connections for 3-phase converter to be started from D.C. side.

and synchronised, while the voltage on the A.C. side differs somewhat from the voltage of the mains, then, at the moment of closing the main switch, a large rush of current will result. The rotary is thus forced to convert a much greater alternating current into a direct current, or *vice versa*, than when normally loaded, with the result that the current may be sufficiently large to open the circuit breakers or set up a pressure surge in the feeders. In such cases, where there are heavy fluctuations in the load, the better procedure is to avoid synchronising altogether, and run the converter up to a speed slightly above synchronism and then open the switches on the D.C. side. If the switches on the polyphase side be then immediately closed, thus connecting the armature to the polyphase supply, the machine will pull itself into

step, when of course the D.C. switches can again be closed. If the pressure of the direct-current mains fluctuates very little from its normal value, the starting up of a converter from the D.C. side becomes an exceedingly simple matter, and should always be adopted whenever possible. In cases where all the converters may at any time be shut down, and no main battery is used in connection with the D.C. mains, the direct current for starting the first rotary may be obtained from either an auxiliary battery or a small induction motor generator set, the induction motor being supplied with power from the A.C. mains.

2. *Starting from the A.C. side.*—This method of starting is applicable only to polyphase rotaries, and consists in switching the alternating-current supply on to the slip rings. The polyphase current flowing through the armature winding sets up a rotating magnetic field, and the torque due to the eddy currents in the solid parts of the pole shoes and hysteresis in the iron is sufficient to start the armature rotating on the principle of the induction motor, with the difference that in this case the primary is caused to revolve. At starting the E.M.F. between the commutator brushes is alternating with a frequency proportional to the difference between the armature speed and that of synchronism. A direct-current voltmeter or incandescent lamps connected across the commutator brushes will indicate by their pulsations the approach of the converter to synchronism. When synchronism is approximately reached the field circuit is then closed, and the armature falls into step with the rotating magnetic field, and the machine runs as a synchronous motor, taking its no-load current from the mains. This method of starting has the disadvantage that the polarity of the D.C. side is not definite, but depends upon the moment at which the converter comes into synchronism. In order to obtain the right polarity it is best to have a moving coil voltmeter connected across the D.C. terminals, the + terminal of the voltmeter being connected to the + of the converter. As the armature approaches the synchronous speed the voltmeter needle begins to move slowly from zero to far over the normal position, and when the speed is almost that of synchronism the field switch should be closed at the moment the voltmeter pointer is at its highest reading. The converter then falls into step, excites itself and has the proper polarity. The main switches on the D.C. side can then be closed and the load put on the converter. A second method for obtaining the right polarity consists in reversing the shunt winding if the polarity, as shown by the voltmeter, be wrong. With the existing direction of rotation and reversed field winding the exciting current demagnetises the field. The rotating field produced by the supplied alternating current is therefore displaced 90 degrees with respect to the field magnet system. From this it follows that on reversing the field winding the converter armature slips back 90 degrees with respect to the revolving field. If the field winding be now reversed for

a second time the armature slips back another 90 degrees with respect to the rotating field, and as the converter magnetises itself again the polarity of the terminals will have become reversed.

During starting, which usually takes from a few seconds to one minute or more, the field windings act like the secondary circuit of a transformer, and since the number of field turns is usually very much larger than the number of armature turns, very high voltages ranging from 3000 to 6000 volts may be induced, and unless special precautions are taken this pressure may be sufficient to rupture the insulation between the winding and earth. For this reason the field magnet

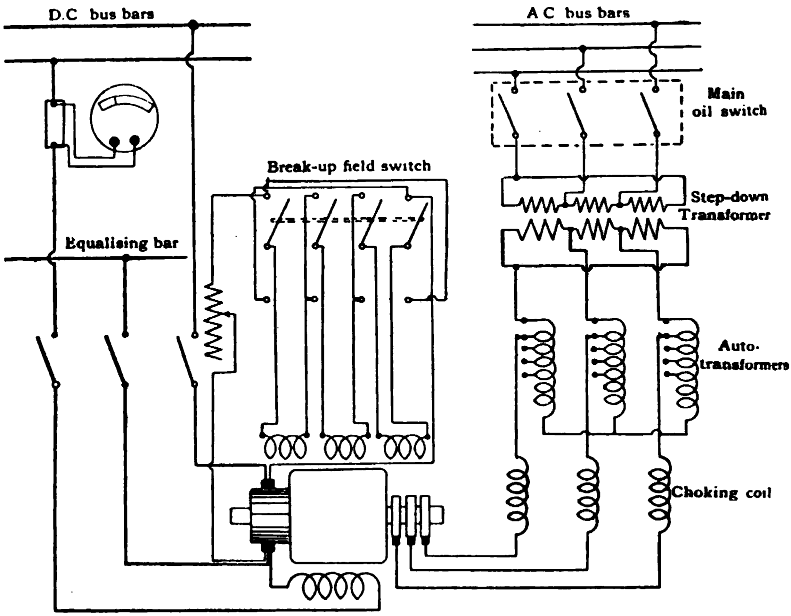

FIG. 326.—Connections for 3-phase converter to be started from A.C. side.

winding should be divided into three or four sections, which by means of a "break-up" switch are on open circuit when running up to synchronism, and are closed at the moment synchronism is reached. Such a switch is indicated in Figure 326.

The disadvantage of this mode of starting is that in order to develop the necessary torque the armature may take from the supply mains a current 3 or 3.5 times in excess of the normal value. Now, since this current is almost entirely wattless, the pressure regulation of the polyphase system will be seriously disturbed unless the voltage is applied to the slip rings in a number of increasing steps, such as would be obtained by using an auto-transformer having from three to five tappings. By this means the starting current can be reduced to that

at normal full-load, but this, of course, involves additional switch gear and more intricate connections.

The scheme of connections for a 3-phase converter suitable for starting up from the A.C. side is shown in Figure 326. The starting switch and auto-transformer on the A.C. side enables the pressure to be applied to the slip rings in a number of steps. The 4-pole field switch enables the polarity, if wrong, to be reversed, and also provides a means of dividing the shunt winding into three sections until the converter reaches synchronism. This method of starting is only used where no suitable direct-current supply is available, or where it is desirable to save a special starting motor or motor-generator set. For if the load on the generating plant be small, and if converters in other places are running from the same network, the starting of a converter from the A.C. side may easily set up current surges which would throw the other converters out of step.

3. *Starting by an auxiliary motor.*—Of the three methods available, this is by far the simplest and least troublesome, and, although it involves the additional cost of the starting motor for each converter, it has been used to some considerable extent. For the starting motor a small induction motor of from 5 to 10 per cent. of the converter output is always used, the armature being keyed to an extension of the main shaft. The auxiliary motor would be designed with two poles less than the converter, and with such a large slip that the converter rotates at almost synchronous speed when normally excited. Should the losses of the converter be an insufficient load to bring down the speed of the induction motor to that of exact synchronism, arrangements should be made to load the converter as a D.C. generator, thereby increasing the slip of the motor until synchronism is reached. The synchronising arrangements and switch gear would be the same as when the converter is started from the D.C. side, and when phase and voltage equality is obtained the switch on the A.C. side is closed and the induction motor disconnected from the source of power.

Parallel Working of Converters.

In large installations rotary converters have to work in parallel not only with the A.C. generators, but also on the D.C. side, where in addition there may be a direct-current generator or floating battery, as in supply schemes for tramways and railways. In order that a floating battery may work well in parallel with converters it is necessary that the converters, when a constant voltage is applied to the slip rings, should have a fairly large voltage drop between no-load and full-load; for, if the volts drop be too small, the converter and not the battery will take up the fluctuations in the load. A large voltage drop, however, for the same reasons that apply to synchronous motors, affects adversely the

overload capacity of converters. If they have not to work in parallel with a battery a small voltage drop is better, as the machine can then, for the same heating, give about 25 per cent. greater output than the corresponding D.C. generator. When a converter has to operate in parallel with a shunt excited dynamo, both machines should be designed to give the same voltage drop, and thus allow the load always to divide itself equally between the two machines. If two or more converters receive alternating current from the same transformers, and are also paralleled on the D.C. side, the direct current may easily divide on the individual brush sets of the various converters, so that, for any one machine, more current flows through the brushes of one polarity than flows through the brushes of the other polarity. As this may lead to serious sparking at the commutator, the practice of feeding each converter from a separate set of transformers should always be adopted.

When compound converters are operated in parallel, the terminal of each machine common to the armature and series field winding should be connected through a switch to a common equalising bus bar, in exactly the same way as ordinary compound direct-current generators are connected. This equalising bar, shown in Figures 325 and 326, serves to equalise the division of the load on all the machines. Before paralleling a compound converter on the D.C. side the switch to the equalising bar should first be closed, so that the series coils receive current from the converters already at work. Immediately after that the positive main switch may be closed, and by strengthening the shunt excitation a part of the load can then be gradually transferred from the other converters to the newly switched-in machine.

A converter, when working in parallel with a battery or D.C. generator, may attain to a dangerously high speed if a large voltage drop suddenly occurs on the A.C. side,—say, arising out of a short circuit. Under such conditions the converter continues to run as a shunt motor from the D.C. side, and pumps a large lagging alternating current back into the fault. The resultant armature reaction weakens the main field, and the speed goes on increasing inversely as the strength of the field. The converter must on this account be protected with a reverse current cut out on the D.C. side, otherwise a dangerously high speed may be attained.

Hunting of Converters.

Since a rotary converter is essentially a synchronous motor, the phenomena of hunting, along with the consequent current surges between the converter and the A.C. generators of the system, is liable to be set up whenever sudden variations in the load or other disturbances in the line occur. In fact, on account of their comparative small armature reaction, converters are much more liable to hunt than

synchronous motors. Trouble from hunting has in some cases assumed a very acute form, which at times has made parallel working quite impossible. When the prime movers, which drive the alternators supplying current to the network, have not a constant angular velocity, the converters, to remain strictly in synchronism, must respond to the oscillations forced upon the system. Of course, the converter cannot do so perfectly, with the result that at one instant it lags behind the synchronous position and so takes more current. The increase of current sets up a synchronising torque which causes the armature to accelerate, and, on account of its momentum, to pass the synchronous position. The rotary thus acts for an instant as an alternating current generator, returning current to the supply mains. The increased load on the armature introduces a retarding torque tending to slow it down, with the result that the armature tends to oscillate about its mean synchronous position. According to the relative inertia of the rotating parts of the generators and converters, these superposed oscillations may increase in amplitude and set up current surges of considerable magnitude either between generators and rotary or between several rotaries, leading in some cases to the machine being pulled out of step.

In a polyphase converter the multi-phase currents circulating in the armature winding give rise to a rotating magnetic field which, since the armature rotates synchronously, will, under steady conditions of working, remain stationary with respect to the field system. When hunting takes place the oscillations of the armature about the synchronous position cause this field to oscillate also. This in turn induces an oscillating E.M.F. in the armature conductors, and, in the case of the coils undergoing commutation, sets up large circulating currents which will lead to violent sparking at the brushes.

The liability to hunt depends greatly upon the engines driving the alternators, for, if these have not sufficient fly-wheel effect to keep down the forced oscillations, resonance may occur with the synchronous converters connected to the network, and oscillations when once started will go on increasing until a converter is pulled out of step. To diminish the possibility of trouble from this cause, the periodic time of the free oscillations of the converter, as expressed by $T_1 = 0.0082\, R_c \sqrt{\frac{\Sigma mr^2}{KW.\sim.Q}}$,* should not approach too closely the periodic time of the forced oscillations $T_2 = \frac{30}{R_g . n_c}$, where, for the converter

R_c = synchronous speed in R.P.M.

Σmr^2 = moment of inertia of the armature in kg./metres.²

Q = ratio of alternating current at short circuit to that at normal load;

* Compare with equation 96, p. 359.

and for the considered prime mover

R_g = speed in R.P.M.
n_c = number of cranks (not in line).

If T_1 comes within more than 50 per cent. of T_2, then either the moment of inertia Σmr^2 or the ratio Q must be altered to give a lower value of T_1. Assuming Σmr^2 to be fixed as would be the case in a machine of standard construction, the value of T_1 can only be reduced by increasing the value of Q,—*i.e.*, by increasing an overload capacity of the converter. Now, for a synchronous motor the load at which the motor will drop out of step increases—within certain limits—with the field excitation, which increase, in the case of a rotary converter, will be almost in direct proportion to the radial depth of the air-gap. Hence for a given converter the larger the air-gap the more remote will be the likelihood of trouble arising from hunting.

A compound winding on the field magnets tends to facilitate hunting; for any alteration in the ampère-turns of the series winding due to a variation in the D.C. load is not accompanied by an instantaneous change of flux, the time required for the flux to obtain a value normal to the new conditions depending upon the degree to which the iron of the magnetic circuit is magnetised. Hence some machines may respond more quickly than others, and thus the voltage of the various converters in parallel may differ and current oscillations be started. On account of the danger of hunting being greatly increased in a compound converter, the regulation of the voltage by either an A.C. or D.C. booster is to be preferred.

Should the E.M.F.'s of the alternating mains and the converter have different wave shapes, high-frequency currents will circulate between the converter and the generators supplying the system, and these may be able to start phase swinging should the load on the converters be subjected to rapid fluctuations. The higher frequency currents will be greater the smaller the reactance of the whole electric circuit with respect to them, and if such currents are likely to arise it is best to obtain the necessary voltage regulation by means of choking coils, as their high reactance tends to damp down the currents due to the harmonics in the E.M.F. waves.

In order to render rotary converters free from swinging troubles, and the resulting deleterious effect upon the commutation, it is necessary that the free oscillations should be rapidly damped out. This may be affected either by

(1) Fitting heavy amortisseurs into the pole shoes, if these be laminated, or,

(2) By connecting the solid pole shoes together with copper or bronze bridges. Closed conducting circuits are thus formed for the induction of currents which damp out the oscillations of the

magnetic field. The adoption of such damping arrangements will of course lower the efficiency by an amount ranging from 1 to 1.5 per cent., but this is of very little consequence compared with the importance of steady running and freedom from hunting.

Static Balancers for Three-Wire System.—Where a rotary converter or D.C. dynamo supplies current to a 3-wire system it may not always be convenient to employ a direct-current balancer, and in such cases it becomes necessary to connect the middle wire of the system to a point having a potential midway between that of the positive and negative brushes. Theoretically this point could be obtained from an auxiliary set of brushes placed intermediate between the main brushes. But, any variation in the magnitude of the load or out-of-balance current will cause the position of neutral voltage to alter, and unless the position of the auxiliary brushes be correspondingly adjusted serious sparking will

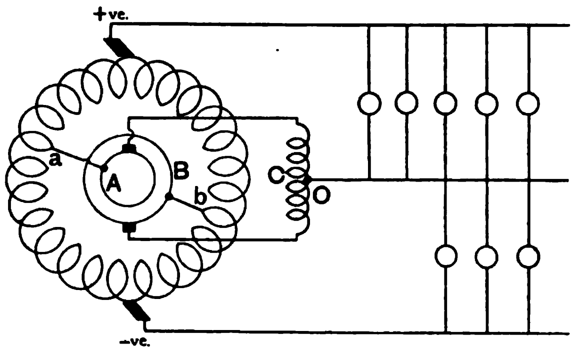

FIG. 327.—Principle of static balancer.

result. This method of dealing with the out-of-balance current is therefore of no practical utility.

Another method has, however, been proposed by Dolivo Dobrowolsky,* in which the middle wire of the system is connected to the middle point of the windings of a choking coil, excited by an alternating current drawn from the armature winding. In the case of converters working from transformers the secondary winding of the latter may be used in place of the choker, the star point being connected to the neutral wire. The principle underlying Dobrowolsky's method is illustrated in Figure 327, where the balancer is applied to a 2-pole machine. There two slip rings A, B, from which connections are made to two points *a*, *b*, in the armature winding, which for the case under consideration are diametrically opposite. The choking coil or static balancer C is simply a low-resistance coil wound on a laminated closed magnetic circuit resembling a transformer. The

* *E.T.Z.*, 1895, p. 323.

middle point O of the coil is connected to the middle wire of the system, while the ends of the coil are connected across the slip rings A, B. When the converter is in operation an alternating E.M.F. is produced between the slip rings having a frequency $=\frac{Rp}{60}$, and a maximum value, on a sine wave assumption, equal to the E.M.F. between the brushes on the commutator. The application of this alternating E.M.F. to the terminals of the choking coil will give rise to a magnetising current, the effective value of which is

$$I_m = \frac{E_s}{2\pi \sim L}$$

where E_s is the effective E.M.F. between the slip rings and L_c the inductance of the coil C.

From Figure 327 it is obvious that the middle point of the choking coil will be at a potential intermediate between that of its terminals, and therefore the slip rings. Now, the tappings from the armature winding to the slip rings are always symmetrically situated with respect to the $+ve$ and $-ve$ brushes. Hence the middle point O will have a fixed potential with respect to the commutator brushes, and the voltage between this point and either brush will be equal to one-half of that between the main brushes. The out-of-balance current on entering the static balancer divides equally between each half, as with this subdivision the balancer offers the least apparent resistance. The magnetising effect produced in one half neutralises that of the other, the resultant magnetising effect of the out-of-balance current in the balancer itself is zero. If the choking coil is of the core type the windings of each half of C will have to be distributed equally on the two limbs otherwise a uni-directional field will be produced by the out-of-balance current, causing increased iron losses and excessive magnetising current. The path of this field is through the cores and back *via* the air, clamping bolts fixing the top and bottom yokes, and iron case if provided.

With the simple arrangement shown in Figure 327 the out-of-balance current will cause an unequal distribution of current in the armature, and therefore unequal heating. Should the $+ve$ side be the more heavily loaded the current in that half of the armature on the same side of the tappings as the $+ve$ brush will be greater than the current in the remaining conductors by about half of the out-of-balance current. In order to obtain a more uniform distribution of the out-of-balance current in the armature, and therefore diminished heating, 2-phase (Figure 328) static balancers have been adopted for this purpose. With a 2-circuit or wave winding one tap per slip ring is only required, the armature winding being divided into as many parts by the tappings as there are collector

rings, independent of the number of poles. With a multiple-circuit winding the number of taps per ring are equal to the number of pairs of poles, and should be symmetrically situated with regard to the latter.

The unequal distribution of the out-of-balance current in the armature winding will give rise to unequal armature reaction on the field, and this will have a maximum effect with the single-phase arrangement, but a decreasing influence as the number of slip rings are increased. Increasing the number of slip rings—*i.e.* supplying the out-of-balance current to the armature at a greater number of points—causes a more rapid oscillation of the flux due to the armature reaction, but as the amplitude of the flux oscillations is much reduced the resultant field is more constant. In some cases the oscillation due to

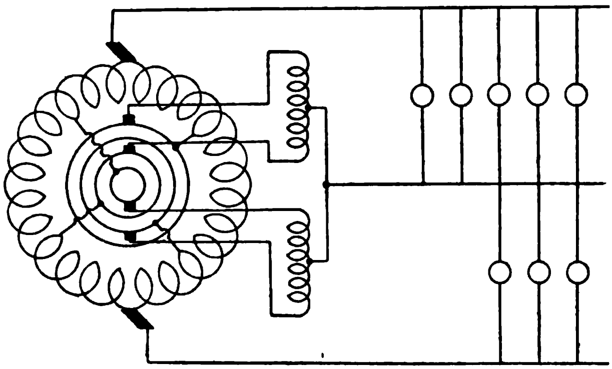

FIG. 328.—Principle of static balancer.

this reaction has shown itself by a flickering of the lamps connected to the circuit.

The size of a static balancer depends upon the amount of unbalanced power,—*i.e.* it depends upon the maximum difference between the load on the two sides of the 3-wire system. If E_c denote the voltage between the middle wire and the outside conductor, and I the maximum current in the outside conductors, then the converter will be designed for a full-load output of

$$W = 2\ E_c I \text{ watts}$$

Now, let I_o denote the maximum out-of-balance current, which should not exceed 10 to 20 per cent. of I, then for a single-phase balancer the effective voltage across the slip rings $= \frac{2\ E_c}{\sqrt{2}} = \sqrt{2}\ E_c$. As the out-of-balance current divides equally between both halves of the winding, the current in the balancer winding $= I_o/2$,—*i.e.* on the assumption that the

losses are negligible. The volt-ampère capacity of the balancer is therefore

$$W_o = \sqrt{2}\, E_c \cdot \frac{I_o}{2} = \frac{E_c I_o}{\sqrt{2}}$$

and
$$\frac{W_o}{W} = \frac{E_c I_o}{\sqrt{2}} \cdot \frac{1}{2\, E_c I} = \frac{1}{2\sqrt{2}} \cdot \frac{I_o}{I} = 0.35\, \frac{I_o}{I}$$

With a maximum out-of-balance current of 20 per cent., $I_o/I = 0.2$, the volt-ampère capacity of the balancer will be 7 per cent. of the volt-ampère capacity of the rotary, and as an auto-transformer of ratio unity is one half the size of a transformer of the same volt-ampère capacity, the size of the transformer will only be 3.5 per cent. of the volt-ampère capacity of the converter. When an *m*-phase balancer is used, each will have a capacity $\frac{1}{m}$ that of a single-phase one.

Comparison of Rotary Converters and Motor Generators.—*Prime Cost.*—In the working of rotary converters it will nearly always be necessary to transform the high voltage of the supply mains to a lower voltage convenient for the converter; hence, in the comparison of rotary converters with motor generators, the transformers required for the former must also be taken into consideration. The first cost of motor generator sets is usually somewhat higher than that of rotary converters, particularly in the larger sizes. The actual difference, however, is not very great, as will be seen from Table XXXII., which gives the relative costs of the various sizes. This table is based on quotations by the same manufacturer of rotary converter and motor generator sets of 25 and 60 cycles. The unit is taken as a 25 ~ converter with the necessary transformers and regulators.

TABLE XXXII.*—COMPARATIVE COSTS OF ROTARY CONVERTERS AND MOTOR GENERATORS.

Capacity in K.W.	Rotaries with Transformers and Regulators.		25 ~ Synchronous Motor Generator Set.	60 ~ Induction Motor Generator Set.
	25 ~	60 ~		
250–300	1	0.95	1.00	0.95
500	1	1.00	1.05	1.05
1000	1	1.05	1.05	1.10

Efficiency.—The efficiency of a converter will generally be higher than that of the motor generator, because with the latter there is the

* From paper, by Wm. Eglin, read before section E of the International Electrical Congress of St. Louis, 1904.

combined transformation of electrical energy in the motor into mechanical energy, and then in the generator again into electrical energy, while with the converter the change from one form of current to the other takes place direct, and only the momentary difference of the two currents flows in the armature winding. The copper loss is therefore smaller, but this is somewhat neutralised by the losses occurring in the transformers and regulators. The following table shows the efficiency of a 400-K.W. 60 ~ rotary converter and a 400-K.W. 60 ~ motor generator set with both high- and low-voltage motors. These tests, made on machines built by the same firm, show that even at high frequencies the rotary converter is more efficient than the motor generator set.

TABLE XXXIII.

Full-load Efficiency of 400-*K. W.*, 450-*R.P.M.*, 60 ~ *2-phase Rotary Converter and Motor Generator.*

Rotary converter with transformer	90.5 per cent.
Motor generator with 220-volt induction motor and transformers.	84.7 ,,
Motor generator with 5000-volt induction motor .	86.9 ,,

Commutation.—Since very little field distortion occurs with increasing load, the conditions for good commutation are more favourable with the converter. The converter is therefore better adapted for coping with such sudden and large fluctuations in the loads as are encountered in traction systems.

INVERTED ROTARIES.

As mentioned in the introduction to this chapter, rotary converters, besides converting from alternating current to direct, may also be used to convert from direct to alternating, in which case they are referred to as "Inverted Rotaries." Though the use of the latter is somewhat limited, they are, when special cases are met with, a distinct advantage. For example, in a low-tension direct-current system a district at some considerable distance from the generating station may be supplied by converting from direct to alternating, transmitting as alternating at high tension and then re-converting to direct current. Or in a station with D.C. generators for short-distance supply and alternators for long-distance supply, a converter may be used as the connecting link to transfer the load from the direct to alternating generators, or *vice versa*, and thus be operated either way according to the load on the system.

When converting from alternating to direct, the converter can only have one speed, namely, the synchronous speed; any alteration of the exciting current has no influence upon the speed, but merely alters the

power factor of the alternating-current circuit. When, however, a converter is operated as an inverted rotary, and is not connected in parallel with other alternators on the A.C. side, the speed of the converter as a direct-current motor depends upon the excitation, and varies inversely as the strength of the field magnets. The field strength will depend upon the magnitude and phase relation of the alternating current when the inverted rotary has a constant field excitation. A lagging current weakens the field and thus increases the speed, while a leading current has the opposite effect. Hence, if a lagging current be taken from an inverted rotary, as, for instance, in starting an induction motor, the armature reaction will weaken the field, and thus the speed of the armature will increase, and so also will the frequency. An increase of frequency may, however, increase the lag of the current, thus further weakening the field and increasing the speed. Under the conditions the acceleration may be so rapid as to get beyond the control of the field rheostat, and so the speed may become dangerously high. Inverted rotaries, if excited from a constant voltage supply, must therefore be protected with some device which opens the main switches on the D.C. and A.C. sides whenever the speed comes within the danger limit. Another method is to separately excite the converter from a small exciter mechanically driven from the armature shaft, for in that case any increase of speed will raise the exciter volts and so increase the direct magnetising M.M.F. at a greater rate than the demagnetising M.M.F., thus putting a check on the speed.

When an inverted rotary is operated in parallel with alternators of much greater capacity the danger of racing does not exist, for under these conditions a change of field excitation does not alter the speed, but merely changes the phase relation of the alternating current delivered by the converter. That is, the converter not only receives power from the direct-current supply and delivers it—less the losses—to the alternating system, but at the same time it receives a wattless current from the alternating system, lagging at under-excitation, leading at over-excitation, and can in the same way as an ordinary converter or synchronous motor be employed to compensate for wattless currents in other points of the system.

CHAPTER XV

DESIGN OF ROTARY CONVERTERS

BESIDES stating the normal output of the D.C. side, the specification for a converter should also include the E.M.F., frequency, and number of phases of the supplied alternating current, and the problem which presents itself is to design a machine which will meet the above requirements without undue sparking at the commutator, and with a temperature rise at continuous full-load not exceeding about 45° C.

Frequency.—Theoretically, rotary converters can be designed for any frequency, but from considerations of practical working the limit of successful operation of these machines is reached at 50 cycles. Above this frequency such severe limitations are imposed upon the design, especially that of the commutator, that the only solution to the problem of converting from alternating current to direct lies in the employment of motor generator sets. Above a frequency of 50 ∼ there are two outstanding difficulties in the way of successful working: (1) good parallel running of two or more rotaries becomes extremely difficult, owing to the ease with which oscillations can be started; and (2) the question of commutation becomes a very difficult one to solve in a satisfactory manner.

As already pointed out, in connection with alternators, the lower the frequency the better the parallel running; for with a given angular displacement the phase difference between the E.M.F. waves of the various machines is smaller. The same argument applies equally to the case of the rotary converter, for in order to put the high-frequency machine on the same basis as the low-frequency machine in this respect, it would be necessary to have the same number of poles in each machine, and this would mean that a machine operating at 50 cycles would have to run at double the speed of a machine of the same output operated at 25 cycles. This is, of course, impossible, on account of the excessive peripheral speed that would result, the principal difficulty arising at the commutator. If, on the one hand, the peripheral speed of the commutator be too great, the satisfactory collection of the current with carbon brushes is rendered very difficult owing to the vibration of the latter; while, on the other hand, if the diameter be diminished below a certain limit the segments would become too thin to satisfy mechanical

requirements. The higher the direct current-pressure the greater the difficulties in this respect.

The net result is that, with high frequencies, the number of field poles, diameter of armature, and number of commutator segments have to be increased beyond that required for a low-frequency machine. All these features, namely, crowded poles, high peripheral speeds, thin commutator segments, short distance between brushes, etc., are wholly unfavourable to successful parallel running and good commutation. It is for these reasons that a frequency of 25 ~ has become more or less standard for rotary converter work, and at this frequency well designed machines should not give the slightest trouble. Between 30 and 50 cycles their performance may still be good under favourable conditions of good engines, high speeds, and low voltages; but above 50 ~ the use of rotary converters, on units of large size, is practically out of the question.

Number of Poles.—Since a rotary converter is a synchronous machine, the number of pairs of poles bears a definite ratio to the frequency and number of revolutions per minute of the armature, thus

$$p = \frac{60 \sim}{R}$$

The peripheral speed of the armature as expressed by

$$v = \frac{\pi DR}{6000} = \frac{\pi D}{2p} \cdot \frac{2pR}{6000} = \frac{\tau \sim}{50} \text{ metres per second,}$$

will be greater the larger the values of pole pitch τ and the frequency $\sim$. The same relation also holds good between the pole pitch τ_c and the peripheral speed v_c at the commutator, namely,

$$v_c = \frac{\tau_c \sim}{50} \text{ or } \sim = \frac{50\, v_c}{\tau_c}$$

Hence, in designing converters for fairly high frequencies, it becomes necessary to make the peripheral speed as high as possible, while the pole pitch is kept fairly low. There is, however, a lower limit to the value of τ_c, settled from the fact that for good commutation the mean P.D. per segment should not exceed about 15 volts; and because, for mechanical reasons, the thickness of a commutator segment should not be less than about 3 mm. For instance, if, in a 550-volt, 50 ~ converter having a multiple-circuit armature, the volts per segment be limited to 15, 37 segments per pole pitch will then be required. Further, supposing each segment, including insulation, measures 4 mm., this will necessitate a pole pitch of

$$\tau_c = 37 \times 4 = 150 \text{ mm.} = 15 \text{ cms.}$$

and a commutator peripheral speed

$$v_c = \frac{15 \times 50}{50} = 15 \text{ metres per second.}$$

With converters for 40 to 50 periods it is quite good practice to work with peripheral speeds somewhat higher than this, the upper limit being reached at $v_c = 18$ metres per second. Beyond this speed the carbon brushes tend to chatter, which, if allowed to go too far, may be the cause of serious sparking. The number of pairs of poles with which to design the machine can be settled as follows :—

Let E denote the voltage between brushes on D.C. side, e_{per} the mean volts per segment permissible, t thickness of commutator segment in cms., and t_1 thickness of mica insulation in cms., then

$$\frac{E}{e_{per}} = \text{number of segments per pole.}$$

$$\frac{E}{e_{per}} \cdot (t+t_1) = \tau_c = \text{pole pitch at commutator,}$$

and $$v_c \nless \frac{E}{e_{per}} \cdot (t+t_1) \cdot \frac{\sim}{50}$$

Further, if Q denote the density at which the current can be collected from the commutator, n_s = number of segments covered by brush arc, and l the length of all the brushes per spindle; then, assuming one brush spindle per pole and a direct-current output of I amperes,

$$I = p \cdot Q \cdot l \cdot (t+t_1) \cdot n_s$$

Substituting for $(t+t_1)$ the expression $\frac{e_{per}}{E} \cdot \frac{50\,v_c}{\sim}$, there is obtained

$$I \ngtr p \cdot Q \cdot l \cdot \frac{e_{per}}{E} \cdot \frac{50\,v_c}{\sim} \cdot n_s$$

or, the number of pole pairs

$$p \nless \frac{I \cdot E \cdot \sim}{50 \cdot Q \cdot l \cdot e_{per} \cdot v_c \cdot n_s} = \frac{1000\,\text{K.W.} \cdot \sim}{50 \cdot Q \cdot l \cdot e_{per} \cdot v_c \cdot n_s}.$$

where K.W. denotes the kilowatts output from the D.C. side. This equation shows, as would naturally be expected, that the number of poles will increase with the output K.W. and the frequency $\sim$; and, if the following numerical values be inserted,

$$Q = 5 \text{ ampères per cm.}^2 \text{ (page 450)}, \quad e_{per} = 15 \text{ volts}$$
$$n_s = 3, \quad v_c = 18 m/sec, \quad \text{and } l = 20 \text{ to } 30$$

the number of poles for 500-volt machines of various outputs and frequencies may be chosen somewhat as follows :—

TABLE XXXIV.

Rated Output.	25 $\sim$	50 $\sim$
100– 200 K.W.	4 poles	6– 8 poles
300– 500 ,,	6 ,,	8–10 ,,
600– 800 ,,	8 ,,	12–16 ,,
1000–1200 ,,	12 ,,	20–24 ,,
1500 ,,	16 ,,	...

Output Coefficients and Main Dimensions.—When the output is limited by thermal considerations, and not by the question of commutation, it has been shown in the previous chapter that for a given machine,

$$\text{Output as a converter} = \frac{1}{\sqrt{\beta}} \times \text{ output as D.C. generator,}$$

i.e., output as D.C. generator $= \sqrt{\beta}$ K.W.,

where K.W. denotes the output as a converter. The dimensions of a converter armature can therefore be arrived at in the same way as those of an ordinary direct-current generator to be designed for an output $\sqrt{\beta}$ K.W., terminal voltage E, and normal current $\sqrt{\beta}\ .\ \mathrm{I}$.

For a direct-current armature winding the terminal voltage is

$$\mathrm{E} = \mathrm{E}_i' = 4\ \mathrm{T}\ .\sim .\ \Phi \times 10^{-8} = 4\ \mathrm{T}\ .\ \frac{\mathrm{R}p}{60}\ .\ \Phi \times 10^{-8} \text{ volts,}$$

where T denotes the number of turns in series per circuit and Φ the magnetic flux per pole. Multiplying both sides of the above equation by $\sqrt{\beta}$ I, there is then obtained the further equation

$$\mathrm{E}\ .\ \mathrm{I}\ .\sqrt{\beta} = 4\ \mathrm{T}\ .\ \mathrm{I}\sqrt{\beta}\frac{\mathrm{R}p}{60}\ .\ \Phi \times 10^{-8} = 0.66\ \mathrm{I}\ .\ \mathrm{T}\ .\ p\ .\ \Phi\ .\ \mathrm{R}\sqrt{\beta} \times 10^{-9}$$

Now, let

D = diameter of armature core at air-gap in cms.
L_g = gross length of armature core in cms.
l_i = ideal pole length $\cong \mathrm{L}_g$.
b_i = ideal pole breadth $\cong \dfrac{\pi \mathrm{D} \sigma}{2p}$.
σ = ratio of pole arc to pole pitch.
q = number of armature circuits in parallel.
B_g = average flux density in air-gap.
AC = ampère-conductors per cm. of armature periphery,
then,

$$\mathrm{B}_g = \frac{\Phi}{b_i\ .\ l_i} = \frac{2p\Phi}{\pi \mathrm{D}\ .\ \sigma\ .\ \mathrm{L}_g}$$

$$i.e.\ p\Phi = \frac{\pi\ .\ \mathrm{D}\ .\ \mathrm{L}_g\ .\ \mathrm{B}_g\ .\ \sigma}{2}$$

$$\text{and AC.} = \frac{2\ \mathrm{T}\ .\ q}{\pi \mathrm{D}}\ .\ \frac{\mathrm{I}\sqrt{\beta}}{q} = \frac{2\mathrm{T}\ .\ \mathrm{I}\sqrt{\beta}}{\pi \mathrm{D}}, \text{ or I.T} = \frac{\pi \mathrm{D}\ .\ \mathrm{AC}}{2\sqrt{\beta}}$$

When these values for $p\Phi$ and IT are substituted in the equation for EI $\sqrt{\beta}$, there is obtained a further equation

$$\mathrm{EI}\sqrt{\beta} = 0.66\ .\ \frac{\pi .\ \mathrm{D}\ .\ \mathrm{AC}}{2\sqrt{\beta}}\ .\ \frac{\pi\ .\ \mathrm{D}\ .\ \mathrm{L}_g\ .\ \mathrm{B}_g\ .\ \sigma}{2}\ \mathrm{R}\sqrt{\beta}\ .\ 10^{-9}$$

$$i.e.,\ \mathrm{KW.}\sqrt{\beta} = \mathrm{D}^2\ .\ \mathrm{L}_g\ .\ \mathrm{R} \times \mathrm{AC}\ .\ \mathrm{B}_g\ .\ \sigma \times 0.162 \times 10^{-12}$$

The equation to the output coefficient can now be written down as

$$\xi = \frac{\mathrm{KW.}\sqrt{\beta}}{\mathrm{D}^2 . \mathrm{L}_g . \mathrm{R}} = \frac{\mathrm{AC} . \mathrm{B}_g . \sigma}{\sqrt{\beta}} \times 8{\cdot}162 \times 10^{-12}$$

$$i.e.\ \mathrm{D}^2 . \mathrm{L}_g = \frac{\mathrm{KW.}}{\xi . \mathrm{R}} = \frac{\mathrm{KW.}\sqrt{\beta}}{\mathrm{R} . \mathrm{B}_g . \mathrm{AC} . \sigma} \times 6\ 2 \times 10^{11}.$$

In order to obtain as nearly as possible a sinusoidal field curve, σ is generally made equal to about 0.65 or 0.70 (*vide* page 216). The exact proportions in which AC and B_g are chosen depends upon the conditions under which the converter has to operate. If the converter has to be designed with a large overload capacity as a synchronous motor, B_g is chosen relatively large and AC relatively small.

If, on the other hand, it is desirable that the motor should

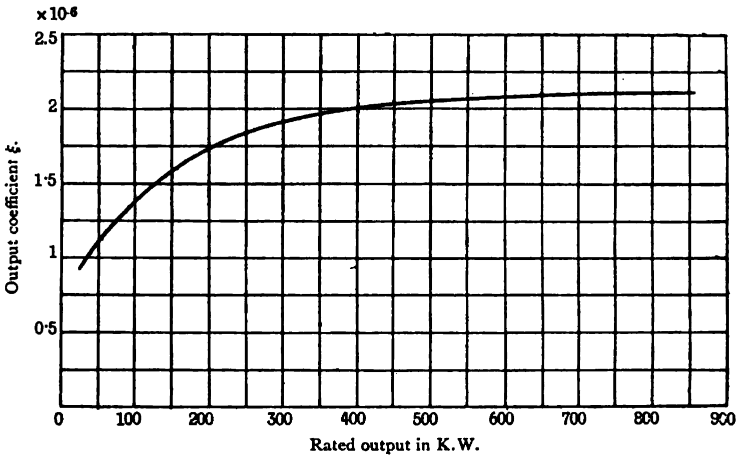

FIG. 329.—Output coefficient of rotary converters.

not have too great a synchronising power, AC is made larger and B_g smaller.

In converters of modern design the value of AC usually ranges from 150 to 250 ampère-conductors per cm., while B_g lies between 7000 and 10,000 respectively. The values of $\sqrt{\beta}$ will be as set forth on page 433.

In obtaining the main dimensions of a converter, it is usual to start from the formula

$$\mathrm{D}^2\mathrm{L}_g = \frac{\mathrm{KW.}\sqrt{\beta}}{\xi . \mathrm{R}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

The value for the output coefficient ξ to be inserted in this formula should be deduced from a number of previously built machines, but in the absence of such data the curves of Figure 329 may be used to obtain a first approximation. The values of output coefficients given

in this curve will be approximately the same as those for D.C. generators.

When the value of D^2L_g has been provisionally settled, the next step is to determine the best diameter and length of core. As in the case of alternators, the peripheral speed which is usually regarded as permissible may form the basis for the settlement of the core diameter. A high peripheral speed is somewhat desirable in order to reduce the weight and size of the machine for a given output, but is limited by consideration of mechanical strength depending upon the way in which the armature is constructed. In armatures of ordinary construction the peripheral speed ranges from 20 to 30 metres per second, and 23 or 24 may be considered an average. The speed R being known, the diameter D can be determined by settling on a suitable value of v, thus $D=\frac{6000v}{\pi R}$. Knowing D, the length of the core is expressed by

$$L_g=\frac{KW.\sqrt{\beta}}{\xi . D^2R}$$

Another method of settling D and L_g is to take

$$L_g \cong b_i = 0.65\,\tau = \frac{2}{\pi}\,.\,\tau = \frac{2}{\pi}\,.\,\frac{\pi D}{2p} = \frac{D}{p}$$

and when this value is substituted in equation (127)

$$D^3=\frac{p\,.\,KW.\ \sqrt{\beta}}{\xi.\ R}$$

from which there is obtained the diameter D, pole pitch $\tau=\frac{\pi D}{2p}$, length $L_g=\frac{D}{p}$, and the peripheral speed $v=\frac{\pi DR}{60}$. Should the latter not come within the limits mentioned above, a more convenient value should be assigned to L_g and the diameter D then obtained, thus

$$D=\sqrt{\frac{D^2L_g}{L_g}}$$

It may be necessary in this way to make several trial calculations until there are obtained convenient values of D, L_g, and v. When L_g has been settled the number of radial air ducts should then be fixed on the basis of one for every 7 or 8 cms. of gross length. The radial depth of the air-gap is chosen almost the same as in D.C. generators, and varies from 4 mm. in small machines to 8 or 9 mm. in large machines, according to the curve of Figure 330. The pole shoe arc is in nearly every case made equal to 0.7 τ, as this, when the poles are well chamfered, gives an almost sine distribution of magnetic flux. In some cases the pole and pole shoes are a solid steel casting, and though the solid pole shoes tend to facilitate the weakening of all harmful field pulsations and the damping out of any free oscillations, yet the

increased eddy current loss resulting therefrom is of such a magnitude as to have an appreciable effect upon the efficiency of the machine. For this reason the more usual practice is to adopt laminated pole shoes and to fit *amortisseurs* into them. The magnetic system is then sketched out in a similar manner as for a D.C. generator, and if the plan of the pole shoe approximates to a square a pole of circular cross section is to be preferred in order to keep down the mean length per turn for the copper. Settling on a maximum flux density in the pole core of about 15,000 lines per cm.^{2}, and assigning a suitable value to the leakage factor of the magnet system, an approximate value of the magnetic flux entering the armature can now be estimated. If, as is usually the case, the magnet ring be of cast steel, then the cross-section of the yoke should be designed for a fairly low induction, say

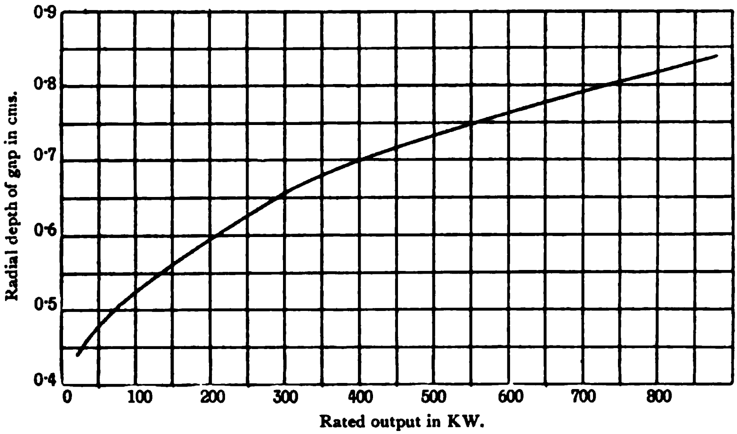

FIG. 330.—Radial depth of air-gap for rotary converters.

12,000 lines per cm.^{2} This is necessary in order to obtain a magnet ring which will have the necessary mechanical strength. The length of the pole core will ultimately be settled from the amount of space required for the field copper, but as a preliminary this may be given a value of between 25 and 30 cms.

Of the magnetic circuit, the radial depth of armature core is the only dimension that remains to be settled.

An assumed depth of slot has to be selected for the present, and this may be taken as ranging from 3.5 to 5.0 cms., according to the size of the machine. To derive the radial depth of core that is required below the tooth roots it is necessary to know the value of flux density permissible, and this will be settled on the same basis as for alternators. (See Table XXIV, page 389).

Armature Winding.—When the outline of the magnetic circuit has been provisionally determined, and an estimate made of the

magnetic flux entering the armature per pole, the next step is to decide upon the armature winding. The number of turns in series per circuit is given by

$$T=\frac{E_i' \times 10^8}{4 \cdot \sim \Phi}$$

the induced voltage E_i' being taken as equal to the terminal voltage E at normal full-load plus 3 per cent. to allow for the volts drop in winding and at brush contacts,—*i.e.*, $E_i'=1.03E$. If q denote the number of armature circuits in parallel, and I the current output from the D.C. side, then the fluctuating current in each conductor is equivalent to a steady current of the amount

$$I_{eq}=\frac{I\sqrt{\beta}}{q}$$

For outputs greater than 100 K.W. multiple circuit windings are, as already stated, always employed, in which case the number of armature circuits will be the same as the number of poles. If, for such a winding, $C=2qT$ denote the total number of armature conductors, then pitch at rear end of armature $=y_R=\frac{C}{2p}\pm 1$, and pitch at commutator end of armature $=y_F=\frac{C}{2p}\mp 1$.

In order to obtain a perfectly symmetrical winding, the number of turns per circuit T should be such that $\frac{2T}{m}$ is a whole number, where m denotes the number of slip rings.

Should such a condition not be fulfilled when the initial value obtained for T is adopted, the relation between T and Φ should then be adjusted so as to meet this requirement.

When the equivalent current per circuit does not exceed 70 or 80 ampères, wire of circular cross-section will be the most convenient to use, but for currents exceeding 100 ampères rectangular wire is necessary, as otherwise the space wasted by interstices becomes excessive. The current density in the armature conductors is really determined by the permissible I^2R loss, and the following values represent good practice.

TABLE XXXV.

Equivalent Current, I_{eq}.	Equivalent Current Density per $Cm.^2$
0– 40 ampères	360 ampères
40– 60 ,,	320 ,,
60– 90 ,,	290 ,,
90–150 ,,	260 ,,
150–200 ,,	240 ,,

Slots and Teeth.—The number of slots n_s should be chosen in such a way as to give from 600 to 800 ampère-conductors per slot without causing the flux density in the teeth to be raised to too high a value. In settling the slot dimensions the better plan is to start from the necessary cross-section of iron required at the tooth root, and this should be such that the apparent flux density at this section does not exceed about 22,000 lines per cm.^{2} in normal 25 ~ converters. With machines for higher frequencies the limit set to the hysteresis or iron loss on the teeth may necessitate a smaller value of tooth induction, because otherwise the required efficiency may not be obtained or the heating of the machine may be too great. With an assumed slot depth ranging from 3.5 to 5.0 cms., according to the size of the machine, the width of iron t_1 at tooth root is first determined, and from a knowledge of the tooth pitch t_p the slot width is given by $s = t_p - t_1$. The necessary thickness of slot insulation is then determined, and the conductors dimensioned so that they can get into the slot. If a convenient solution is not, however, possible, the slot dimensions must be altered, and if this is insufficient the iron length also. Press-spahn, leatheroid, and micanite are the insulating materials generally used for the lining of the slots,—these materials either being used separately or made up so as to form a composite insulation.

The thickness of the lining ranges from 1.0 mm. in 250-volt machines to 1.5 mm. in machines for 600 volts.

Commutator and Brushes.—The commutator, which is designed essentially to provide sufficient bearing surface for the brushes, should also have the requisite cooling surface to radiate the heat generated in virtue of the I^{2}R and friction loss.

The number of segments N_s should be such that the mean voltage between adjacent segments does not exceed about 15 volts, and to meet this requirement it will nearly always be necessary with usual voltages to have only one turn per commutator segment. If t and t_1 denote the width of segment and mica insulation respectively, then the commutator diameter will be

$$D_c = \frac{N_s(t + t_1)}{\pi} \text{ cms.}$$

D_c should never be allowed to exceed 80 per cent. of the armature diameter, as otherwise the peripheral speed may be too great for the carbon brushes. The normal values of t range from 4 to 6 mm., but for mechanical reasons should never be less than 3 mm. For machines up to 600 volts on the D.C. side the insulation between segments has a standard thickness of 0.7 mm.

According to the grade of brush employed and nature of the armature winding, the contact area between the brushes and the commutator must be such that the current density ranges from 5 to 7 ampères per cm.^{2} In multiple-circuit windings no equalising rings can

be used as in the case of D.C. generators, and as a result of this the current may not divide itself equally between the various armature circuits, and some brush sets may hence be considerably overloaded. With multiple-circuit armatures it is therefore advisable to choose the current density of the brushes somewhere in the neighbourhood of the lower limit. In selecting brushes of suitable dimensions one is generally confined to certain standards, which become necessary in order that the same brush holders should be used for different sizes of machines. The particular brush selected should not, however, have too great an arc of contact, as, otherwise, the number of turns undergoing commutation simultaneously may be too great to ensure sparkless collection of the current. The length of a brush in the direction of rotation should therefore be limited, so that not more than three segments are covered by the brush. A length of commutator is then selected so that the required number of brushes can be conveniently arranged on each spindle.

The probable temperature rise of the commutator should now be estimated, and if it should exceed about 45° C. the assumed dimensions of commutator must be correspondingly altered.

Collector Rings.—The collector rings with their brush gear form a very important part of the converter, and, owing to the heavy currents which have to be dealt with, must be very carefully designed. It is owing to the possibility of trouble arising at this part that converters supplied with current from a 3-phase network are often only provided with three slip rings, even although the armature heating of the same machine wound for six phases is considerably less. For this purpose very soft carbon brushes are desirable, as they can then be worked at a current density approaching 20 ampères per cm.^{2} When the contact surface has been calculated, it should be divided up suitably for a number of normal brushes. The collector ring is then given a sufficiently large diameter, so that the brushes can be arranged conveniently round it, the width of the ring being slightly in excess of that of the brush. In calculating the current carried by the slip ring it must be remembered that any wattless current must also be taken into account.

Field Winding.—The field ampère-turns at no-load and full-load, denoted by AT_0 and AT respectively, are next calculated, and if the converter is to be compounded the shunt winding will be given by AT_0 ampère-turns, corresponding to the no-load voltage and the series winding, and

$$AT - \frac{E_0}{E} AT_0 \text{ ampère-turns}$$

corresponding to the normal full-load current I, where E_0 and E denote the terminal voltage at no-load and full-load respectively. The space to be allotted to it having been settled, the calculation of the

shunt winding is then carried out in the usual way, according to the formula

$$W = \frac{2 \times 10^{-6} \cdot L \cdot AT^2}{A_x \cdot F_c} \quad \text{(See p. 391.)}$$

The shunt coils will invariably be wound with d.c.c. round wire, for which the space factor F_c may be taken as ranging from 0.45 to 0.55. In calculating the series winding it is usual to assume a current density of about 130 to 150 ampères per cm.,[2] and to use this as a basis for obtaining the size of conductor required.

Since the armature reaction cannot be calculated with very great accuracy, and the magnetic properties of the iron on the saturation of which the compounding depends are for the most part not exactly known, from 10 to 20 per cent. is sometimes added to the calculated value of the ampère-turns at full-load. A diverter, which is nearly always connected in parallel with the series winding, would then be relied upon for adjusting the amount of compounding when the machine is completed. In order to compensate for any variation of voltage due to the alteration of the temperature of the exciting coils, a regulating resistance is generally connected in series with the shunt coils.

Design of a 400-K. W., 25 ~, 3-phase Compounded Rotary Converter.

Specification—

Rated output from D.C. side = 400 K.W.
Frequency of supplied alternating current = 25 ~.
D.C. terminal volts at no-load = 500.
D.C. terminal volts at full-load = 550.
Full-load efficiency $\nless$ 95 per cent.
Temperature rise of any part $\ngtr$ 45° C.

D.C. voltage to be regulated by means of a compound field winding and choking coils in series with the lines.

Assuming a sinusoidal distribution of potential over the commutator, the voltage between slip rings at no-load $= E_{so} = \frac{E}{\sqrt{2}} \sin \frac{\pi}{m} = \frac{500}{\sqrt{2}} \sin 60$ $= 307$ volts. And at normal full-load $= E_s \simeq \frac{550}{\sqrt{2}} \sin 60 = 337$ volts.

Increase of slip ring voltage from no-load to full-load $= \delta E_s = 30$ volts.

Direct-current output at normal load $= I = \frac{400 \times 1000}{550} = 730$ ampères.

Assuming the iron, friction, and excitation loss to be 3.5 per cent. of the output $h = 1.035$ and watt component of the line current

$$= I_w = 2\frac{\sqrt{2} \cdot I \cdot h}{m} = \frac{2\sqrt{2} \cdot 730 \cdot 1035}{3} = 710 \text{ ampères.}$$

Let it be assumed, with consideration for the overload capacity of the machine as a synchronous motor, that the reactance voltage $I_{sw}r_{xc}$ in each choking coil resulting from the watt component of the line current I_{sw} has not to exceed 30 per cent. of the slip ring voltage E_s, then

$$I_{sw}r_{xc} = 0.3\ E_s \quad \text{(See p. 448.)}$$

$$i.e.,\ r_{xc} = \frac{0.3 \times 337}{710} = 0.14 \text{ ohm.}$$

With this reactance the alteration of the wattless line current from no-load to full-load is

$$\delta I_{swl} = \frac{1}{r_{xc}}\left(\delta E_s + \frac{I_{sw}^2 r_{xc}^2}{2E_s}\right)$$

$$= \frac{1}{0.14}\left(30 + \frac{710^2 \times 0.14^2}{2 \times 337}\right) = 320 \text{ ampères.}$$

Suppose that the average load on the converter is that corresponding to three-quarter normal full load, then, in order to obtain the minimum heating of the armature, the power factor should be unity at this load. Hence at three-quarter full-load the wattless current should be zero. In order to obtain this the wattless line current at no-load should be chosen as

$$I'_{swl} = 0.75 \times 320 = +240 \text{ ampères,}$$

and at full-load

$$I_{swl} = -0.25 \times 320 = -80 \text{ ampères.}$$

the signs + and – indicating a lagging and leading current respectively.

Wattless current in each armature circuit at no-load

$$= I'_{wl} = \frac{I'_{swl}}{m\ .\ 2 \sin\frac{\pi}{m}} = +\frac{240}{3\ .\ 2\ .\ 0.866} = +46 \text{ ampères.}$$

and at full-load

$$I_{wl} = \frac{-80}{3\ .\ 2\ .\ 0.866} = -15 \text{ ampères.}$$

Voltage required at secondary terminals of step-down transformer from equation 126

$$= E_t \simeq E_s - I_{swl}r_{xc} + \frac{I_{sw}^2 r_{xc}}{2E_s} = 337 - 710 \times 0.14 + \frac{-80^2 \times 0.14^2}{2 \times 337} = 348 \text{ volts}$$

At full-load

$$\frac{\text{Wattless current}}{\text{Load current}} = \frac{I_{swl}}{I_{sw}} = \frac{80}{710} = 0.112 = k.$$

Hence value of β is given by

$$\beta = 1 + \frac{8(h^2 + k^2)}{m^2 \sin^2\frac{\pi}{m}} - \frac{16h}{\pi^2}$$

$$= 1 + \frac{8\ (1.035^2 + 0.112^2)}{3^2 \sin^2 60} - \frac{16 \times 1.035}{\pi^2} = 0.6$$

Main Dimensions.—For a machine of this size and frequency $p=3$ will be a suitable number of pole pairs; hence the armature speed in revolutions per minute will be

$$R=\frac{60\sim}{p}=\frac{60\times 25}{3}=500$$

From the curve in Figure 329 the output coefficient $\xi=2\times 10^{-6}$; hence

$$D^2L_g=\frac{\text{K.W.}\sqrt{\beta}}{\xi\,.\,R}=\frac{400\times 0.77}{2\times 10^{-6}\times 500}=3.1\times 10^5$$

Assuming a peripheral speed of 24 metres per second, armature diameter $=D=\dfrac{6000\times 24}{\pi\times 500}\simeq 90$ cms.

Gross length of core $=L_g=\dfrac{3.1\times 10^5}{90^2}=38$ cms.

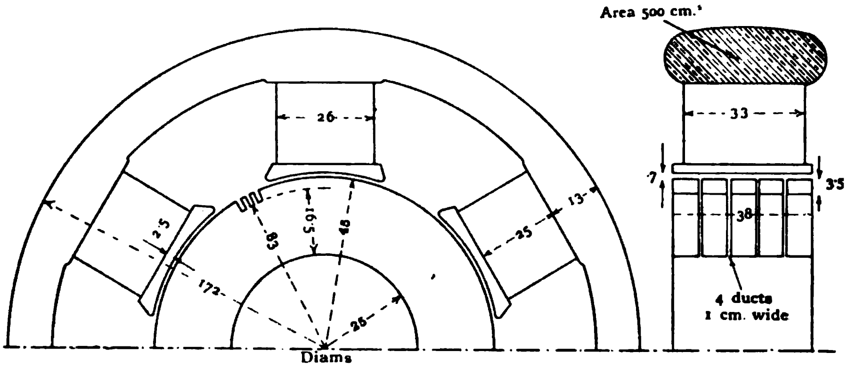

FIG. 331.—Magnetic circuit of 400-K.W. rotary converter (all dimensions in cms.). Lengths of magnetic path (per pole).

With 4 ventilating ducts each 1 centimetre wide the net length of iron $=L_n=(38-4)\,0.9=31$ cms.

From Figure 330, radial depth of air-gap $=0.7$ cm.

Pole pitch at face $=\tau=\dfrac{\pi\times 91.4}{6}=48$ cms.

Circumferential length of pole shoe $=0.7\times 40=33$ cms. The pole shoe is laminated, the iron stampings being riveted together between two brass plates by copper rods which also serve as damping coils. A dimensioned drawing of the magnetic circuit is given in Figure 331, the pole cores and magnet yoke being of cast steel.

Pole shoe height under centre of pole $=2.5$ cms.

Length of pole core $=25$ cms.

Cross section of pole $=26\times 33=860$ cms.^{2}

Settling on a maximum flux density in the pole core of 14,000 lines per cm.^{2}, the flux in each pole at full-load $=860\times 14{,}000=12\times 10^6$ lines.

With a flux density in the yoke of 12,000 lines per cm.^{2}, the cross-sectional area of yoke $= \frac{12 \times 10^6}{2 \times 12000} = 500$ cms.^{2}

If, for the present, a leakage factor $= 1.15$ be assumed, then the flux entering the armature per pole will be approximately

$$= \Phi = \frac{12 \times 10^6}{1.15} = 10.4 \times 10^6 \text{ lines.}$$

From page 389, permissible flux density in armature core $= 10,000$; hence radial depth of armature core below teeth

$$= \frac{\Phi}{2 \times 10000 \times L_n} = \frac{10.4 \times 10^6}{2 \times 10000 \times 31} = 16.5 \text{ cms.}$$

Assumed depth of slot $= 3.5$ cms.

Internal diameter of armature stampings $= 50$ cms.

Armature Winding.—Terminal voltage at full-load $= 550$ volts.

Assuming 3 per cent. of the induced voltage to be consumed in the armature, the total voltage required to be induced at full-load

$$= E_t = 1.03 \times 550 = 567 \text{ volts}$$

Number of turns in series per circuit

$$= T = \frac{E_t \times 10^8}{4 \sim \Phi} = \frac{567 \times 10^8}{4 \times 25 \times 10.4 \times 10^6} = 55$$

In order that the number of turns may be the same in all the phases of the winding it is necessary that the quantity $\frac{2T}{m}$ should be a whole number. With $T = 55$ this condition will not be obtained; hence the number of turns per circuit must, say, be increased to 57, and the armature flux per pole reduced correspondingly to 10×10^6 lines.

Number of conductors per circuit $= 114$.

Total number of conductors $= C = 684$.

Winding pitch at rear end of armature $= y_R = \frac{684}{6} + 1 = 115$.

Winding pitch at commutator end of armature $= y_F = \frac{684}{6} - 1 = 113$.

Each slip ring will be connected to three points in the winding, which are distant from each other by $\frac{684}{3} = 228$ conductors; thus

Slip ring A connected to conductors Nos. 1, 229, and 457.

" B " " " 77, 305, and 533.

" C " " " 153, 381, and 609.

Total direct-current output at full-load $= 730$ ampères.

Current per circuit $= \frac{730}{6} = 122$ ampères.

Equivalent current per circuit $= 122 \sqrt{\beta} = 122 \times 0.77 = 94$ ampères.

From page 472, equivalent current density $= 260$ ampères per cm.^{2}

Slots and Teeth—Total ampère conductors on armature $= 684 \times 122 = 83{,}000$.

Assuming 600 to 800 ampère conductors per slot, number of slots $= 140$ to 104.

In order to come between these limits with an even number of conductors per slot, it is necessary to have 114 slots and 6 conductors in each.

Assumed depth of slot $= 3.5$ cms.

As the apparent flux density of tooth roots should not exceed 22,000 lines per cm.², the area of cross-section of iron required per pole at tooth roots should not be less than

$$\frac{10 \times 10^6}{22{,}000} = 455 \text{ cms.}^2$$

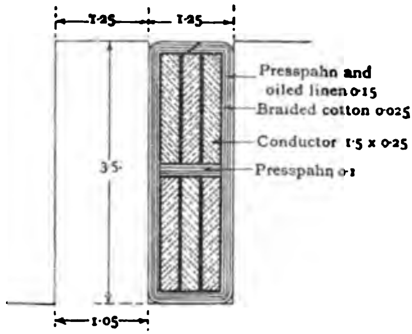

FIG. 332.—Slot for 400-K.W. connector.

Diameter of circle passing through bottom of slots $= 90 - 7 = 83$ cms.

Pitch of teeth at root $= \dfrac{\pi \times 83}{114} = 2.3$ cms.

Number of teeth per pole $= \dfrac{114}{6} = 19$.

Number of teeth directly under pole shoe $= 0.65 \times 19 = 12.4$.

Allowing for fringing, number of teeth carrying the flux from each pole $= 14$.

Width of tooth at root $= \dfrac{455}{31 \times 14} = 1.05$ cm.

[Nett length of iron in tooth $= L_n = 31$.]

Width of slot $= 2.3 - 1.05 = 1.25$ cms.

Pitch of teeth at armature periphery $= \dfrac{\pi \times 90}{114} = 2.5$ cms.

Width of tooth at top $= 2.5 - 1.25 = 1.25$ cm.

For a 550-volt machine the thickness of insulation for lining the slots will be 0.15 cms. Assuming the conductors in each slot to be arranged 3 abreast and 2 deep, and that the rectangular wire is covered with braided cotton 0.025 cm. in thickness, the width of conductor

$$= \frac{1.25 - (2 \times 0.15) - 3(2 \times .025)}{3} \cong 0.25 \text{ cm.};$$

and depth of conductor $= \dfrac{3.5 - 0.4}{2} = 1.50$ cm.

Section of conductor $= 1.5 \times 0.25$ cm.²

By inserting a strip of press-spahn between top and bottom conductors, the slot of the depth selected—namely, 3.5 cms.—will then be conveniently filled. A dimensioned sketch for the slots and the teeth is given in Figure 332.

Commutator and Brushes.—With one armature turn per segment the number of commutator segments $= \frac{C}{2} = \frac{684}{2} = 342$. Assuming for the width of segment $t = 0.5$ cms., and using 0.7 mm. mica for insulation between segments, the diameter of commutator will be

$$D_c = \frac{N_s (t + t_1)}{\pi} = \frac{342 \times 0.57}{\pi} = 62 \text{ cms.}$$

The peripheral speed corresponding to this diameter is 16.5 metres per second, a value which is not excessive.

If the brushes be worked at a current density of about 5 ampères per cm.^{2}, the contact area necessary for either the $+ve$ or $-ve$ brushes $= \frac{730}{5} = 146$ cms.^{2}—*i.e.*, about 48 cms.^{2} per brush spindle. In order to be narrower than the pitch of 4 segments, the circumferential width of a brush will be about 2.0 cms. A very convenient size of brush is one having a length of about 4 cms.; hence, with a contact area per brush of 2×4 cms.^{2}, six brushes will be required per spindle. The useful length of the commutator must therefore be about 30 cms. Brush pressure adjusted to 100 grammes per cm.^{2}

Collector Rings.—Total wattless current at full-load = 80 ampères.

Total current entering each slip-ring $= \sqrt{710^2 + 80^2} = 710$ ampères. Soft carbon brushes will be used to lead the current into the rings, and since these can be worked at a current density of about 20 ampères per cm.^{2}, the contact area required per ring $= \frac{710}{20} \cong 40$ cms.^{2} Arrangements will be made for 4 brushes of $2.5 \times 4 = 10$ cms.^{2} per ring. The voltage drop under 2 brushes in series $\cong 1.2$ volts; *i.e.*, 0.6 volt per ring. The I^2R loss per collector ring is therefore $740 \times 0.6 \cong 450$ watts. With a diameter of ring = 35 cms. the surface speed

$$= \frac{\pi \times 35 \times 500}{6000} \cong 9.5 \text{ metres per second.}$$

If the pressure on the brushes be adjusted to 0.1 kg. per cm.^{2}, and a coefficient of friction for carbon of $\mu = 0.25$ be assumed, then the friction loss per ring becomes

$$W = 9.81 \,.\, A \,.\, v\, P \,.\, \mu \text{ watts.} \quad \text{(See p. 452.)}$$
$$= 9.81 \times 40 \times 9.5 \times 0.1 \times 0.25 = 93 \simeq 100 \text{ watts.}$$

Total loss per ring $= 450 + 100 = 550$ watts.

Assuming a heating coefficient $K_r = 0.9$, and a temperature rise limited to 45° C., then the surface required per ring

$$= A_r = K_r \times \frac{W}{T^\circ(1+0.1v_r)} = 0.9 \cdot \frac{550}{45(1+0.1\times 9.5)} = 560 \text{ cms.}^2;$$

hence, width of each ring $= \dfrac{560}{\pi \times 35} = 5.0$ cms.

Armature Reaction per Phase r_x.—Inductance due to slot leakage is

$$L_s = 0.4\,\pi\, C^2 \cdot \frac{l_{in}}{q} \cdot \left(\frac{d}{3s} + \frac{d_1}{s}\right) \times 10^{-8} \quad \text{(See p. 255.)}$$

where $l_i = 2(38-4) = 68$ cms., $s = 1.25$ cm., $C = \dfrac{2T}{m} = \dfrac{2\times 57}{3} = 38$,

$$q = \frac{n_s}{p \cdot m} = \frac{114}{3\times 3} \cong 12.$$

In a rotary converter winding a group of conductors belonging to any given phase are located in either the upper or lower portions of the slots, and for this reason the values of d and d_1 in the above equation should each be made equal to one-half the slot depth; *i.e.*, $d = d_1 = 1.75$ cm.

$$L_s = 0.4\pi \cdot 38^2 \cdot \frac{68}{12} \cdot \left(\frac{1.75}{3\times 1.25} + \frac{1.75}{125}\right) \times 10^{-8} = 19.0 \times 10^{-5} \text{ henries}$$

Inductance due to tooth head leakage is

$$L_a = 0.92\,\frac{C^2}{q} \cdot l_{in} \cdot \Delta' \cdot 10^{-8}$$

In calculating Δ' only those slots which lie between the pole tips need be considered; hence,

$$q = \frac{0.30\tau}{t_p} = \frac{0.30\times 48}{2.5} \cong 6$$

$$\text{and } \Delta' = \log\left(1 + \frac{\pi t_p}{s}\right) + 3.15 + 6 \log \frac{r}{5t_p}$$

$$t_p = 2.5 \text{ and } r = \tau \cdot \frac{p}{p+1} = 48 \cdot \tfrac{3}{4} = 36$$

$$\text{Hence } \Delta' = \log\left(1 + \frac{\pi\times 2.5}{1.25}\right) + 3.15 + 6 \log \frac{36}{5\times 2.5} = 6.8$$

$$\text{and } L_a = 0.92 \times \frac{38^2}{12} \times 68 \times 6.8 \times 10^{-8} = 51 \times 10^{-5} \text{ henries.}$$

Inductance due to end leakage is

$$L_e = 0.46\, l_e \cdot C^2 \cdot \left[\log \frac{l_e}{d_s} - 0.5\right] \times 10^{-8}$$

$$l_e \cong 2.5\tau = 2.5 \times 48 = 120 \text{ cms.,}$$

$$d_s = 6$$

$$L_e = 0.46 \times 120 \times 38^2 \left[\log \frac{120}{6} - 0.5\right] \times 10^{-8} = 64 \times 10^{-5} \text{ henries.}$$

Total inductance per phase

$$L = 134 \times 10^{-5} \text{ henries}$$

Hence $r_x = 2\,\pi{\sim}L = \dfrac{2\,\pi \times 25 \times 134}{10^6} = 0.21$ ohm.

Excitation at No-load.—At no-load the E.M.F. to be induced in each armature circuit is

$$E_{io} = E_o - \sqrt{2}\,.\,3\,.\,I'_{wl} r_x$$
$$= 500 - 1.414 \times 140 \times 0.21 = 460 \text{ volts,}$$

for which there is required a flux

$$\Phi_o = \frac{E_{io} \times 10^8}{4T\sim} = \frac{460 \times 10^8}{4 \times 57 \times 25} = 8.3 \times 10^6 \text{ lines.}$$

With an assumed leakage factor of 1.15 the flux to be generated in each pole at no-load $= 8.3 \times 10^6 \times 1.15 = 9.5 \times 10^6$ lines. The ampère-turns required to drive this flux through the magnetic circuit have been calculated and tabulated below. In working out the air-gap ampère-turns the ideal pole breadth has been assumed to be

$$b_i \simeq 1.1 \times \text{pole-shoe arc.}$$

	Cross-sectional Area (Cms.²).	Flux Density.	Ampère-turns per Cm.	Length in Cms. per Pair of Poles.	Ampère-turns per Pair of Poles.	Ampère-turns per Pole.
Armature core .	500	8000	2.5	40	100	50
Teeth . . .	455*	17500	75	7	500	250
Air-gap . .	$\left\{\begin{matrix} b_i \times l_i \\ = 36 \times 35 \end{matrix}\right\}$	6500	$[k_3 = 1.1]$	2×0.7	8200	4100
Pole core . .	850	11000	12	50	600	300
Yoke . . .	500	9200	8	75	600	300

Ampère-turns required per pole $= AT_{ol} = 5000$.

The magnetising ampère-turns due to the wattless current

$$AT_M = 0.45\,.\,k_2\,.\,m\,.\,T_1\,I'_{wl}\,\frac{\sin\frac{\pi}{2}\sigma}{\frac{\pi}{2}\sigma}$$

$$= 0.45 \times 0.83 \times 3 \times \frac{2 \times 57}{3} \times 46 \times 0.81 = 1600$$

Field magnet ampère-turns at no-load

$$AT_o = AT_{ol} - AT_M = 5000 - 1600 = 3400.$$

* Area at bottom of teeth.

Excitation at Full-load.—Resistance of the armature winding at 60° C.,—*i.e.*, allowing for a rise of 40° C.

$$= R_a = \frac{1.7 \times 10^{-6} \, . \, k \, . \, l_a \, . \, T \, (1 + 0.004 \, T^\circ)}{q_c \, . \, a}$$

$k = 1.3$, $T = 57$, $T^\circ = 40$, $a = 1.5 \times 0.25 = 0.375$, $q_c = 6$,
and $l_a \cong 2 \, L_g + 2.5\tau = 2 \times 38 + 2.5 \times 48 = 76 + 120 \simeq 200$ cms.

$$R_a = \frac{1.7 \times 10^{-6} \times 1.3 \times 200 \times 57 \, (1 + 0.004 \times 40)}{6 \times 0.375} = 0.013 \text{ ohm.}$$

E.M.F. to be induced at full-load (see p. 449).

$$E_i = E + I(\sqrt{\beta} R_a + R_s) + E_b - \sqrt{2} I_{wl} r_x.$$

$E = 550$, $I = 710$, $\beta = 0.6$, $I_{wl} = -15$, $r_x = 0.21$.
$E_b = 1.75$ (from Figure 323).

Assume for the present that $R_s \cong 0.1 R_a = 0.0013$ ohm.

$$E_i = 550 + 710(\sqrt{0.6} \times 0.013 + 0.0013) + 1.75 + \sqrt{2} \times 15 \times 0.21$$
$$= 564 \text{ volts.}$$

This value agrees very closely with the assumed value $E_i = 567$ on page 480. From page 480 armature flux per pole at full-load $= 10.4 \times 10^6$ lines, for which there is required the following ampère-turns.

	Flux Density.	Ampère-turns per Cm.	Ampère-turns per Pair of Poles.	Ampere turns per Pole.
Armature core	10000	5	200	100
Teeth	21000	460	3200	1600
Air-gap	8400	...	10000	5000
Pole core	14000	27	1400	700
Yoke	12000	16	1200	600

Ampère-turns required per pole $= AT_1 = 8000$.
Demagnetising ampère-turns due to the wattless current

$$AT_{DM} = 0.45 \times 0.83 \times 3 \times \frac{2 \times 57}{3} \times 15 \times 0.81 \cong 500.$$

Field magnet ampère-turns at full-load

$$AT = AT_1 + AT_{DM} = 8000 + 500 = 8500.$$

Field Windings.—Shunt ampère-turns at no-load $= 3400$.

Shunt ampère-turns at full-load $= AT_{sh} = 3400 \times \frac{550}{500} = 3750$.

Series ampère-turns at full-load $= AT_{se} = 8500 - 3750 = 4750$.
Assumed space factor of shunt coil $= F_{csh} = 0.5$.
Assumed space factor of series coil (strip winding) $= F_{cse} = 0.75$.
Length of field winding space (shunt + series) $= 24$ cms.

On basis of equal current density in both windings, ratio of length available for shunt coil to length available for series

$$= \frac{AT_{sh}}{AT_{se}} \cdot \frac{F_{cse}}{F_{csh}} = \frac{3750}{4750} \cdot \frac{0.75}{0.5} = 1.18 \text{ cms.}$$

Hence

Length for shunt coil $= 13$ cms.

Length for series coil $= 24 - 13 = 11$ cms.

Depth of winding space assumed as 4 cms.

Shunt Coils.—Watts per coil $= W_{sh} = \dfrac{2 \times 10^{-6} \,.\, L \,.\, AT^2_{sh}}{A_x \,.\, F_{csh}}$

$L = 1.4$ metre, $A_x = 1.3 \times 0.4 = 0.52$ dcm.^{2}

$$W_{sh} = \frac{2 \times 10^{-6} \times 1.40 \times 3750^2}{0.52 \times 0.5} = 150 \text{ watts.}$$

On the assumption that 10 per cent. of the voltage applied to the field winding is absorbed in the shunt regulating resistance,

$$\text{Volts per spool} = \frac{550 - 50}{6} = 83.$$

Shunt exciting current $= I_{sh} = \dfrac{150}{83} = 1.8$ ampères.

Number of shunt turns per pole $= \dfrac{3750}{1.8} = 2100.$

Sectional area of wire $= \dfrac{52 \times 0.5}{2100} = 0.0125$ cm.^{2}

Nearest standard wire = No. 18 $= 0.0117$ cm.^{2}

Actual current density $= \dfrac{1.8}{0.0117} = 154$ ampères per cm.^{2}

Resistance of shunt winding $= \dfrac{500}{1.8} = 280$ ohms.

Series Coils.—Assuming about 20 per cent. of main current to be passed through a diverter, current in the series coils at full-load = 600 ampères.

Number of series-turns per pole $= \dfrac{4750}{600} \simeq 8.$

Cross-section of winding space $= 11 \times 4 = 44$ cms.^{2}

Cross-sectional area of conductor $= \dfrac{44 \times 0.75}{8} = 4.1$ cms.^{2}

For such a heavy current, strip winding will have to be used; hence size of series conductor = 4 strips in parallel each 4×0.255 cm.^{2}

Thickness of press-spahn insulation between turns $= 0.3$ mm.

Losses, Heating, and Efficiency at Full-load.—

(1) *Armature.*—Mass of armature teeth $= 1.15 \times 3.5 \times 31 \times 114 \times 0.0078 = 110$ kgs.

Mean flux density $\simeq 20{,}000$.

From Figure 241, $c = 0.29$, $\beta = 1.35$.

Iron loss in teeth = kgs . $\sim c \,.\, \beta = 110 \times 25 \times 1.35 \times 0.29 = 1100$ watts,

Mass of armature core $= \frac{\pi(83^2 - 50^2)}{4} \times 31 \times 0.0078 = 840$ kgs.

Flux density = 10,000, hence $c = 0.1$
Iron loss in core $= 840 \times 25 \times 1.35 \times 0.1 \simeq 3000$ watts.
Total core loss $= 1100 + 3000 = 4100$ watts.
Armature copper loss $= \beta I^2 R_a = 0.6 \times 710^2 \times 0.013 = 4000$ watts.

Copper loss within slots $= 4000 \times \frac{38}{120} = 1300$ watts.

Total loss between core flanges $= W_a = 5300$ watts.
Cooling surface of armature

$$= A_a = \frac{\pi}{4}(90^2 - 50^2)\,6 + 38 \times \pi\,(90 + 50) = 43{,}200 \text{ cms.}^2$$
$$= 432 \text{ sq. dcms.}$$

Temperature rise $= T^\circ = K'_a \,.\, \frac{W_a}{A_a(1 + 0.1v)}$

$v = 24$ m./sec.; hence, assuming $K_a = 10.5$,

$$T^\circ = 10.5 \times \frac{5300}{432(1 + 2.4)} = 36^\circ \text{ C.}$$

(2) *Commutator.*—Loss due to brush contact resistance $= W_{cr} = I \,.\, E_b$ $= 710 \times 1.75 = 1250$ watts.

Friction loss $= W_{cf} = 9.81\ v_c \,.\, P \,.\, A \,.\, \mu$.

$A = 8 \times 6 \times 6 = 380$ cms.^{2}, $v = 16.5$ m./sec., $P = 0.1$ kg., and μ (assumed) $= 0.25$.

$W_{cf} = 9.81 \times 16.5 \times 0.1 \times 380 \times 0.25 = 1500$ watts.
Total commutator loss $= W_c = 1300 + 1500 = 2800$ watts.
Radiating surface $= A_c = \pi D_c L_c = \pi \times 62 \times 30 = 5900$ cms.^{2}
$= 59$ dcms.^{2}

Temperature rise $= T^\circ = 2.1 \times \frac{2800}{59(1 + 1.65)} = 38^\circ$ C.

Collector Rings.—From page 482, loss per ring = 550 watts.
Total loss at collector rings $= 3 \times 550 = 1650$ watts.
Excitation Loss—

Shunt exciting current = 1.8 ampère.
Loss in shunt circuit (resistance included) $= 550 \times 1.8 = 1000$ watts.
Loss in resistance $= 0.1 \times 1000 = 100$ watts.
Loss in shunt coils = 900 watts, *i.e.* 150 watts per coil.
Resistance of series winding at 60° C.

$$R_3 = \frac{1.7 \times 10^6 \,.\, l \,.\, T_{se} \,.\, (1 + 0.004\ T^\circ)}{a}$$

$l = 140$ cms., $T_{se} = 6 \times 8 = 48$, $a = 4.1$, $T^\circ = 40$.

$$R_3 = \frac{1.7 \times 10^{-6} \times 140 \times 48(1 + 0.004 \times 40)}{4.1} = 0.0032$$

Loss in series winding $= I^2R_s = 600^2 \times 0.0032 = 1150$ watts.
Loss in diverter $= 110 \times 600 \times 0.0022 = 210$.
Total loss in series $= 1360$.
Loss per series coil $= 230$ watts.
Total excitation loss per coil $= W_m = 150 + 230 = 380$ watts.
Radiating surface per coil $= A_m = 37$ dcms.^{2}

$$\text{Estimated temperature rise} = K_m \cdot \frac{W_m}{A_m} = 4 \times \frac{380}{37} = 40^\circ \text{ C.}$$

Losses—

Armature iron	= 4100 watts
„ I^2R	= 4000 „
Commutator I^2R	= 1300 „
„ friction	= 1500 „
Collector ring	= 1650 „
Shunt excitation	= 1000 „
Series excitation	= 1360 „
Bearing friction and windage	= 4000 „
Total . . .	18910 „

Total estimated loss $\simeq$ 18.9 K.W.

$$\text{Hence full-load efficiency} = \frac{400}{400 + 18.9} = 95.5 \text{ per cent.}$$

The efficiency required from the specification is 95 per cent., hence a slight margin is left for contingencies, such as eddy current losses in amortisseurs, which itself will amount to about 0.5 per cent.

Examples of Designs

In Table XXXVI. there is tabulated the design data of three rotary converters having the following outputs:—

Design No. 1.	165 K.W.
„ No. 2.	500 „
„ No. 3.	700 „

The voltage of machines Nos. 1 and 2 is regulated by a compound winding along with a series resistance. In the 700-K.W. converter the regulation is obtained by means of a synchronous booster, the armature of which is connected between the slip rings and the armature winding of the converter.

[Table

TABLE XXXVI.—DESIGN DATA OF THREE ROTARY CONVERTERS.
(All Dimensions in Cms.)

Design Number	1	2	3
Detail Drawings	Plate XV.	...	Plate XVI.
Manufacturer	Crompton & Company.	British Westinghouse Company.	General Electric Company.
Specification—			
Rated output in K.W.	165	550	700
Number of phases	3	3	6
Speed in R.P.M.	500	500	500
D.C. terminal volts (no-load)	500	500	460/550
,, ,, (full-load)	550	550	460/550
A.C. terminal volts (full-load)	350	345	326/390
Full-load current (D.C. side)	300	1,000	1,270
,, ,, (A.C. side)	290	950	600
Number of poles	6	8	12
Frequency	25	$33\frac{1}{3}$	50
Armature Core—			
External diameter	66	1c6	140
Internal diameter	36	64	97
Gross length between flanges	18	32	27
Number of vent. ducts	1	3	3
Nett length of core	15	27.0	21
Number of slots	66	96	252
Armature Winding—			
Conductors per slot	10	8	4
Turns in series per circuit	165	48	42
Number of circuits	2	8	12
Size of conductor (bare)	2.3×0.126	1.9×0.2	...
Slot space factor	0.3	...	...
Direct current per circuit (ampères)	150	125	...
Equivalent current per circuit (ampères)	120	95	...
Equivalent current density (ampères per cm.^{2})	400	250	...
Mean length per turn	140	...	...
Resistance of winding (ohms)	0.08	0.0047	...
Turns per commutator segment	1	1	1
Commutator—			
Diameter	50	75	75
Length	13.3	29	27
Number of segments	329	384	504
Width of segment at periphery	0.4	0.53	0.46
Brushes (D.C. side)—			
Number of spindles	6	8	12
Brushes per spindle	3	5	6
Width of brush	2.5	4.4	3.8
Length of brush arc	1.2	1.9	1.3
Current density in ampères per cm.^{2}	5.8	6.0	7
Volts drop over $+ve$ and $-ve$ brushes	1.8	1.7	2.25

TABLE XXXVI. (*continued*)—

Design Number	1	2	3
Detail Drawings	Plate XV.	...	Plate XVI.
Manufacturer	Crompton & Company.	British Westinghouse Company.	General Electric Company.
Brushes (A.C. side)—			
Spindles per ring	1	1	1
Brushes per spindle	2	3	2
Width of brush	2.5	5.0	5.0
Length of brush arc	1.9	3.1	2.0
Current density in ampères per cm.²	30	20	30
Air-Gap—			
Radial depth	0.47	0.8	0.5
Field Magnet Iron—			
Pole pitch at air-gap	35	42	37
Pole arc ÷ pole pitch	0.65	0.75	0.67
Width of pole parallel to shaft	20	30.5	Diam. of pole core = 25
Breadth of pole normal to shaft	16	32.5	
Radial length of pole and shoe	22	30.4	22.5
Cross-sectional area of yoke	400 cm.²	700 cm.²	900 cm.²
Poles of	Laminated iron	Laminated iron	Cast steel
Magnet ring of	Cast iron	Cast steel	Cast steel
Shunt Field Winding—			
Turns per coil	1640	1200	1050
Size of wire (bare)	diam. = 0.142	diam. = 0.205	diam. = 0.234
Radial depth of winding	5	7	3.7
Cross-section of winding space	5 × 16	7 × 15	3.7 × 20
Space factor	0.35	0.37	0.6
Mean length per turn	100	160	95
Full-load current (ampères)	3.5	6	7
Current density in ampères per cm.²	220	185	162
Resistance (warm) in ohms	108	75	...
Series Field Winding—			
Turns per coil	5	3½	...
Size of wire (bare)	3(3.15 × 0.16)	5 × 1.3	...
Full-load current (ampères)	275	1000	...
Current density in ampères per cm.²	180	155	...
Resistance (warm) in ohms	0.0035	0.001465	...
Magnetic Data (full-load)—			
Flux per pole	3.54×10^6	8.6×10^6	...
Leakage coefficient	1.15	1.15	...
Flux density in armature core	12,000	8,500	...
,, ,, teeth (maximum)	19,000	...	...
,, magnet poles	13,500	10,000	...
,, ,, yoke	5,000	14,000	...
,, at air-gap	8,000	8,400	...

TABLE XXXVI. (*continued*)—

Design Number	1	2	3
Detail Drawings	Plate XV.	...	Plate XVI.
Manufacturer	Crompton & Company.	British Westinghouse Company.	General Electric Company.
Losses and Efficiency (full-load)—			
Core loss in watts	5,000	6,350	...
Armature I^2R loss in watts	3,500	2,900	...
Commutator I^2R loss in watts	550	1,960	...
Collector ring I^2R loss in watts	850		...
Brush friction loss in watts	750	5,740	...
Shunt excitation in watts	1,000	3,230	...
Series excitation in watts	270	1,460	...
Bearing friction and windage loss in watts	1,500	3,960	...
Total loss	13,420	25,600	...
Efficiency	92 %	95.5 %	...
Constants—			
Output coefficient	...	...	...
Armature peripheral speed m/sec.	17.0	27.5	36
Commutator peripheral speed m/sec.	13.0	19.5	19.5
Ampère-conductors per cm.	470	290	230
Air-gap flux density	8,000	8,400	...

TABLE OF PROPERTIES OF COPPER WIRES.

S.W.G.	Diameter. Bare. Mm.	S.C.C. Mm.	D.C.C. Mm.	T.C.C. Mm.	Cross Section. Sq. Cm.	Resistance in Ohms per Kilometre. 0°C.	20°C.	40°C.	60°C.	80°C.	100°C.	Metres per Ohm at 20° C.	Kilograms per Ohm at 20° C.	Ohms per Kilogram at 20° C.	Metres per Kilogram.	Kilograms per Kilometre (Bare).
7/0	12,70	..	..	13,20	1,265	0,1260	0,1360	0,1460	0,1580	0,1690	0,1780	7380	8300	0,0001205	0,89	1126
6/0	11,80	..	..	12,30	1,090	0,1460	0,158	0,1700	0,1820	0,1960	0,2050	6370	6170	0,0001620	1,04	970
5/0	11,00	..	..	11,40	0,950	0,1680	0,1820	0,1960	0,2100	0,226	0,2400	5500	4620	0,0002610	1,10	845
4/0	10,15	..	..	10,65	0,814	0,1960	0,2100	0,2275	0,2450	0,2650	0,2800	4720	3400	0,000294	1,39	720
000	9,44	..	..	9,90	0,705	0,2260	0,2450	0,265	0,2840	0,3050	0,3200	4100	2540	0,000394	1,61	622
00	8,84	..	..	9,30	0,614	0,2590	0,2800	0,300	0,3240	0,3500	0,3680	3600	1950	0,000513	1,84	545
0	8,23	..	..	8,67	0,531	0,2990	0,3200	0,350	0,3750	0,4000	0,4250	3100	1460	0,000685	2,12	475
1	7,62	..	..	8,05	0,456	0,350	0,3750	0,405	0,4350	0,4675	0,4975	2650	1075	0,000930	2,47	405
2	7,00	..	7,35	7,45	0,385	0,4150	0,4450	0,480	0,5150	0,5450	0,5900	2245	770	0,001298	2,92	342
3	6,40	..	6,75	6,85	0,320	0,500	0,5350	0,580	0,6200	0,6000	0,7000	1870	535	0,00187	3,50	285
4	5,90	..	6,25	6,30	0,274	0,585	0,6300	0,680	0,7300	0,7800	0,8300	1590	385	0,00260	4,12	243
5	5,40	..	5,65	5,80	0,228	0,700	0,7550	0,810	0,8700	0,9400	0,9900	1320	269	0,00373	4,95	202
6	4,90	..	5,15	5,25	0,187	0,850	0,920	0,990	1,060	1,140	1,200	1090	180	0,00554	6,00	107
7	4,47	..	4,75	4,85	0,157	1,02	1,10	1,18	1,27	1,36	1,44	915	127	0,00787	7,20	140
8	4,07	..	4,35	4,45	0,1295	1,23	1,32	1,42	1,54	1,65	1,74	760	87	0,01150	8,70	116
9	3,65	..	3,95	4,05	0,105	1,51	1,64	1,76	1,89	2,03	2,15	610	57,5	0,01740	10,70	93,5
10	3,25	..	3,55	3,65	0,0833	1,91	2,05	2,22	2,39	2,56	2,70	485	35,5	0,0281	13,60	74,00
11	2,95	..	3,20	3,30	0,0685	2,30	2,50	2,70	2,90	3,15	3,30	395	24	0,0416	16,50	60,50
12	2,65	..	2,90	3,00	0,0546	2,90	3,15	3,40	3,65	3,90	4,15	320	15,5	0,0645	20,50	48,7
13	2,34	2,50	2,60	2,70	0,0429	3,70	4,00	4,30	4,65	4,95	5,25	250	9,5	0,1050	26,2	38,00
14	2,03	2,20	2,30	2,38	0,0324	4,90	5,30	5,72	6,15	6,55	7,00	189	5,50	0,1820	34,60	29,00
15	1,83	2,00	2,10	2,20	0,0254	6,10	6,55	7,10	7,60	8,10	8,60	152	3,55	0,281	42,70	23,40
16	1,63	1,80	1,90	1,98	0,0208	7,70	8,30	8,90	9,60	10,20	10,80	120	2,23	0,447	54,00	18,50
17	1,42	1,55	1,65	1,75	0,01585	10,00	10,50	11,70	12,50	13,40	14,20	92	1,30	0,766	70,50	14,15
18	1,22	1,346	1,42	1,52	0,0117	13,60	14,75	15,90	17,05	18,30	19,40	68,2	0,707	1,414	96,10	10,39
19	1,017	1,142	1,220	1,32	0,00814	19,6	21,2	22,9	24,5	26,3	27,9	47,3	0,340	2,94	139	7,20
20	0,915	1,017	1,118	1,22	0,00659	24,2	26,2	28,20	30,28	32,5	34,48	38,4	0,222	4,49	171	5,84
21	0,8125	0,915	1,015	1,11	0,00520	30,7	33,1	35,8	38,4	41,2	43,60	30,2	0,139	7,18	217	4,02
22	0,7110	0,812	9,915	1,01	0,00398	40,05	43,4	46,7	50,2	53,8	57,0	23,0	0,0816	12,25	283	3,53
23	0,6100	0,711	0,812	0,914	0,00293	54,3	58,75	63,5	67,9	72,8	77,0	18,0	0,0442	22,60	386	2,59
24	0,5590	0,660	0,762	0,862	0,00245	65,0	70,5	75,8	81,3	87,1	92,5	14,2	0,0308	32,40	460	2,19
25	0,5080	0,610	0,710	0,812	0,00203	78,5	84,6	92,0	98,4	105,0	112,0	11,8	0,0216	47,10	555	1,80
26	0,4570	0,559	0,660	0,762	0,001643	96,8	105,0	113	121	130,0	138,0	9,5	0,0139	71,8	686	1,40
27	0,4160	0,508	0,610	..	0,00136	117	126,0	136	146	157	155	7,95	0,00960	104,0	827	1,21
28	0,3760	0,483	0,585	..	0,00111	143,0	154,2	167	179	192,0	203,5	6,46	0,00639	156,0	1015	0,986
29	0,3460	0,457	0,559	..	0,000932	169,9	185,5	198	212,3	228,0	241,0	5,46	0,00454	220,5	1203	0,834
30	0,3150	0,432	0,533	..	0,000780	204,0	220	238	254,5	274,0	290,0	4,54	0,00314	322	1444	0,692
31	0,2950	0,407	0,508	..	0,000685	233	251	271	291	313	330	3,96	0,00240	416	1653	0,606
32	0,2740	0,381	0,482	..	0,000590	270	292	314	338	361	384	3,45	0,00181	553	1900	0,525
33	0,2540	0,356	0,457	..	0,000508	314	338	368	390	420	446	2,96	0,00132	750	2218	0,453
34	0,2340	0,330	..	..	0,000429	372	402	432	462	496	525	2,50	0,000952	1050	2628	0,381
35	0,2138	0,305	..	..	0,000357	444	481	515	554	598	633	2,09	0,000666	1500	3145	0,318
36	0,1930	0,280	..	..	0,000292	545	587	638	685	738	780	1,71	0,000442	2260	3840	0,261
37	0,1730	0,254	..	..	0,000234	680	734	787	850	910	965	1,37	0,000283	3530	4820	0,208
38	0,1525	0,241	..	..	0,000182	874	940	1018	1089	1168	1240	1,05	0,000172	5820	6170	0,162
39	0,1320	0,216	..	..	0,000137	1165	1257	1354	1456	1562	1650	0,798	0,0000970	10300	8200	0,122
40	0,1220	0,203	..	..	0,000117	1360	1470	1587	1700	1830	1935	0,683	0,0000707	14100	9610	0,104
41	0,1120	0,190	..	..	0,0000980	1620	1750	1880	2025	2178	2300	0,574	0,0000499	20000	11500	0,0874
42	0,1016	0,177	..	..	0,0000812	1960	2121	2280	2460	2628	2785	0,472	0,0000340	29500	13900	0,0720
43	0,0915	..	..	..	0,0000657	2420	2590	2820	3023	3245	3430	0,382	0,0000223	44900	17130	0,0584
44	0,0813	..	..	..	0,0000518	3070	3325	3579	3840	4120	4360	0,302	0,0000139	72100	21700	0,0462
45	0,0711	..	..	..	0,0000398	4000	4330	4660	5020	5380	5710	0,231	0,00000816	122500	28400	0,0353
46	0,0610	..	..	..	0,0000292	5450	5900	6380	6890	7320	7700	0,171	0,00000442	226000	38600	0,0250
47	0,0508	..	..	..	0,0000202	7900	8530	9190	9800	10550	11150	0,118	0,00000213	470000	55500	0,0180
48	0,0406	..	..	..	0,0000130	12250	13280	14270	15380	16450	17400	0,0756	0,000000875	1145000	86750	0,0115
49	0,0305	..	..	..	0,00000730	21800	23590	25400	27390	29400	31000	0,0424	0,000000274	3640000	154800	0,00650
50	0,0254	..	..	..	0,00000505	31300	33950	36600	39400	42200	44300	0,0296	0,000000133	7530000	218500	0,00451

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